FEM20240819

Sunday, August 18, 2024 5:21 PM

Course grade:

HW:

2 term projects:

- Commercial software 15%
- Your own code (trusses and frames) 17%

Final exam (take home)

Outline:

1. Mathematical background:
    a. Weighted Residual Method (WRM)
    b. Weak form
    c. Discretization
    d. Energy methods
    e. Numerical examples
2. Different "1D" element types:
    a. Bar
    b. Beam
    c. Truss
    d. Frame
3. 2D/3D elements:
    a. Numerical integration (quadrature)
4. Code implementation (we'll only do it for 1D elements) -> Matlab, python, C++, ...

40-45% more difficult

more mathematical

& " notations

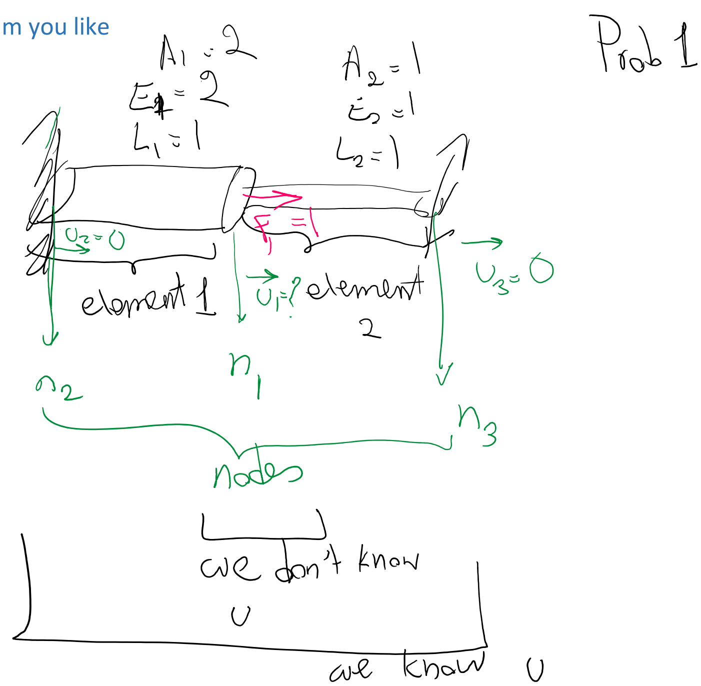
I'll use Ansys, but you can use any commercial program you like

Prob 1

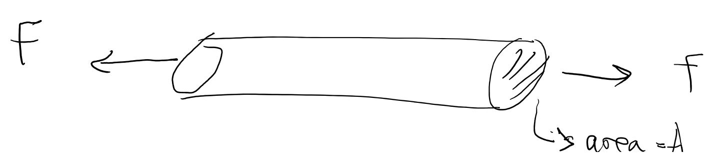
Bar problems using FEM:

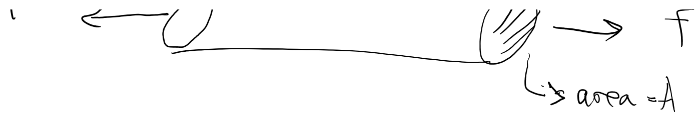
$u_1 = ?, \; u_2 = 0, \; u_3 = 0$

1 unknown



Background

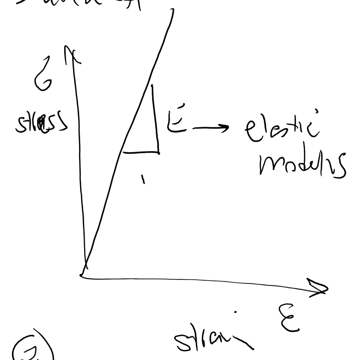




$$\sigma = \frac{F}{A} \quad (1)$$

stress

strain ε

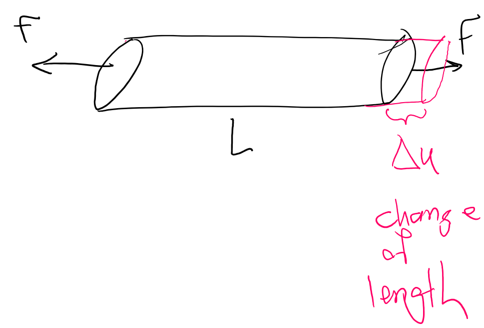


$$\sigma = E\varepsilon$$

$$\Updownarrow$$

$$\varepsilon = \frac{\sigma}{E} \quad (2)$$

$$(1),(2) \Rightarrow \quad \varepsilon = \frac{F}{AE} \quad (3)$$

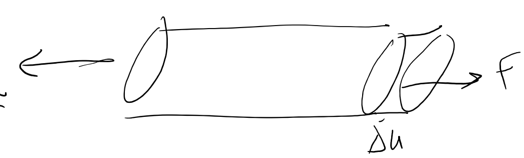


$$\varepsilon = \frac{\Delta u}{L} \iff \Delta u = \varepsilon L \quad (4)$$

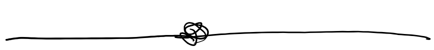
$$(3) \ \& \ (4) \quad \Delta u = \left(\frac{L}{AE}\right) F \Rightarrow$$

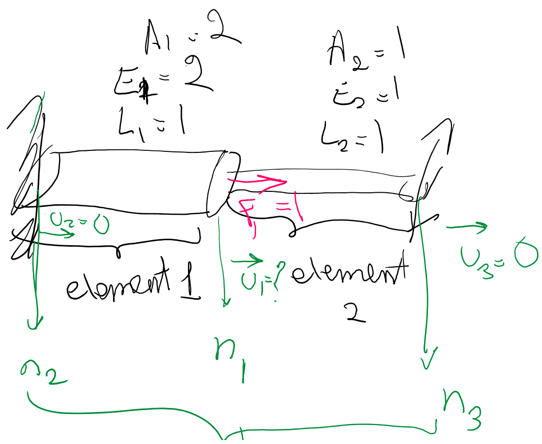
$$(*) \quad \boxed{F = \underbrace{\left(\frac{AE}{L}\right)}_{k} \Delta u}$$

similar to a spring stiffness



$$k_1 = \frac{A_1 E_1}{L_1} = 4 \qquad k_2 = \frac{A_2 E_2}{L_2} = 1$$

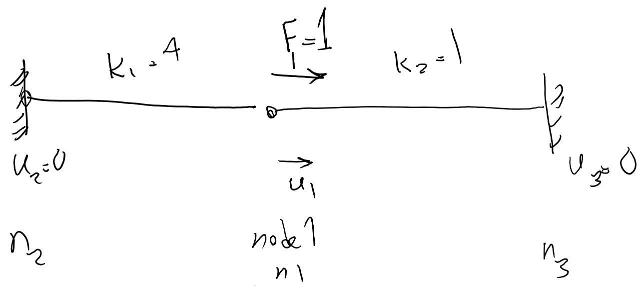


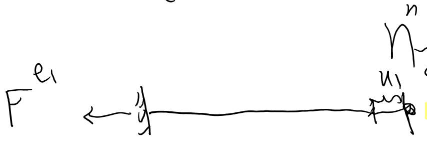


n_2 n_1 n_3

$\underbrace{\qquad\qquad\qquad}_{\text{nodes}}$

$\underline{u_1 = ?}$





$F^{e_1} = k_1 \underbrace{\Delta u^{e_1}}_{u_1} = \underline{4u_1}$

~~we could have done~~ it this way too

F^{e_2} F^{e_2}

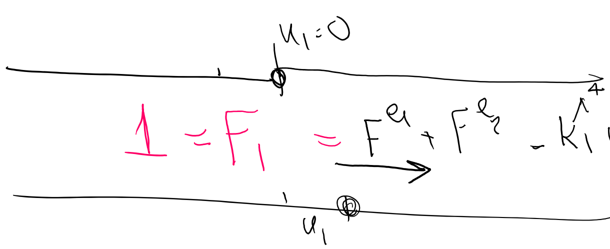
$\Delta u^{e_2} = -u_1$

$F^{e_2} = k^2(-u_1) = -u_1$

$F^{e_2} = k^2(-\Delta u^{e_2})$

$= 1(u_1) = u_1$

$\underline{F_1 = (\overset{4}{k_1} + \overset{1}{k_2})u_1 = 1}$



$1 = F_1 = F^{e_1} + F^{e_2} = \overset{4}{k_1}u_1 + \overset{1}{k_2}u_1 = (k_1 + k_2)u_1$

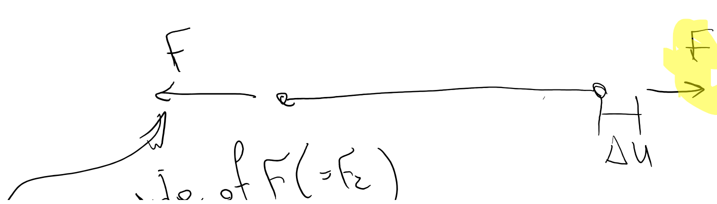
$\boxed{5u_1 = 1}$ $\boxed{u_1 = \frac{1}{5}}$

$F^{e_1} = \frac{4}{5}$

$F^{e_2} = \frac{1}{5}$

Hand Calculation

$\underline{\text{FEM}}$

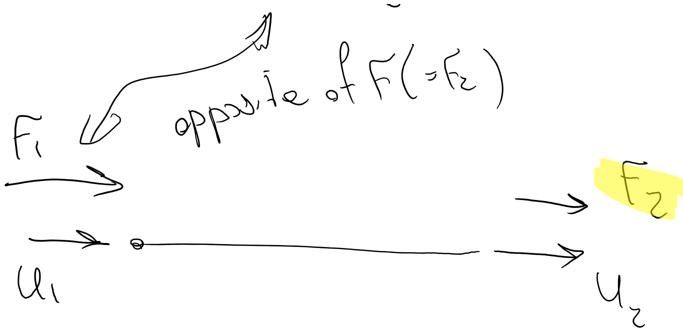


$\boxed{F = k\,\Delta u}$

$k = \frac{AE}{}$

Δu $\qquad \boxed{F = k\,\Delta u}$ ☆

$k = \dfrac{AE}{L}$



$F_2 = $ in terms of u_1 & u_2

$F_2 = k\,\Delta u$

$= k(u_2 - u_1)$

$F_1 = -F_2$

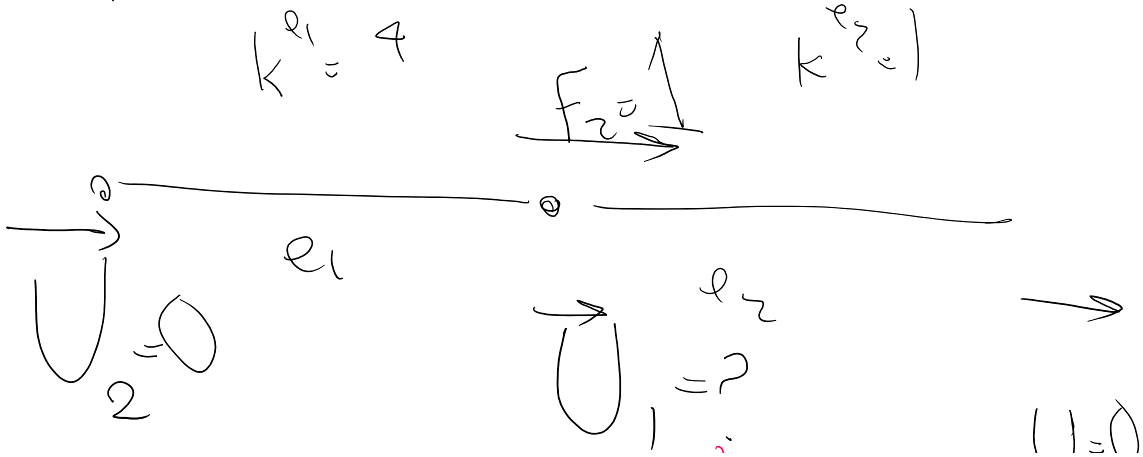
$$\boxed{\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad ☆☆}$$

$$\begin{bmatrix} F_1^{e_1} \\ F_2^{e_1} \end{bmatrix} = k^{e_1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad \begin{bmatrix} F_1^{e_2} \\ F_2^{e_2} \end{bmatrix} = k^{e_2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix}$$

$$F_2 = F_2^{e_1} + F_1^{e_2}$$

Our problem

$k^{e_1} = 4 \qquad k^{e_2} = 1$



$U_2 = 0$ $\qquad U_1 = ?$ $\qquad U_3 = 0$

$$\begin{bmatrix}F_1^{e_1}\\ F_2^{e_1}\end{bmatrix} = \underbrace{k^1}_{4}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}\underbrace{U_1^{e_1}}_{U_2}\\ \underbrace{U_2^{e_1}}_{U_1}\end{bmatrix} \qquad \begin{bmatrix}F_1^{e_2}\\ F_2^{e_2}\end{bmatrix} = k^2\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}\underbrace{U_1^{e_2}}_{U_1}\\ \underbrace{U_2^{e_2}}_{U_3}\end{bmatrix}$$

we'll later see that we can do the assembly as follows

$$F_2^{e_1} + F_1^{e_2} = F_2 = 1$$

$$\begin{bmatrix}F_1^{e_1}\\ F_2^{e_1}\end{bmatrix} = \begin{array}{c|cc} & 2 & 1\\ \hline 2 & & \\ 1 & & \end{array}\begin{bmatrix}k_1 & -k_1\\ -k_1 & k_1\end{bmatrix}\begin{bmatrix}U_1^{e_1}\\ U_2^{e_1}\end{bmatrix} \qquad \begin{bmatrix}F_1^{e_2}\\ F_2^{e_2}\end{bmatrix} = \begin{array}{c|cc} & 1 & 3\\ \hline 1 & & \\ 3 & & \end{array}\begin{bmatrix}k^2 & -k^2\\ -k^2 & k^2\end{bmatrix}\begin{bmatrix}U_1^{e_2}\\ U_2^{e_2}\end{bmatrix}$$

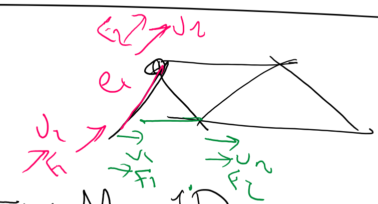
$$\begin{bmatrix}F_1\\ F_2\\ F_3\end{bmatrix} = \begin{array}{c} \\ 1\\ 2\\ 3\end{array}\begin{array}{c}\begin{array}{ccc}1 & 2 & 3\end{array}\\ \begin{bmatrix}k_1+k_2 & -k_1 & -k_2\\ -k_1 & k_1 & 0\\ -k_2 & 0 & +k_2\end{bmatrix}\end{array}\begin{bmatrix}U_1\\ U_2 = 0\\ U_3 = 0\end{bmatrix}$$

$$1 = F_1 = (k_1+k_2)U_1 - k_1 \overset{0}{U_2} - k_2 \overset{0}{U_3}$$

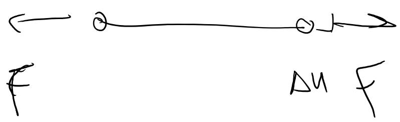
$$\Rightarrow U_1 = \frac{F_1}{k_1+k_2} = \frac{1}{4+1} = \frac{1}{5}$$

$\Rightarrow \quad U_1 = \frac{\quad}{k_1+k_2} = \frac{\quad}{4+1} = \frac{1}{5}$

Trusses

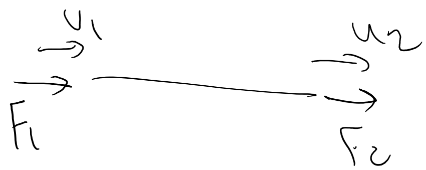


FEM 1D

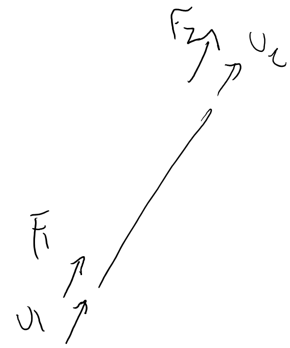


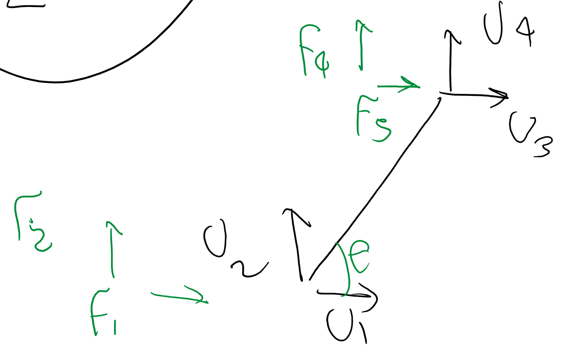
$F = k\Delta u$

$k = \frac{AE}{L}$

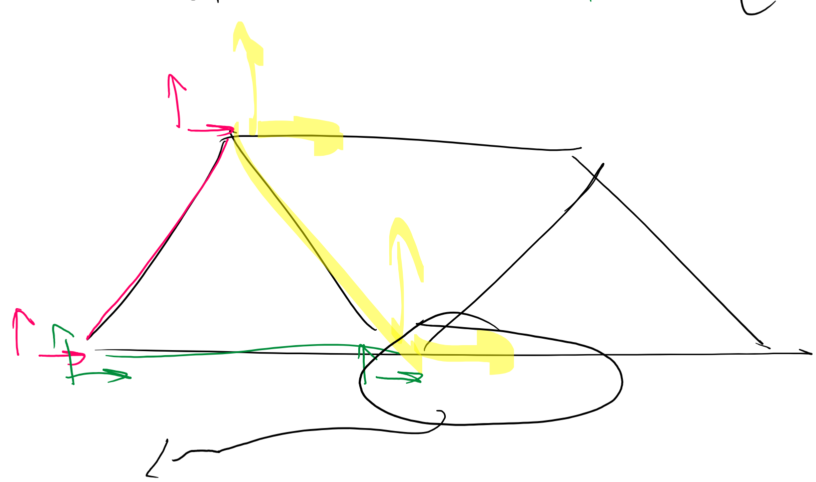


$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$





$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = k \begin{bmatrix} c^2 & cs & - & \\ cs & s^2 & & \\ & & + & \\ - & & & \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$



- The primary unknown (e.g. displacement, temperrature) are the same at that location
- Their "forces" (force,) add up

Engineering perspective

In the first part of the course we'll learn the mathematical perspective of WRS, FEM formulation that can solve even more general PDEs

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Next time, we'll solve a truss and a 3D problem