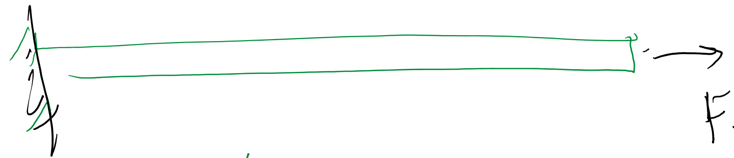
FEM20240826

Monday, August 26, 2024 1:40 PM

$u=0$



$F = EAu' = 0$

Differential equation $(EAu')' + q = 0$

$u' = \frac{du}{dx}$

$E=1 \quad A=1$

$q = x$

$u'' + x = 0 \implies u' = -\frac{x^2}{2} + c_1$

$u'(L) = 0 \to c_1 = \frac{L^2}{2}$

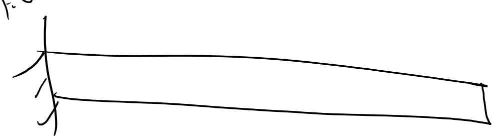
$\implies u = \frac{-x^3}{6} + \frac{L^2}{2}x + c_2$

$u(0) = 0 \to c_2 = 0$

$\boxed{u = -\frac{x^3}{6} + \frac{L^2}{6}}$

In general we can solve this easily

$u(0)=0$



$u'(L)=0$

$$\underbrace{R}_{\text{residual}}(u) = u'' + q = u'' + x^2$$

$R(u) = 0 \iff u$ is the exact solution

What if we write the solution as the sum of some functions we know $\to$ basis function $\to$ we choose them

$$u(x) = a_1 \phi_1(x) + a_2 \phi_2(x)$$

approximate

$$u(x) = \underbrace{a_1}_{\text{unknowns}} \phi_1(x) + \underbrace{a_2}_{} \phi_2(x)$$

unknowns

How about $\quad \phi_1(x) = x \quad \phi_2(x) = x^2 \implies u^h(x) = a_1 x + a_2 x^2$ approximate

$\phi_1(x) = \sin x \quad \phi_2(x) = \sin 2x$

$$R(u) = R(u^h) = u^{h''} + x^2 = 2a_2 \quad x^2$$

satisfy the eqn here

$x=0$ $\quad$ $x = 0.3$ $\quad$ $.75$ $\quad$ $x = L = 1$

$u^h = a_1 \phi_1(x) + \dots + a_n \phi_n(x) \implies$ Plug into

DE $\qquad R(a_1, \dots, a_n)$ residual

or error is satisfying the DE

I can only afford n equations

n equations n unknowns $\implies a_1, \dots, a_n$ obtained

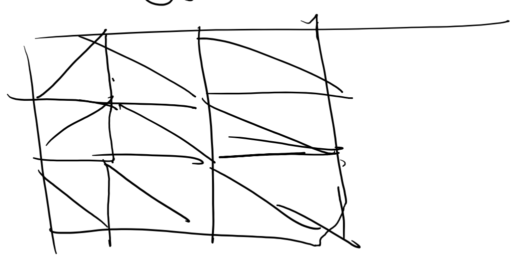
If the problem is linear

$$A_{n \times n} \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} = F_{n \times 1} \implies \vec{a} = A^{-1} F$$

$$u^h(x) = a_1 \phi_1(x) + \cdots + a_n \phi_n(x)$$

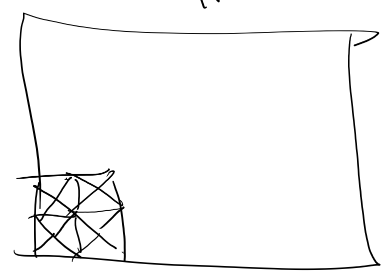
as n increases we get closer to the exact sln

coarse mesh



low n

fine mesh



better sol'n

high n

Balance law $\longrightarrow$ Differential eqn

$\longrightarrow$ Weighted residual statement (WRS) $\longrightarrow$

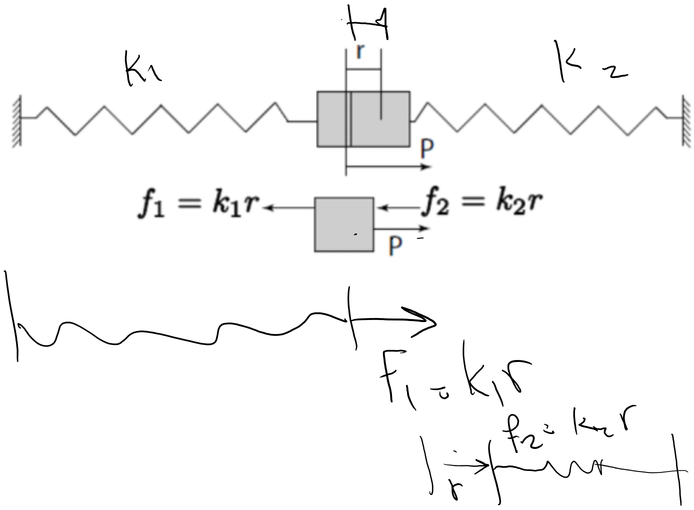
Weak statement

Energy statement $\Longrightarrow$ Weak statement

Balance law

Sum of forces

Balance law



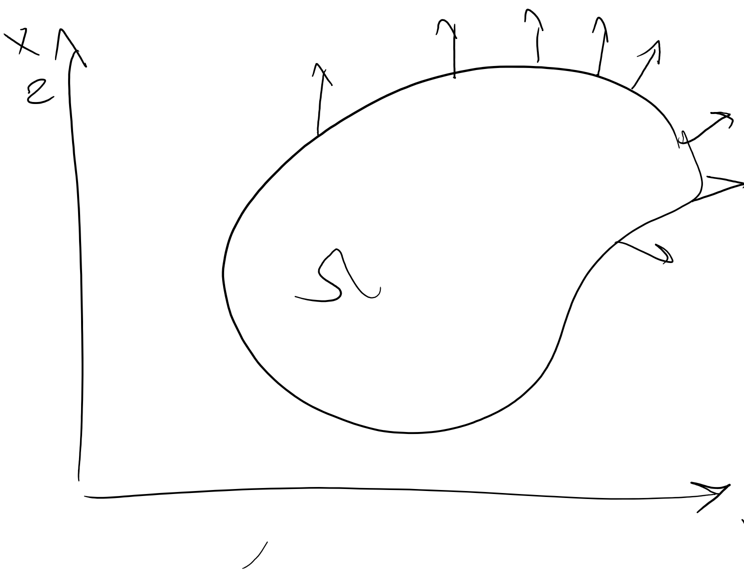
Sum of forces equal to zero is a balance law

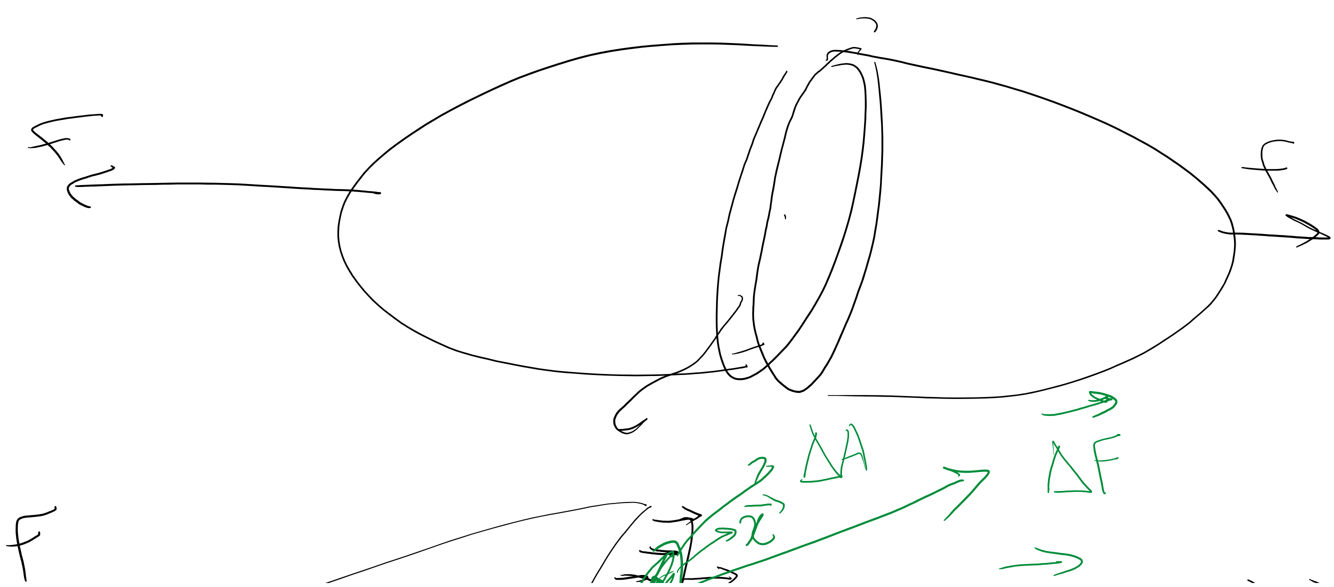
$$\boxed{\sum F = 0}$$

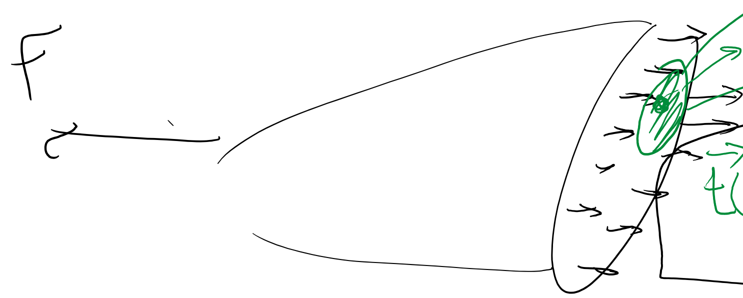
$$-k_1 r - k_2 r + P = 0$$

$$(k_1 + k_2) r = P$$

$$\boxed{r = \frac{P}{k_1 + k_2}}$$

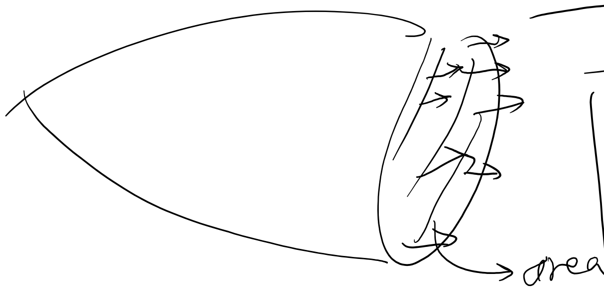




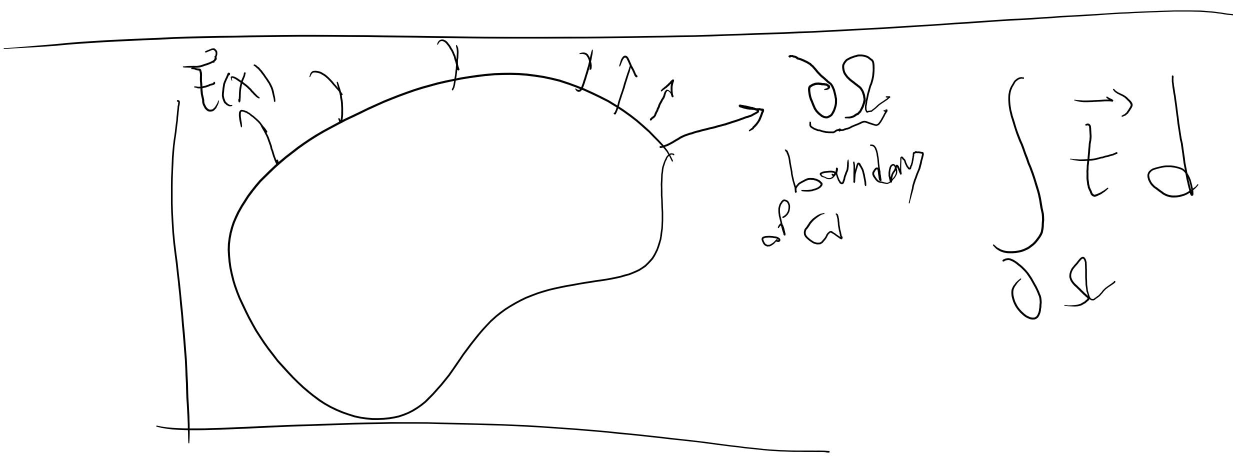


$$\vec{t}(\vec{x}) = \lim_{\Delta A \to 0} \frac{\Delta \vec{F}}{\Delta A} \Rightarrow \Delta \vec{F} = \vec{t}\,\Delta A$$

what's the force acting here



$$\boxed{F = \sum \Delta F = \int_A \vec{t}\, dA}$$



$$\int_{\partial\Omega} \vec{t}\, dA$$