FEM20241106

Wednesday, November 6, 2024 12:53 PM

1D W

$A(x) = 1 + x$

$x_1 = 0$ $x_2 = 1$

$A(\xi) = 1 + x(\xi)$

$\xi_1 = -1$ $\xi_2 = 1$

$$x(\xi) = x_1 N_1(\xi) + x_2 N_2(\xi)$$

$$= 0 \times \left(\frac{1-\xi}{2}\right) + 1 \left(\frac{1+\xi}{2}\right) = \frac{1+\xi}{2}$$

$$A(\xi) = 1 + \frac{1}{2} + \frac{1}{2}\xi = \frac{3}{2} + \frac{1}{2}\xi$$

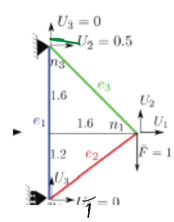
Axial force of the elements:

$$\mathbb{K} = \begin{bmatrix} 0.32+0.221 & 0.24-0.221 & -0.24 \\ 0.24-0.221 & 0.18+0.221 & -0.18 \\ -0.24 & -0.18 & 0.3571+0.18 \end{bmatrix} = \begin{bmatrix} 0.5410 & 0.019 & -0.24 \\ 0.019 & 0.401 & -0.18 \\ -0.24 & -0.18 & 0.5371 \end{bmatrix}$$

$$\mathbf{F} = \mathbf{F}_N + \mathbf{F}_e = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.1105 \\ -0.1105 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.1105 \\ -1.1105 \\ 0 \end{bmatrix} \Rightarrow \boxed{\mathbf{U}_f = \mathbb{K}^{-1}\mathbf{F} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} -0.2123 \\ -3.2950 \\ -1.300 \end{bmatrix}}$$

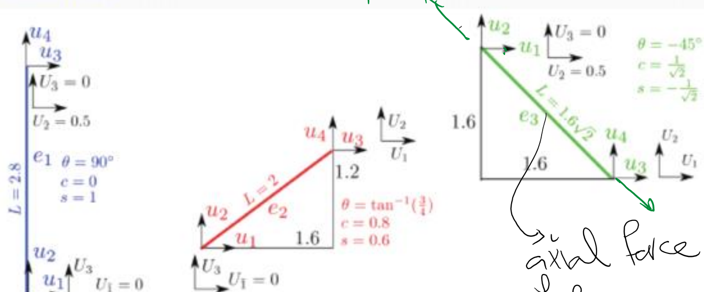
$$U_p = \bar{U} = \begin{bmatrix} 0 \\ .5 \\ 0 \end{bmatrix}$$

$$U_f = \begin{bmatrix} -.2123 \\ -3.2950 \\ -1.200 \end{bmatrix}$$



Truss Example

T^{e_3}



axial force of e_3

eqn ④ last time

$$M^{e_3} = [\bar{2}, \bar{3}, 1, 2]$$

$$U_{e_3} = [.5, 0, -.2123, -3.2950]$$

$$T^{e_3} = \frac{AE}{L}\Big(c(U_3 - U_1) + s(U_4 - U_2)\Big)$$

$$= \frac{1 \times 1}{1.6\sqrt{2}}\left(\frac{1}{\sqrt{2}}(-.2123 - .5) + \frac{-1}{\sqrt{2}}(-3.2950 - 0)\right)$$

$$= +.8064$$

$+.8064$

$= + .806\ T$

$+.8064$

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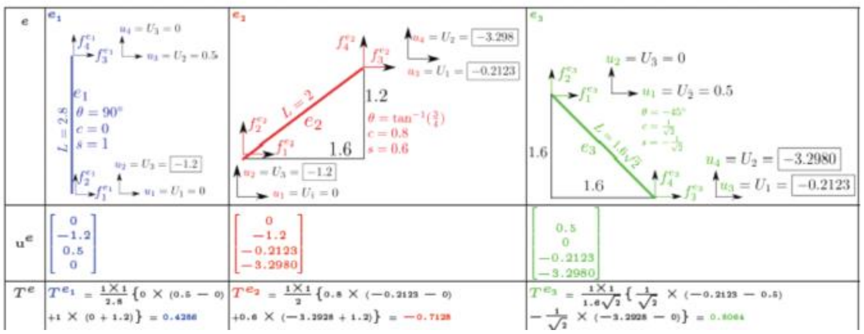
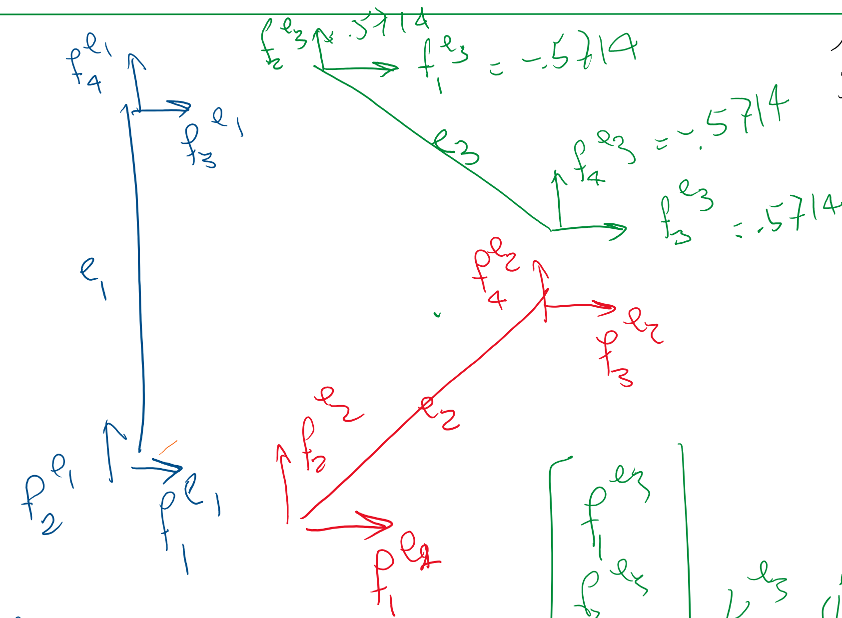
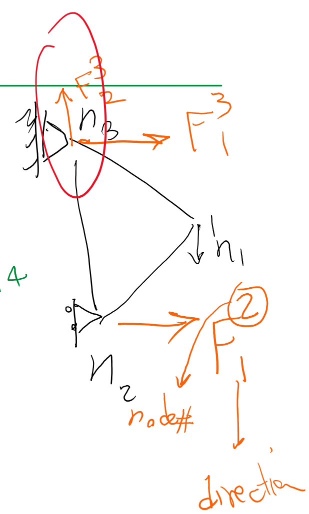
Truss Example: Axial force and element local forces



Support forces:





$$F_1^2 = f_1^{e_1} + f_1^{e_2}$$

$$\begin{bmatrix} f_1^{e_3} \\ f_2^{e_3} \\ f_3^{e_3} \\ f_4^{e_3} \end{bmatrix} = K^{e_3}_{4\times4}\, U^{e_3}_{4\times1} = k^{e_3}\begin{bmatrix} .5 \\ 0 \\ -.2123 \\ -3.29 \end{bmatrix}$$

$$= \begin{bmatrix} -.5714 \\ .5714 \\ +.5714 \\ -.5714 \end{bmatrix}$$

$k^{e_3} u^{e}_{3} =$

$$\begin{bmatrix} 0.221 & -0.221 & -0.221 & 0.221 \\ -0.221 & 0.221 & 0.221 & -0.221 \\ -0.221 & 0.221 & 0.221 & -0.221 \\ 0.221 & -0.221 & -0.221 & 0.221 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0 \\ -0.2123 \\ -3.2980 \end{bmatrix} =$$

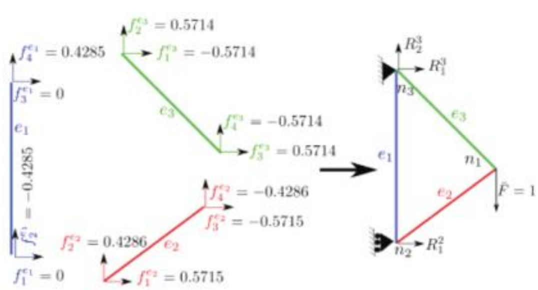
$[-0.5714$

$$\begin{bmatrix} 0.221 & -0.221 & -0.221 & 0.221 \\ -0.221 & 0.221 & 0.221 & -0.221 \\ -0.221 & 0.221 & 0.221 & -0.221 \\ 0.221 & -0.221 & -0.221 & 0.221 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0 \\ -0.2123 \\ -3.2980 \end{bmatrix} =$$

$$\begin{array}{c} 2 \\ 3 \\ 1 \\ 2 \end{array} \begin{bmatrix} -0.5714 \\ 0.5714 \\ 0.5714 \\ -0.5714 \end{bmatrix}$$

	$k^{e_1} a^{e}_{1} =$	$k^{e_2} a^{e}_{2} =$
$-f^{e}$	$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.357 & 0 & -0.357 \\ 0 & 0 & 0 & 0 \\ 0 & -0.357 & 0 & 0.357 \end{bmatrix} \begin{bmatrix} 0 \\ -1.2 \\ 0.5 \\ 0 \end{bmatrix} =$ $\begin{array}{c} 1 \\ 3 \\ 2 \\ 2 \end{array} \begin{bmatrix} 0 \\ -0.4285 \\ 0 \\ 0.4285 \end{bmatrix}$	$\begin{bmatrix} 0.32 & 0.24 & -0.32 & -0.24 \\ 0.24 & 0.18 & -0.24 & -0.18 \\ -0.32 & -0.24 & 0.32 & 0.24 \\ -0.24 & -0.18 & 0.24 & 0.18 \end{bmatrix} \begin{bmatrix} 0 \\ -1.2 \\ -0.2123 \\ -3.2980 \end{bmatrix} =$ $\begin{array}{c} 1 \\ 3 \\ 1 \\ 2 \end{array} \begin{bmatrix} 0.5715 \\ 0.4286 \\ -0.5715 \\ -0.4286 \end{bmatrix}$

Truss Example: Reaction Forces

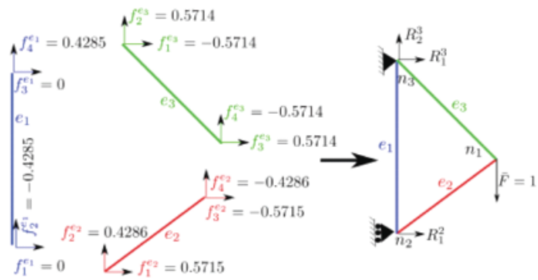


- First, we compute reaction forces by adding up forces from individual elements that contribute to reaction forces:

$$R_1^2 = f_1^{e_1} + f_1^{e_2} = 0 + 0.5715 = 0.5715 \qquad (397a)$$
$$R_1^3 = f_3^{e_1} + f_1^{e_3} = 0 + -0.5714 = -0.5714 \qquad (397b)$$
$$R_2^3 = f_4^{e_1} + f_2^{e_3} = 0.4285 + 0.5714 = 0.9999 \qquad (397c)$$

Not needed but for checking our solution:

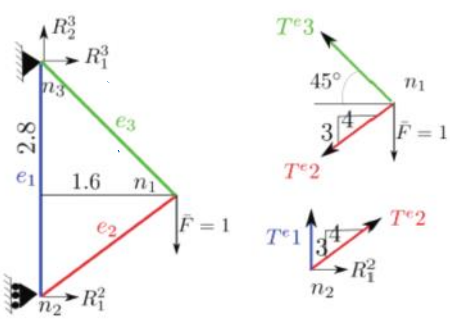
Truss Example: verification of forces at free dofs



- Also, if we want to double-check our calculations on **free dofs**. This step is not needed and it may be done as a verification for hand calculations:

$$F_1^1 = f_3^{e_2} + f_3^{e_3} = -0.5715 + 0.5714 = -0.0001 \tag{398a}$$
$$F_2^1 = f_4^{e_2} + f_4^{e_3} = -0.4286 + -0.5714 = -1 = \bar{F} \tag{398b}$$
$$R_2^2 = f_2^{e_1} + f_2^{e_2} = -0.4285 + 0.4286 = 0.0001 \tag{398c}$$

Truss Example: Direct solution method

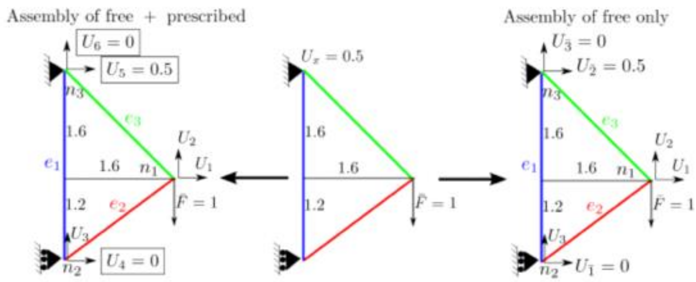


- Since this is a statically determinate structure, we can easily solve the forces and verify our FEM forces.

$$\Sigma F_2 = 0 \quad \Rightarrow \quad R_2^3 - \bar{F} = 0 \quad \Rightarrow \quad R_2^3 = 1 \tag{399a}$$
$$\Sigma M_{n_3} = 0 \quad \Rightarrow \quad 2.8 R_1^2 - 1.6\bar{F} = 0 \quad \Rightarrow \quad R_1^2 = \frac{4}{7} = 0.5714 \tag{399b}$$
$$\Sigma F_1 = 0 \quad \Rightarrow \quad R_1^2 + R_1^3 = 0 \quad \Rightarrow \quad R_1^3 = -\frac{4}{7} = -0.5714 \tag{399c}$$
$$\Sigma F_1 = 0 (@n_2) \quad \Rightarrow \quad R_1^2 + \frac{4}{5} T^e 2 = 0 \quad \Rightarrow \quad T^e 2 = -\frac{5}{7} = -0.7143 \tag{399d}$$
$$\Sigma F_2 = 0 (@n_2) \quad \Rightarrow \quad T^e 1 + \frac{3}{5} T^e 2 = 0 \quad \Rightarrow \quad T^e 1 = \frac{3}{7} = 0.4286 \tag{399e}$$
$$\Sigma F_1 = 0 (@n_1) \quad \Rightarrow \quad -\frac{4}{5} T^e 2 - \frac{1}{\sqrt{2}} T^e 3 = 0 \quad \Rightarrow \quad T^e 3 = \frac{4}{7}\sqrt{2} = 0.8081 \tag{399f}$$

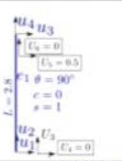
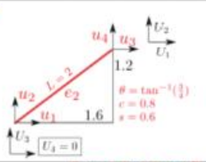
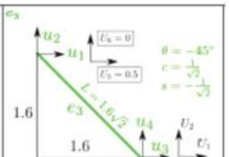
Please read these slides:

Assembly of free + prescribed dofs vs. free only



- All we covered so far was the **assembly of free dof only**.
- We can assemble all dofs (free + prescribed) as shown in figure on the left.
- The numbering of dof when assembling free + prescribed dof is exactly like before with the difference that **we first number free dof followed by prescribed dof** as shown in the figure. For each group (f & p) we start from node n_1 to n_{n_n}.

Truss example: Assembly of global system $(f + p)$

e	e_1	e_2	e_3
			
$\mathbf{k}^e$	$\mathbf{k}^{e_1} = \frac{(1)(1)}{2.8}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$ $= \begin{array}{c} \\ 4 \\ 3 \\ 5 \\ 6 \end{array}\begin{array}{c} 4 \quad 3 \quad\quad 5 \quad 6 \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.3571 & 0 & -0.3571 \\ 0 & 0 & 0 & 0 \\ 0 & -0.3571 & 0 & 0.3571 \end{bmatrix} \end{array}$	$\mathbf{k}^{e_2} = \frac{(1)(1)}{2}\begin{bmatrix} 0.64 & 0.48 & -0.64 & -0.48 \\ 0.48 & 0.36 & -0.48 & -0.36 \\ -0.64 & -0.48 & 0.64 & 0.48 \\ -0.48 & -0.36 & 0.48 & 0.36 \end{bmatrix}$ $= \begin{array}{c} \\ 4 \\ 3 \\ 1 \\ 2 \end{array}\begin{array}{c} 4 \quad 3 \quad\quad 1 \quad 2 \\ \begin{bmatrix} 0.32 & 0.24 & -0.32 & -0.24 \\ 0.24 & 0.18 & -0.24 & -0.18 \\ -0.32 & -0.24 & 0.32 & 0.24 \\ -0.24 & -0.18 & 0.24 & 0.18 \end{bmatrix} \end{array}$	$\mathbf{k}^{e_3} = \frac{(1)(1)}{1.6\sqrt{2}}\begin{bmatrix} 0.5 & -0.5 & -0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 & 0.5 \end{bmatrix}$ $= \begin{array}{c} \\ 5 \\ 6 \\ 1 \\ 2 \end{array}\begin{array}{c} 5 \quad 6 \quad\quad 1 \quad 2 \\ \begin{bmatrix} 0.221 & -0.221 & -0.221 & 0.221 \\ -0.221 & 0.221 & 0.221 & -0.221 \\ -0.221 & 0.221 & 0.221 & -0.221 \\ 0.221 & -0.221 & -0.221 & 0.221 \end{bmatrix} \end{array}$

- Global stiffness matrix:

$$\mathbf{K} = \begin{bmatrix} 0.32+0.221 & 0.24-0.221 & -0.24 & -0.32 & -0.221 & 0.221 \\ 0.24-0.221 & 0.18+0.221 & -0.18 & -0.24 & 0.221 & -0.221 \\ -0.24 & -0.18 & 0.3571+0.18 & 0+0.24 & 0 & -0.3571 \\ -0.32 & -0.24 & 0+0.24 & 0+0.32 & 0 & 0 \\ -0.221 & 0.221 & 0 & 0 & 0+0.221 & 0-0.221 \\ 0.221 & -0.221 & -0.3571 & 0 & 0-0.221 & 0.3571+0.221 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5410 & 0.019 & -0.24 & -0.32 & -0.221 & 0.221 \\ 0.019 & 0.401 & -0.18 & -0.24 & 0.221 & -0.221 \\ -0.24 & -0.18 & 0.5371 & 0.24 & 0 & -0.3571 \\ -0.32 & -0.24 & 0.24 & 0.32 & 0 & 0 \\ -0.221 & 0.221 & 0 & 0 & 0.221 & -0.221 \\ 0.221 & -0.221 & -0.3571 & 0 & -0.221 & 0.5781 \end{bmatrix}$$

- Global force vector. The element vectors that would assemble to $\mathbf{F}_e$ are $\mathbf{f}_e^e = \mathbf{f}_r^e + \mathbf{f}_N^e$ and we do not include $-\mathbf{f}_D^e$ in it as it would be directly taken care of subsequently. Since all = $\mathbf{f}_r^e$ and $\mathbf{f}_N^e$ are zero, $\mathbf{F}_e$ is identically zero.

$$\mathbf{F} = \mathbf{F}_N + \mathbf{F}_e = \begin{bmatrix} 0 \\ -1 \\ 0 \\ R_1^2 \\ R_1^3 \\ R_2^3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ R_1^2 \\ R_1^3 \\ R_2^3 \end{bmatrix}$$

Truss example: Solution of global system $(f + p)$

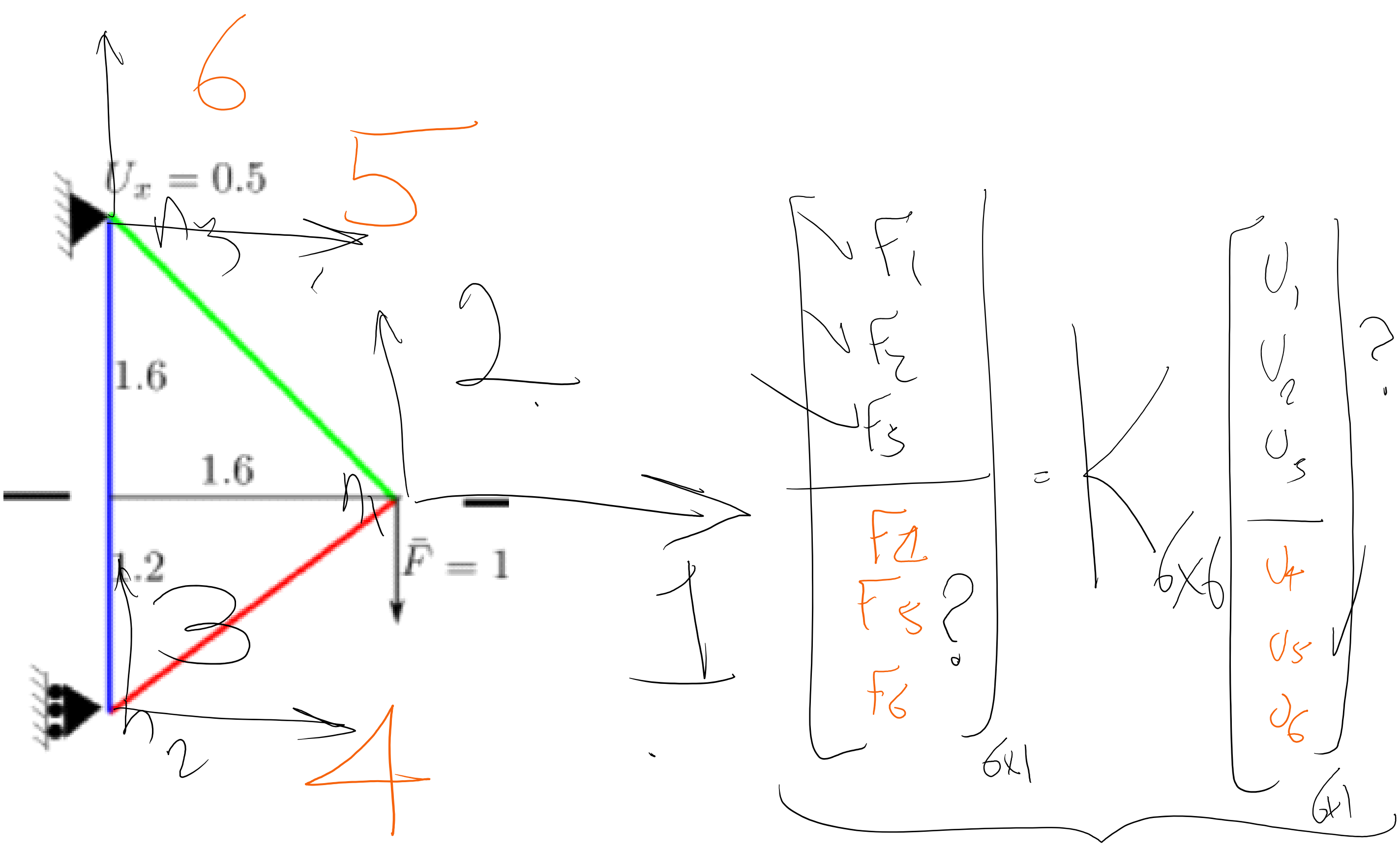
- The global system is,

$\mathbf{KU} = \mathbf{F}$ where

$$\mathbf{K} = \left[\begin{array}{ccc|ccc} 0.5410 & 0.019 & -0.24 & -0.32 & -0.221 & 0.221 \\ 0.019 & 0.401 & -0.18 & -0.24 & 0.221 & -0.221 \\ -0.24 & -0.18 & 0.5371 & 0.24 & 0 & -0.3571 \\ \hline -0.32 & -0.24 & 0.24 & 0.32 & 0 & 0 \\ -0.221 & 0.221 & 0 & 0 & 0.221 & -0.221 \\ 0.221 & -0.221 & -0.3571 & 0 & -0.221 & 0.5781 \end{array}\right] = \left[\begin{array}{c|c} \mathbf{K}_{ff} & \mathbf{K}_{fp} \\ \hline \mathbf{K}_{pf} & \mathbf{K}_{pp} \end{array}\right]$$

$$\mathbf{F} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_6 \\ F_6 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 0 \\ R_1^2 \\ R_1^3 \\ R_2^3 \end{bmatrix} = \left[\begin{array}{c} \mathbf{F}_f \\ \hline \mathbf{F}_p \end{array}\right]$$

- The unknown quantities $\mathbf{F}_p$ and $\mathbf{U}_f$ are highlighted.



Truss example: Solution of global system $(f + p)$

- To solve the system in previous slid, $\mathbf{KU} = \mathbf{F}$, we observe,

$$\mathbf{F}_f = \mathbf{K}_{ff}\mathbf{U}_f + \mathbf{K}_{fp}\mathbf{U}_p \quad (400a)$$
$$\mathbf{F}_p = \mathbf{K}_{pf}\mathbf{U}_f + \mathbf{K}_{pp}\mathbf{U}_p \quad (400b)$$

- Sizes of these matrices and vectors are:

$\mathbf{F}_f : n_f \times 1$	$\mathbf{F}_p : n_p \times 1$		
$\mathbf{K}_{ff} : n_f \times n_f$	$\mathbf{K}_{fp} : n_f \times n_p$	$\mathbf{K}_{pf} : n_p \times n_f$	$\mathbf{K}_{pp} : n_p \times n_p$
$\mathbf{U}_f : n_f \times 1$	$\mathbf{U}_p : n_p \times 1$		
$n_f = \#$free dof	$n_p = \#$ prescibed dof		

Dirichlet force

- In this particular problem $n_f = 3$ and $n_p = 3$ but clearly they can be distinct.
- To solve the unknowns $\mathbf{U}_f$ and $\mathbf{F}_p$ in (400), we first solve for $\mathbf{U}_f$ in (400a). Subsequently, ~~we plug $\mathbf{U}_f$ in (400b):~~

$F_D = -K_{fp}\, a_p$

$$\mathbf{U}_f = \mathbf{K}_{ff}^{-1}\left(\mathbf{F}_f - \mathbf{K}_{fp}\mathbf{U}_p\right) \quad \text{step A} \quad (401a)$$
$$\mathbf{F}_p = \mathbf{K}_{pf}\mathbf{U}_f + \mathbf{K}_{pp}\mathbf{U}_p \quad \text{step B} \quad (401b)$$

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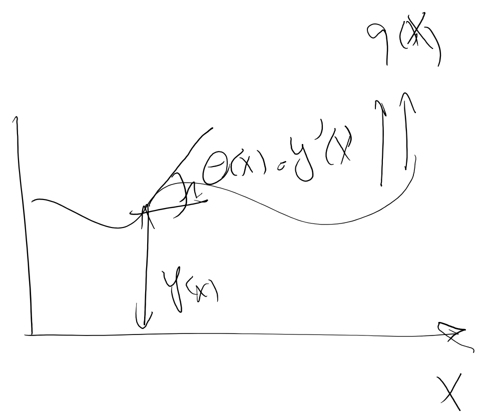
Beam problem: FEM formulation

- Higher order differential equation.

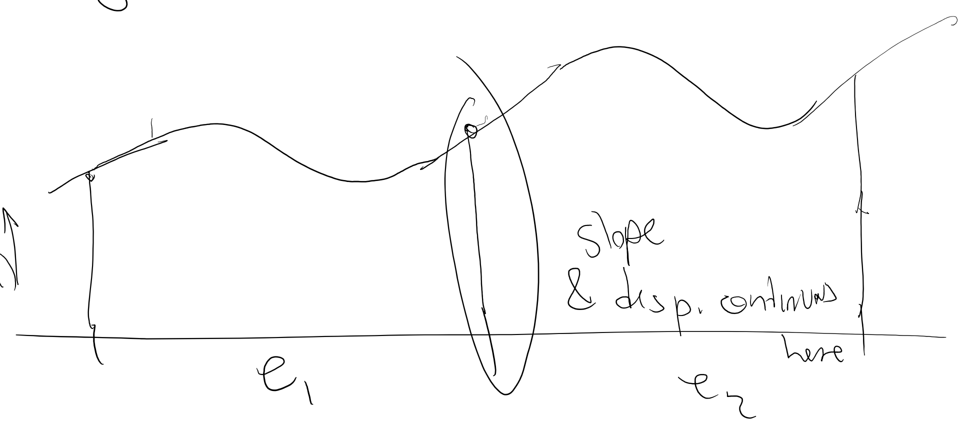
ODE $\frac{d^2}{dx^2}\left(EI\frac{d^2y}{dx^2}\right) + q = 0$



$\underline{M = 4} \Rightarrow m = \frac{M}{2} = \underline{2}$

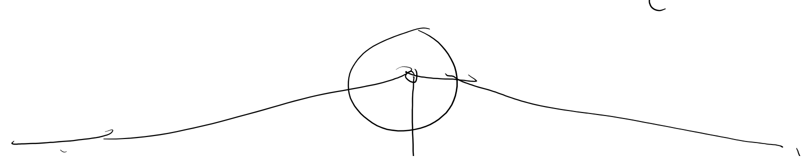
C^{m-1} global continuity is needed

$\underline{C^1}$



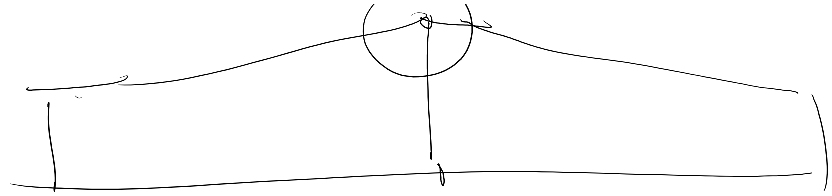
beam $y(x)$

bar $u(x)$

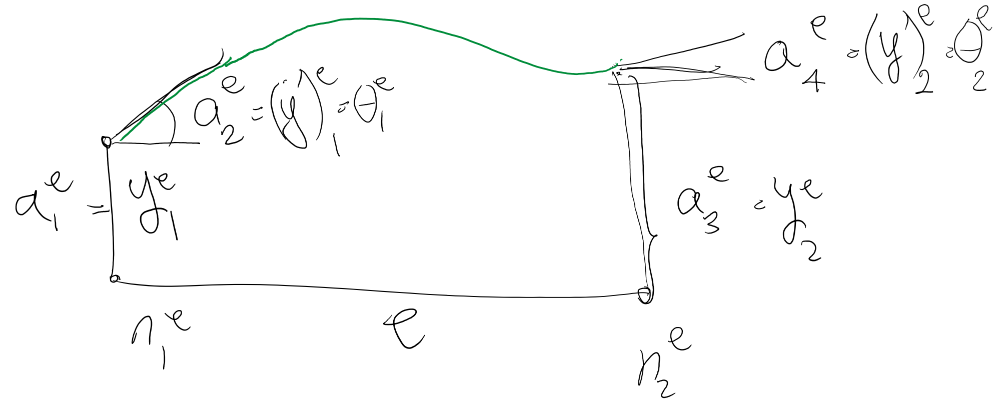


$u(x)$

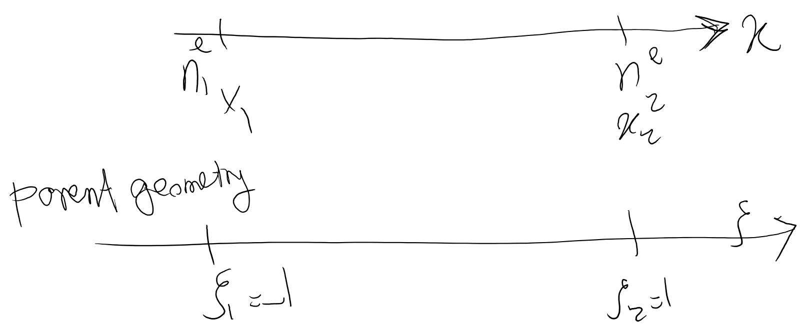
$\longrightarrow C^0$



$\longrightarrow$ for a beam problem to ensure C^1 global continuity we need y & $\Theta = y'$ as dofs at each node



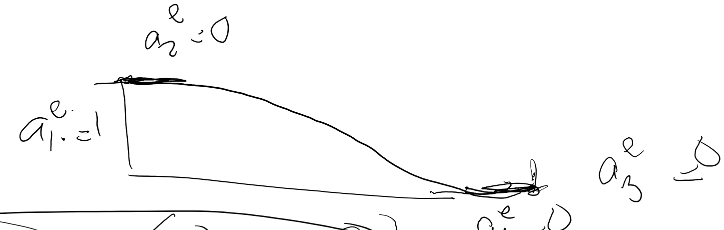
What about the shape functions



N^1: it takes value 1 @ dof 1 & zero elsewhere

4 coefficients

$\Rightarrow$ 4 conditions



4 conditions

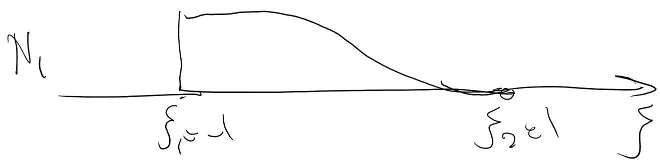
$$\underline{N^1(\xi)} = \alpha_0 + \alpha_1 \xi + \alpha_2 \xi^2 + \alpha_3 \xi^3 + \alpha_4 \xi^4 + \alpha_5 \xi^5 \dots$$

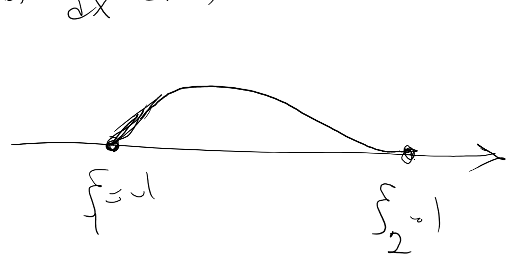
polynomial in terms of ξ

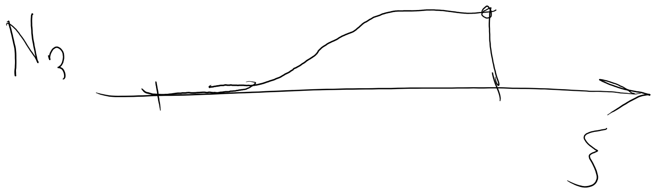
$N(\xi) = \alpha_0 + \alpha_1\xi + \alpha_2\xi^2 + \alpha_3\xi^3 + \alpha_4\xi^4 + \alpha_5\xi^5 \dots$

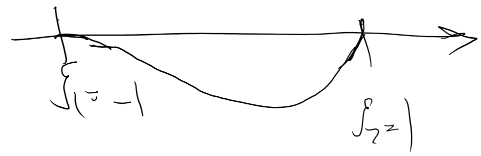
polynomial in terms of ξ

Shape functions for the beam element $\theta(\xi_1=-1) = \frac{dN_2}{dX}(\xi_1=-1)$









Formula for $y(\xi) = N_2(\xi)$

$a_1 = 0 \quad : a_1 = y(\xi=-1) = N_2(\xi=-1) = 0$

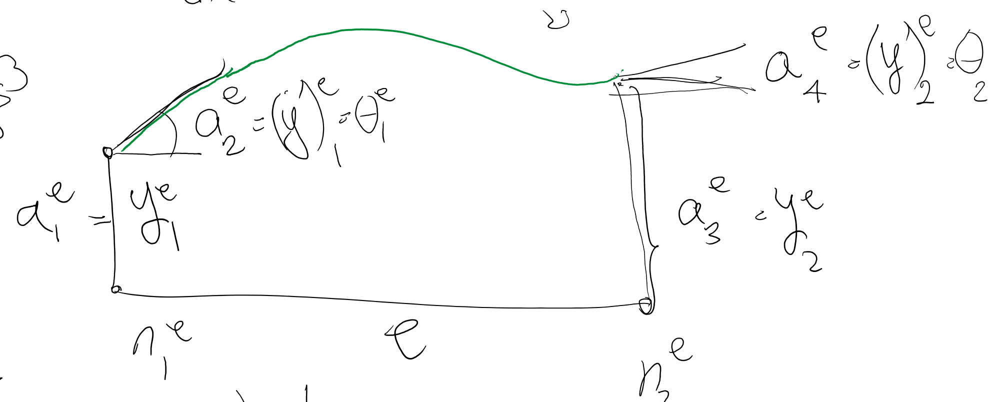
$a_2 = 1 \quad \theta(\xi=-1) = \frac{dy}{dX}(\xi=-1) = \frac{d}{dX}N_2(\xi=-1) = 1$

$a_3 = 0 \quad a_3 = y(\xi=1) = N_2(\xi=1) = 0$

$a_4 = 0 \quad a_4 = \frac{dy}{dX}(\xi=1) = \frac{dN_2}{dX}(\xi=1) = 0$

$N_2(\xi) = \alpha_0 + \alpha_1\xi + \alpha_2\xi^2 + \alpha_3\xi^3$

we'll find values of $\alpha_0, \alpha_1, \alpha_2, \alpha_3$

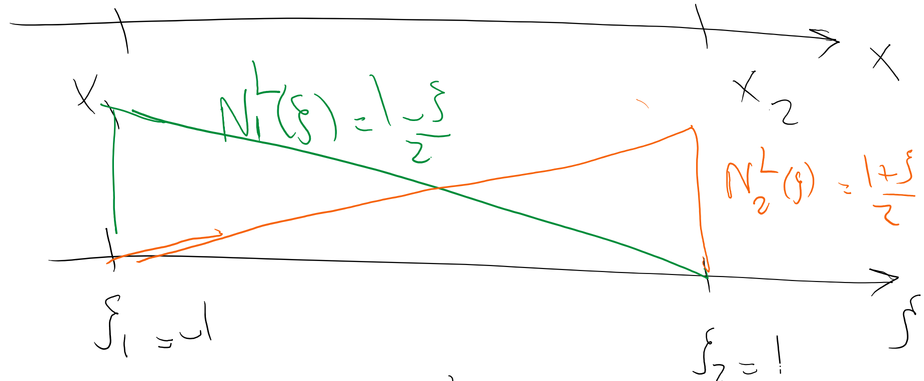


$\frac{dN_2(\xi)}{dX} = \frac{dN_2(\xi)}{d\xi} \cdot \frac{d\xi}{dX} = \frac{dN_2(\xi)}{d\xi} \frac{1}{\frac{dX}{d\xi}}$ $\quad x(\xi)$ we need this

$x(\xi)$

$x(\xi)$

$x(\xi)$



$x(\xi) = A + B\xi$ bad :-(

$x(-1) = x_1$

$x(1) = x_2$

$x(\xi) = x_1 N_1^L(\xi) + x_2 N_2^L(\xi)$

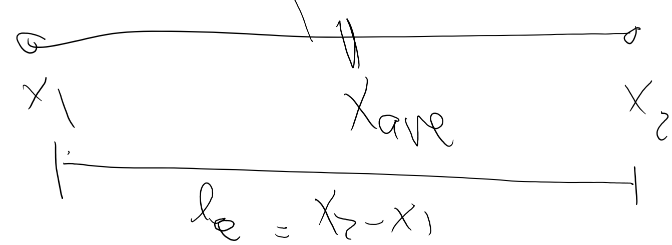
$x(\xi_1) = x_1 \underbrace{N_1^L(\xi_1)}_{1} + x_2 \underbrace{N_2^L(\xi_1)}_{0} = x_1$

$x(\xi_2) = x_2$

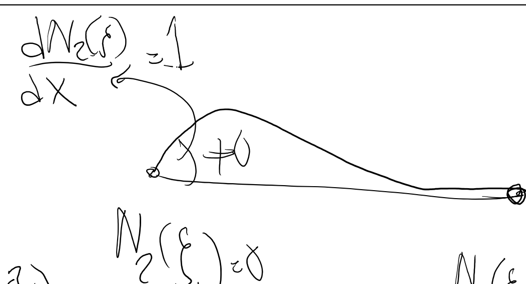
$$x(\xi) = x_1\left(\frac{1-\xi}{2}\right) + x_2\left(\frac{1+\xi}{2}\right) = \underbrace{\frac{x_1+x_2}{2}} + \left(\frac{x_2-x_1}{2}\right)\xi$$

$$x(\xi) = x_{ave} + \frac{L_e}{2}\xi$$

$$J = \frac{dx(\xi)}{d\xi} = \frac{L_e}{2}$$



Summary for $N_2(\xi)$



$N_2(\xi_2) = 0$

$\frac{dN_2}{dx}(\xi_2) = 0$

$$N_2(\xi) = \alpha_0 + \alpha_1\xi + \alpha_2\xi^2 + \alpha_3\xi^3$$

$$\frac{d}{dx}N_2(\xi) = \frac{1}{\left(\frac{dx}{d\xi}\right)}\frac{dN_2}{d\xi} = \frac{1}{\left(\frac{L_e}{2}\right)}\left(\alpha_1 + 2\alpha_2\xi + 3\alpha_3\xi^2\right)$$

$$N_2(\xi_1 = -1) = 0 \Longrightarrow \alpha_0 + (-1)\alpha_1 + (-1)^2\alpha_2 + (-1)^3\alpha_3 = 0$$

$$\frac{dN_2}{dx}(\xi_1 = -1) = 1 \Longrightarrow \frac{2}{l}\left(\alpha_1 + 2(-1)\alpha_2 + 3(-1)^2\alpha_3\right) = 1$$

$$\frac{dN_2}{dx}(\xi_1 = -1) = 1 \Rightarrow \frac{2}{L_e}\left(\alpha_1 + 2(-1)\alpha_2 + 3(-1)^2\alpha_3\right) = 1$$

$$N_2(\xi_2 = 1) = 0 \Rightarrow \alpha_0 + (1)\alpha_1 + (1)^2\alpha_2 + (1)^3\alpha_3 = 0$$

$$\frac{dN_2}{dx}(\xi_2 = 1) = 0 \quad \frac{2}{L_e}\left(\alpha_1 + 2(1)\alpha_2 + 3(1)^2\alpha_3\right) = 0$$

- Similarly for N_2:

$$\left.\begin{aligned} N_2(\xi=-1) &= 0 \\ \frac{dN_2}{d\xi}(\xi=-1) &= \frac{L^e}{2} \\ N_2(\xi=1) &= 0 \\ \frac{dN_2}{d\xi}(\xi=1) &= 0 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} \alpha_0 - \alpha_1 + \alpha_2 - \alpha_3 &= 0 \\ \alpha_1 - 2\alpha_2 + 3\alpha_3 &= \frac{L^e}{2} \\ \alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 &= 0 \\ \alpha_1 + 2\alpha_2 + 3\alpha_3 &= 0 \end{aligned}\right\} \Rightarrow \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \frac{L^e}{8}\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

$$\boxed{N_2(\xi) = \frac{L^e}{8}(1 - \xi - \xi^2 + \xi^3)}$$

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We'll do this for N1, N3, N4

FEM formulation of beam elements: Shape functions

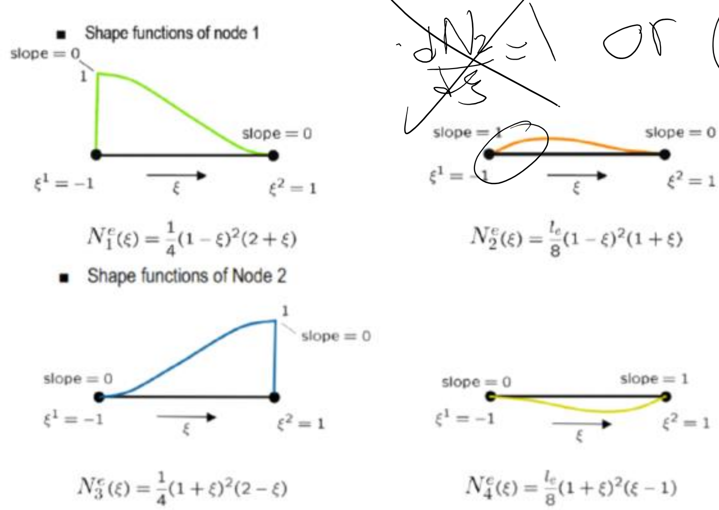

figure from F. Cirak

$\cancel{\frac{dN_2}{d\xi} = 1}$ or $\frac{dN_2}{dx} = 1$

$\frac{dy}{dx}$ to be continuous

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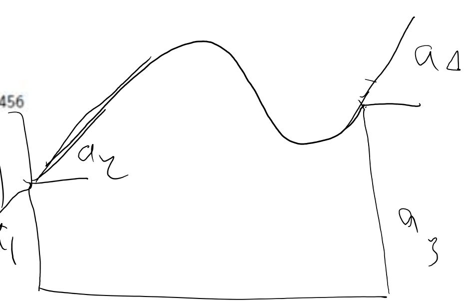
$$y(\xi) = a_1 N_1(\xi) + a_2 N_2(\xi) + a_3 N_3(\xi) + a_4 N_4(\xi)$$



$\xi = \xi_1 = -1$

$$y(\xi_1) = a_1$$

$$\frac{dy}{dx}(\xi_1) = \theta(\xi_1) = a_2$$

$$y(\xi_2) = a_3$$

$$\frac{dy}{dx}(\xi_2) = a_4$$