2022/09/06

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Another set of results showing the effect of in-situ defects

Fracture stress: discrepancy between theory and experiment

$$\sigma_{th} = \sqrt{\frac{E\gamma}{a_0}}$$

σ_c σ_f

measured strength

	a_0 [m]	E [GPa]	σ_{th} [GPa]	σ_b [MPa]	σ_{th}/σ_b
glass	$3 * 10^{-10}$	60	14	170	82
steel	10^{-10}	210	45	250	180
silica fibers	10^{-10}	100	31	25000	1.3
iron whiskers	10^{-10}	295	54	13000	4.2
silicon whiskers	10^{-10}	165	41	6500	6.3
alumina whiskers	10^{-10}	495	70	15000	4.7
ausformed steel	10^{-10}	200	45	3000	15
piano wire	10^{-10}	200	45	2750	16.4

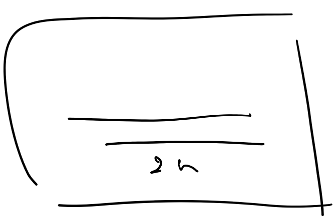
high $\sigma_c >> \sigma_f$

as we get to finer length scales we get close to the theoretical estimate

From last time

$$\sigma_f = \sqrt{\frac{\kappa_0}{4a}}\,\sigma_c$$

σ_c → theoretical estimate



$\sigma_f \downarrow$ as $\frac{1}{\sqrt{a}}$

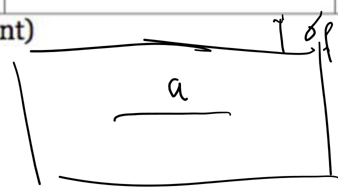
we want to experimentally verify this

Griffith's verification experiment

- Glass fibers with artificial cracks (much larger than natural crack-like flaws), tension tests

	Crack Length, $2a$ mm	Measured Strength, σ_f MPa	$\sigma_f\sqrt{a}$ MPa$\sqrt{m}$
sample 1	3.8	6.0	0.26
sample 2	6.9	4.3	0.25
sample 3	13.7	3.3	0.27
sample 4	22.6	2.5	0.27

(Data from the Griffith experiment)



$$\sigma_f > \sqrt{\frac{x_0}{4a}}\,\sigma_c \rightarrow$$

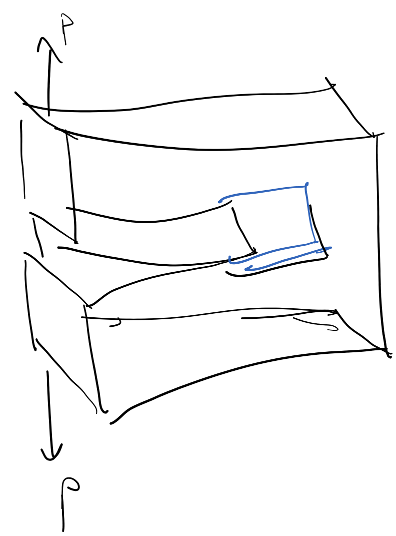
$$\sigma_f\sqrt{a} = \sqrt{\frac{x_0}{4}}\,\sigma_c = \text{const}$$

So, this experiment verifies the power of -0.5 of strength versus crack length.

We had the stress-based (using stress concentration concept)
explanation why sigma_f << sigma_c

Energy-based explanation:

4.1.3. Cause of discrepancy:
2. Energy approach



W = external work

$$\dot{W} = \dot{U}_e + \dot{U}_p + \dot{U}_k + \dot{U}_\Gamma$$

$\dot{U}_e$ → Internal elastic energy

$\dot{U}_p$ → Internal plastic energy

$\dot{U}_k$ → kinetic energy

$\dot{U}_\Gamma$ → fracture energy for breaking atomic bond

$\underbrace{\dot{U}_e + \dot{U}_p}_{\text{internal energy}}$

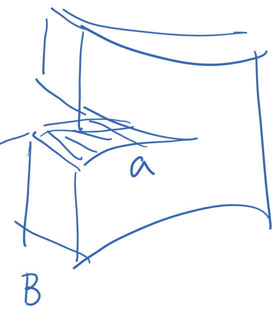
Simplification:
1. Ignore plastic part (acceptable for more brittle fracture)
2. Ignore dynamic effect -> Kinetic energy (rate) is almost zero → quasi-static

$$\dot{W} = \dot{U}_e + \dot{U}_\Gamma \rightarrow \dot{U}_\Gamma = -(\dot{U}_e - \dot{W})$$

potential ← $\Pi = U_e - W$

potential energy $\Pi = U_e - W$

$$\frac{dU_\Gamma}{dt} = -\frac{d\Pi}{dt}, \quad \Pi = U_e - W$$



$$\frac{d(\cdot)}{dt} = \frac{d(\cdot)}{da}\left(\frac{da}{dt}\right)$$

$\frac{da}{dt}$ → crack speed

da ← crack length

$$\frac{1}{B}\cdot\frac{dU_\Gamma}{da}\cancel{V_c} = -\frac{1}{B}\frac{d\Pi}{da}\cancel{V_c} \rightarrow \frac{\partial U_\Gamma}{\partial(aB)} = -\frac{1}{B}\frac{\partial \Pi}{\partial a}$$

B → thickness

$\partial(aB)$ → area

$$\underbrace{\frac{\partial U_\Gamma}{\partial A}}_{= 2\gamma_s} = -\frac{1}{B}\frac{\partial \Pi}{\partial a}$$

$$\boxed{2\gamma_s = -\frac{1}{B}\frac{\partial \Pi}{\partial a}} \quad (*)$$

$2\gamma_s$ → material resistance

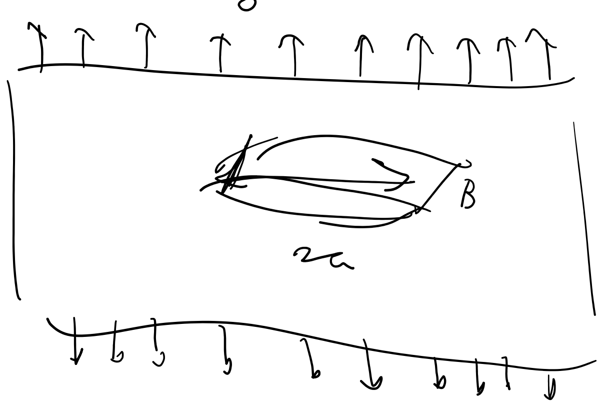
$-\frac{1}{B}\frac{\partial \Pi}{\partial a}$ → energy release rate

Π potential energy $= U_e - W$

Use equation (*) to explain why strength is much lower than theoretical strength

From Inglis solution

2a crack in an infinite domain



$A = (2a)B$

$$\Pi(a) - \Pi(0) = \frac{-\pi a^2 \sigma^2 B}{E}$$

$$\frac{d\Pi}{dA} = \frac{1}{B}\frac{d\Pi}{d2a} = -\frac{\pi a \sigma^2}{E}$$

If the crack can propagate $-\frac{d\Pi}{dA} \geq 2\gamma_s$

the minimum far field stress needed is $\sigma = \sigma_f$ (material strength)

$(\quad \pi a \sigma_0^2)$

$$-\left(-\frac{\pi a \sigma_f^2}{E}\right) = 2\gamma_s \rightarrow$$

(last time)

Energy argument	Stress-concentration argument
$\sigma_f = \sqrt{\dfrac{E\gamma_s}{4a}}$	$\sigma_f = \sqrt{\dfrac{E\gamma_s}{\frac{\pi}{2} a}}$
$\sigma_f = .5\sqrt{\dfrac{E\gamma_s}{a}}$	$\sigma_f \approx .8\sqrt{\dfrac{E\gamma_s}{a}}$

Both approaches predict the same formula for the strength, except the factor upfront.

Recall $\sigma_c = \sqrt{\dfrac{E\gamma_s}{x_0}} \gg \sqrt{\dfrac{E\gamma_s}{4a}} = \sigma_f$

How can we incorporate plasticity (with some approximation)

Energy equation for ductile materials

Plane stress

$$\sigma_c = \sqrt{\frac{2E\gamma_s}{\pi a}}$$ Griffith (1921), ideally brittle solids

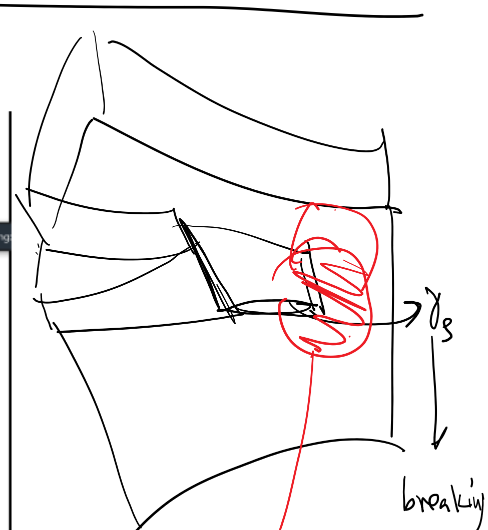
modification

$$\boxed{\sigma_c = \sqrt{\frac{2E(\gamma_s + \gamma_p)}{\pi a}}}$$ Irwin, Orowan (1948), metals

γ_p plastic work per unit area of surface created

$\gamma_p \gg \gamma_s$

$\boxed{\gamma_p \approx 10^3 \gamma_s \quad \text{(metals)}}$



$\gamma_s \rightarrow \gamma_s + \gamma_p$ → plastic energy dissipated around the

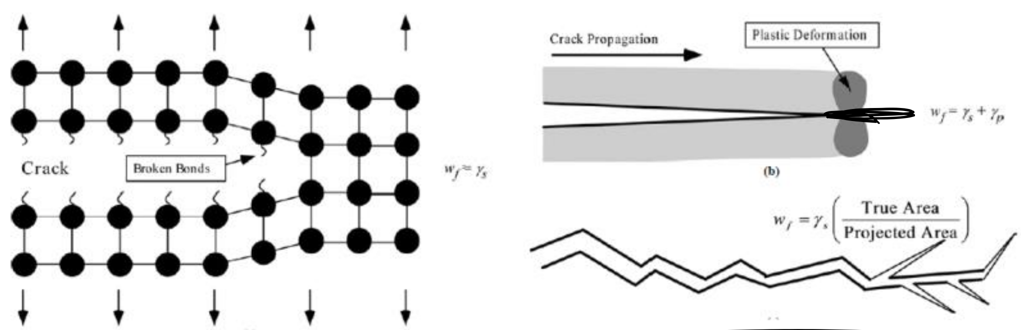
$\gamma_s \to \gamma_s + \gamma_p$ dissipated around the crack tip

γ_s ↓ breaking atomic bonds

Generalization of Energy equation

$$\sigma_f = \sqrt{\frac{2E w_f}{\pi a}}$$

- w_f: Fracture energy from plastic, viscoelastic, or viscoplastic effects
- w_f can also be influenced by crack meandering and branching
- Caution: If nonlinear displacement regions are large enough this equation is not accurate as it is based on linear elastic solution ($\Pi = \Pi_0 - \frac{\pi\sigma^2 a^2 B}{E}$)



Energy release rate

here ↓ per unit area of crack growth

Irwin 1956 $\boxed{G \equiv -\frac{d\Pi}{dA}}$ a.k.a Crack extension force
Crack driving force

when the crack grows $-\frac{d\Pi}{dA} \geq \underbrace{2\gamma_s}_{R \text{ resistance}}$

$G = -\frac{d\Pi}{dA}$

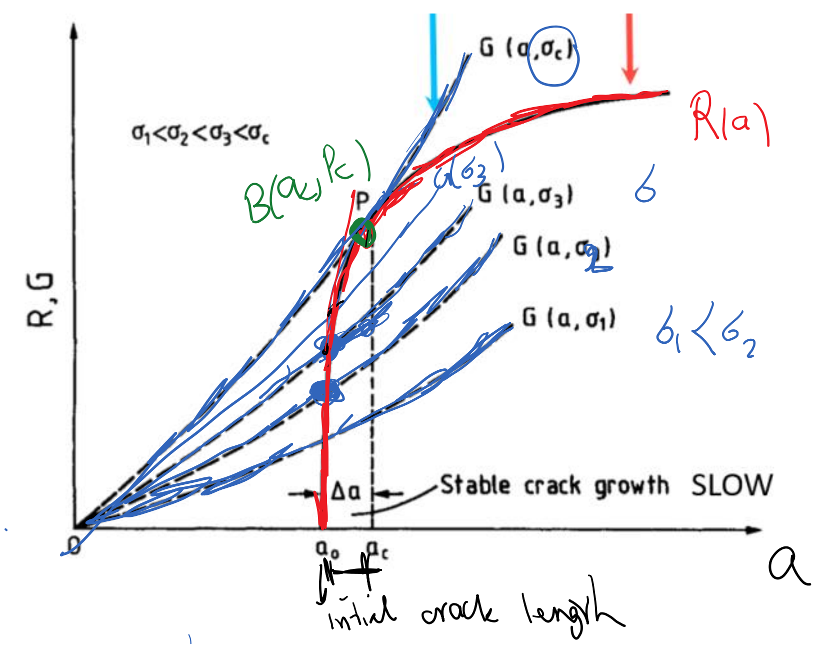
$< R = 2\gamma_s$ crack won't propagate

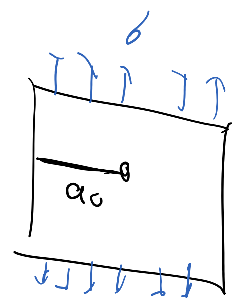
$= R$ propagate

$> R$ " in dynamic mode

$G > R$ " in dynamic mode

G, R plots (curves)



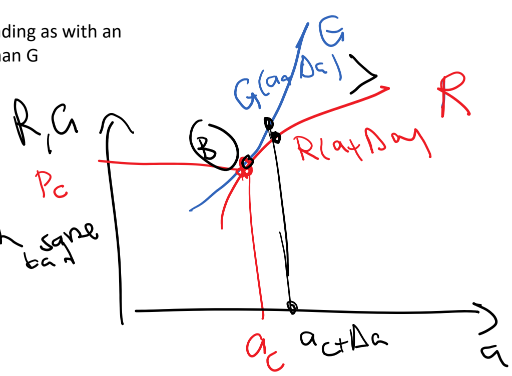


$\sigma_1 < \sigma_2 < \sigma_3 < \sigma_4 < \sigma_c$

σ_3: crack propagation initiation stage: $G = R$

The crack will not grow with this sustained loading as with an increment of crack length R becomes larger than G

σ_4 same



for continued propagation with same load

$G(a+\Delta a) > R(a+\Delta a)$

initially

$G(a) = R(a)$

$$\lim_{\Delta a \to 0} \frac{G(a+\Delta a) - G(a)}{\Delta a} > \frac{R(a+\Delta a) - R_a}{\Delta a}$$

$$\boxed{\frac{dG}{da} > \frac{dR}{da}}$$ condition for unstable crack growth (beyond σ_c, P_c)

$$\boxed{\frac{dG}{da} \geq \frac{dR}{da}}$$ condition for unstable crack growth (beyond $\sigma_{critical}$)

$G = R$

$$\boxed{\frac{dG}{da} < \frac{dR}{da}}$$ stable crack growth, loading should increase for the crack to grow

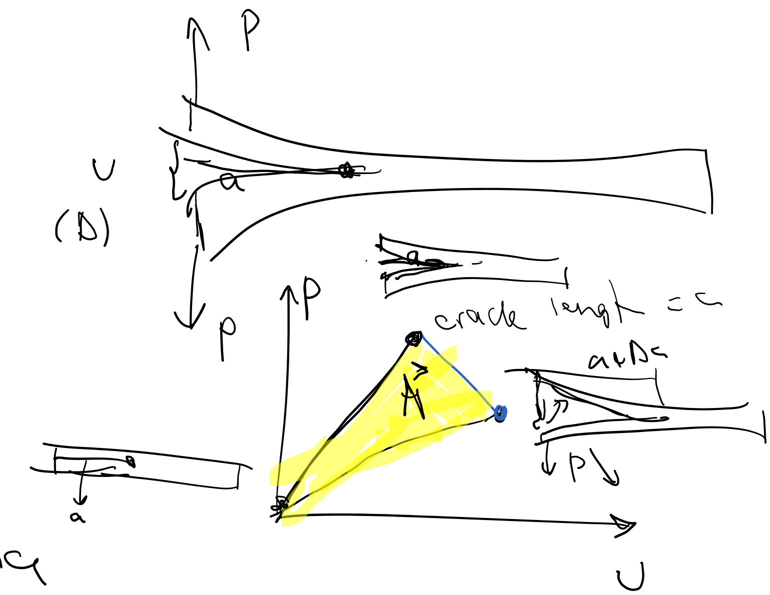
$\sigma_g \to G_c$

How to calculate G?

1. For P-Delta (u) systems

$$G = -\frac{d\Pi}{dA} = \frac{\text{shaded area}}{B} \quad (\Delta)$$

$$= \frac{P^2}{2B}\frac{dC}{da}$$ → Compliance



2. Using Stress intensity factor (SIF)

$$G = \frac{K_I^2 + K_{II}^2}{E'}$$

E' → elastic modulus or ...