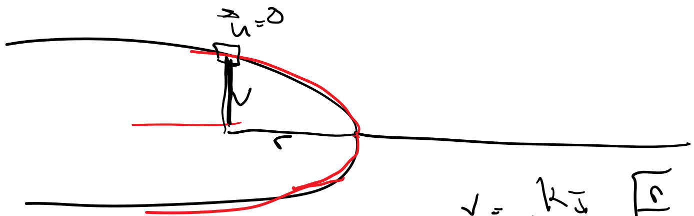
2022/09/22

Thursday, September 22, 2022 2:27 PM

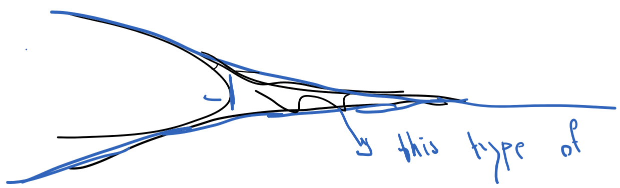
From last time, we were discussing the displacement profile



$$v = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\,(\kappa+1)$$

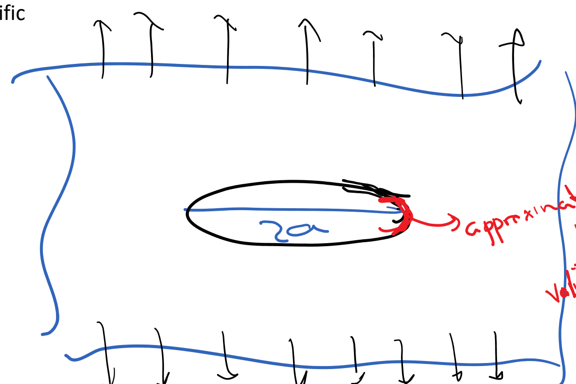
$$\underline{r = Av^2}$$

The real crack opening is like:

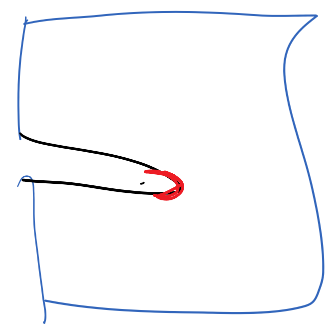


this type of opening removes $\sigma \propto \frac{1}{\sqrt{r}}$ singularity

We want to emphasize the this parabolic opening (LEFM) theory, is the asymptotic opening and the actual crack opening is prob lem specific



approximated by $v \propto \sqrt{r}$ valid for any crack tip



exact crack opening for mid-crack in an infinite domain

$$y = 0, -a \leq x \leq a$$

$$2\mu v = \frac{\kappa+1}{2}\mathrm{Im}\tilde{Z} - y\mathrm{Re}Z \longrightarrow v = \frac{\kappa+1}{4\mu}\mathrm{Im}\tilde{Z}$$

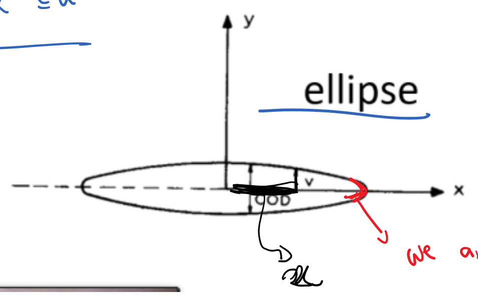
$$Z(z) = \frac{\sigma x}{\sqrt{x^2 - a^2}} \longrightarrow \tilde{Z}(z) = \sigma\sqrt{x^2 - a^2}$$

$$-a \leq x \leq a \quad i = \sqrt{-1} \longrightarrow \tilde{Z}(z) = i(\sigma\sqrt{a^2 - x^2})$$

$$\boxed{v = \underbrace{\frac{\kappa+1}{4\mu}\sigma}_{A}\sqrt{a^2 - x^2}} \rightarrow \left(\frac{v}{A}\right)^2 + x^2 = a^2$$

ellipse

Crack Opening Displacement



we approximated this by a parabola

$$\mathrm{COD} = 2v = \frac{\kappa+1}{2\mu}\sigma\sqrt{a^2 - x^2}$$

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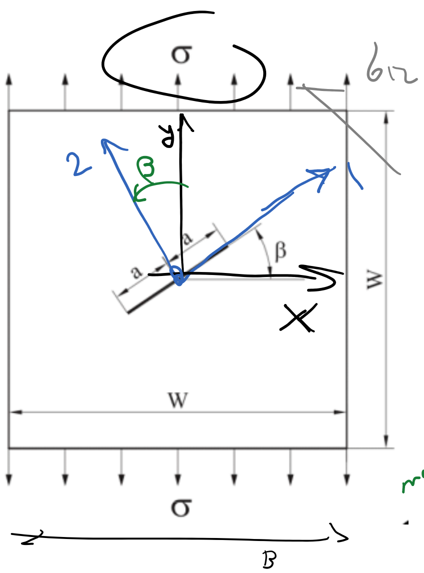
$\hookrightarrow = G = \frac{E}{2(1+\nu)}$

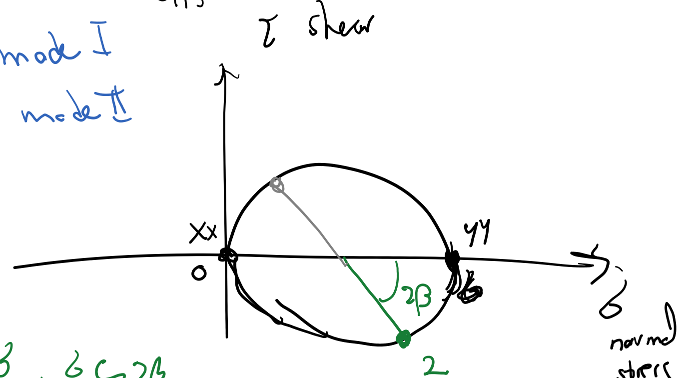
Calculation of stress intensity factors

$a \ll W, B \longrightarrow$ we can use infinite domain SIFs



$\sigma_{22} \longrightarrow$ mode I

$\sigma_{12} \longrightarrow$ mode II



mode I $\swarrow$ $\sigma_{22} = \frac{\sigma}{2} + \frac{\sigma}{2}\cos 2\beta$

$\sigma_{11} = \frac{\sigma}{2} - \frac{\sigma}{2}\cos 2\beta$

$\sigma_{12} = \frac{\sigma}{2}(\sin 2\beta) = \sigma \sin\beta\cos\beta$

Mode II

max @ $\beta = 0$

$K_I = \sqrt{\pi a}$ (far field normal stress) $= \sqrt{\pi a} \cdot \sigma_{22} = \sqrt{\pi a}\,\sigma\cos^2\beta$

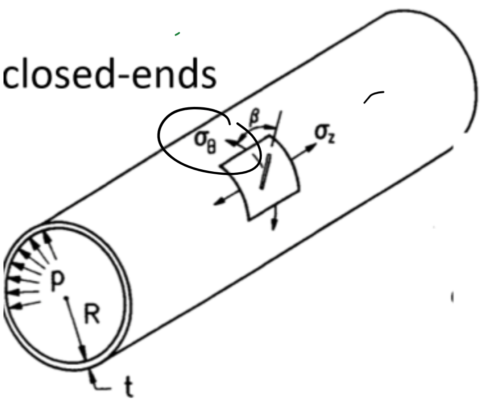
$K_{II} = \sqrt{\pi a}$ (" " shear ") $= \sqrt{\pi a}\,\sigma_{12} = \sqrt{\pi a}\,\sigma\sin\beta\cos\beta$

$\hookrightarrow$ max @ $\beta = 45°$

- A key assumption is that the infinite domain SIF formulas are used. If the crack is long enough, we need to take the domain size into effect.

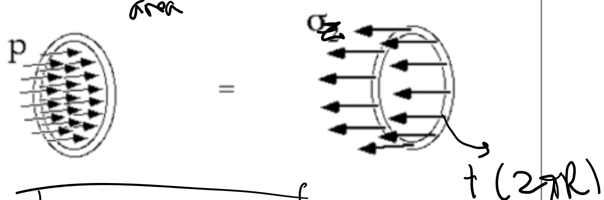
Cylindrical pressure vessel with an inclined through-thickness crack

closed-ends

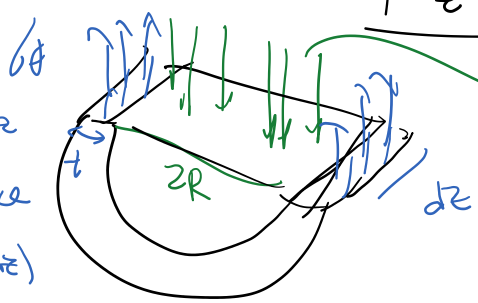


$\frac{R}{t} \geq 10$ thin-walled pressure

$$\underbrace{(\pi R^2)}_{\text{area}} p = (2\pi R t)\sigma_z$$



$$\boxed{\sigma_z = \frac{1}{2}\frac{R}{t}p}$$



Area $= 2t\,dz$

tensile force

$\sigma_\theta\,(2t\,dz)$

Compressive force

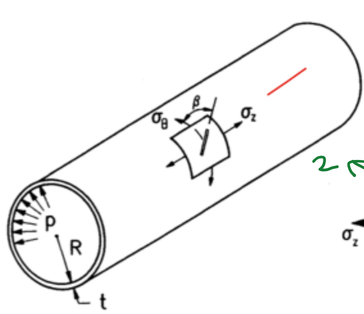
$= (\underbrace{2R\,dz}_{\text{area}})\,p$

$= \sigma_\theta\,(2t\,dz)$

$$\boxed{\sigma_\theta = \frac{R}{t}p = 2\sigma_z}$$

We want to calculate SIFs for a short crack at angle β on the pressure vessel.

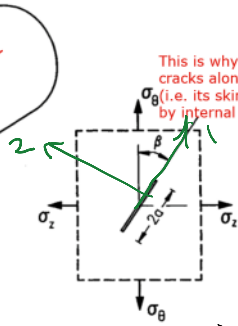
Cylindrical pressure vessel with an inclined through-thickness crack



Equilibrium

$\sigma_\theta = 2\sigma_z$ $= p\frac{R}{t}$

This is why an overcooked hotdog usually cracks along the longitudinal direction first (i.e. its skin fails from hoop stress, generated by internal steam pressure).



$\sigma_{22} = \sigma_{zz}\cos^2\beta + \sigma_{\theta\theta}\sin^2\beta$

$\sigma_{12} = -\sigma_{zz}\sin\beta\cos\beta + \sigma_{\theta\theta}\sin\beta\cos\theta$

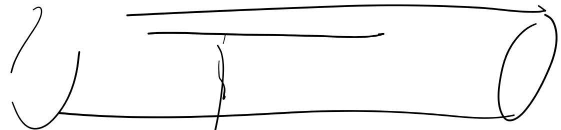
$\rightarrow$ $\sigma_{22} = \frac{PR}{2t}(1+\sin^2\beta)$

$\sigma_{12} = \frac{PR}{2t}\sin\beta\cos\theta$ $\Big\} \rightarrow$

$K_I = \sigma_{22}\sqrt{\pi a} \longrightarrow$ max @ $\beta = \frac{\pi}{2}$

$K_I = \sigma_{22}\sqrt{\pi a} \rightarrow$ max @ $\beta = \frac{\pi}{2}$

$K_{II} = |\sigma_{12}|\sqrt{\pi a}$



↳ preferred crack dir on [illegible]

How to calculate SIF:

- SIF handbook
- Computational methods (e.g. FEM)
- Experimental methods (e.g. photo sensitive material)

Computation of SIFs

- Analytical methods (limitation: simple geometry)

  - superposition methods
  - weight/Green functions
- Numerical methods (FEM, BEM, XFEM)

  numerical solutions -> data fit -> **SIF handbooks**
- Experimental methods
  - photoelasticity

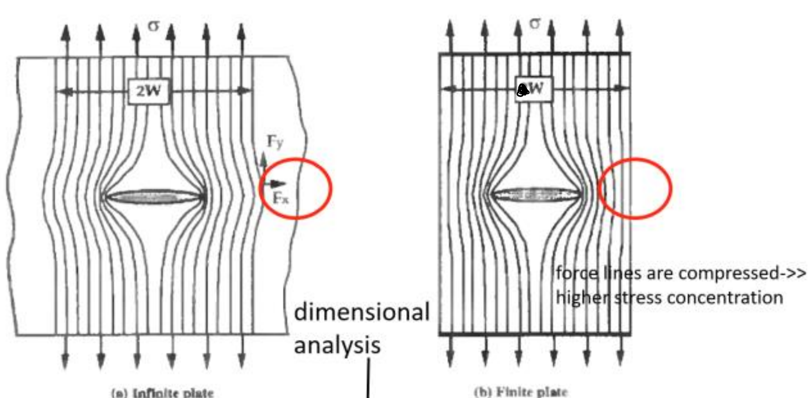
SIF for finite size samples

Exact (closed-form) solution for SIFs: simple crack geometries in an **infinite** plate.

Cracks in finite plate: influence of external boundaries cannot be neglected -> generally, no exact solution

SIF for finite size samples

K_I < K_I



(a) Infinite plate (b) Finite plate

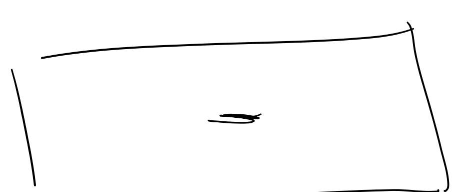
geometry/correction factor [-]

$$K_I = f(a/W)\sigma\sqrt{\pi a} \qquad a \ll W : f(a/W) \approx 1$$

For this mid crack example we have an analytical solution

$$f(a,W) = \sqrt{\frac{1}{\cos\frac{\pi a}{W}}} = \sqrt{\sec\frac{\pi a}{W}}$$

$$a \ll W \rightarrow f(a,W) \rightarrow 1$$

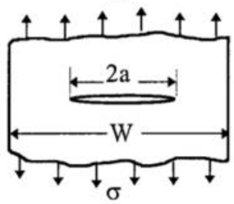


$a \ll W \longrightarrow f(a,W) \longrightarrow 1$

$a \rightarrow W \qquad \sec\frac{\pi a}{W} = \sec\frac{\pi \frac{W}{2}}{W} \longrightarrow 0$

$f(a,W)$ & $K_I \nearrow \infty$

2. Centre crack in a strip of finite width



$$K_I = \sqrt{\sec\frac{\pi a}{W}}\,\sigma\sqrt{\pi a}$$

secant function

$$\sec\theta = \frac{1}{\cos\theta}$$

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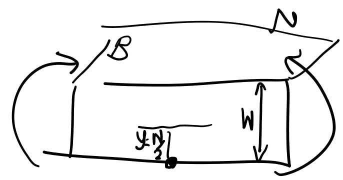
Typical convention in providing SIF correction factors

Ex

σ_{max} w/o crack $\qquad \sigma_{max} = \frac{My}{EI}$

for $y = \frac{W}{2}$

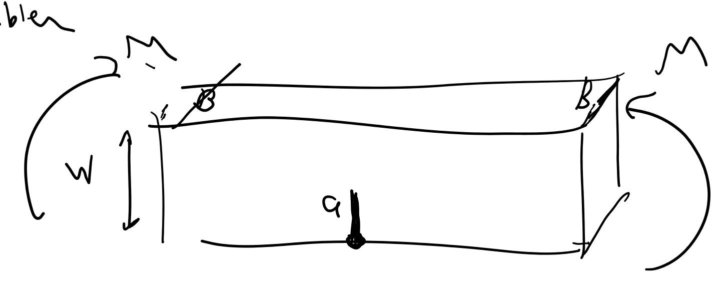


$$\sigma_{max} = \frac{M\frac{W}{2}}{E\frac{1}{12}BW^3} = \frac{6M}{BW^2}$$

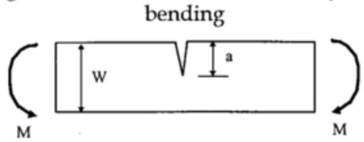
$$K_I = f\left(\frac{a}{W}\right)\sigma_{max}\sqrt{\pi a}$$

correction factor depending on geometry

maximum tensile (mode I) stress for this problem



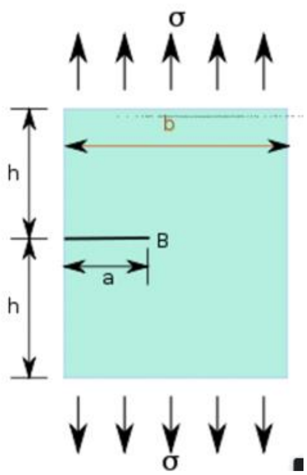
5. Edge crack in a beam of width *B* subjected to bending



$$K_I = f\left(\frac{a}{W}\right)\sigma\sqrt{\pi a} \quad \text{where } \sigma = \frac{6M}{BW^2}$$

a/W	f(a/W)
0.1	1.044
0.2	1.055

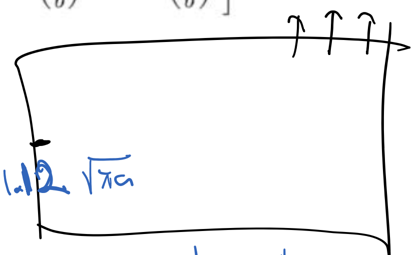
0.3	1.125
0.4	1.257
0.5	1.500
0.6	1.915



$h/b \geq 1$ and $a/b \leq 0.6$

$$K_I = \sigma\sqrt{\pi a}\left[1.12 - 0.23\left(\frac{a}{b}\right) + 10.6\left(\frac{a}{b}\right)^2 - 21.7\left(\frac{a}{b}\right)^3 + 30.4\left(\frac{a}{b}\right)^4\right]$$

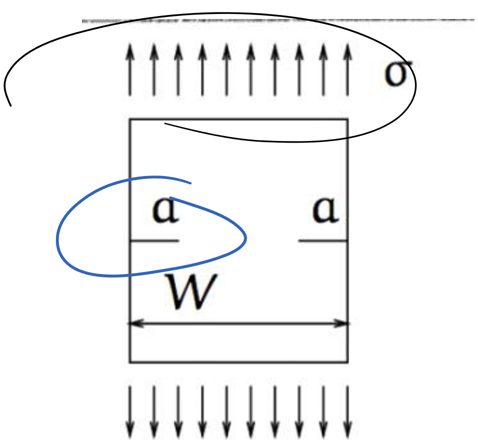
what about $a \ll b$



$K_I = 1.12\sqrt{\pi a}$

for a very short crack on the edge

12% higher than a very short mid crack

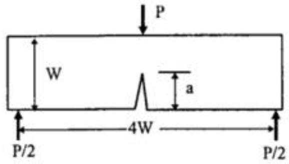


K_I $\quad \frac{a}{W} \to 0 \quad$ $1.12\,\sigma\sqrt{\pi a}$

$$K_I = \sigma\sqrt{a}\left[1.12\sqrt{\pi} + 0.76\frac{a}{W} - 8.48\left(\frac{a}{W}\right)^2 + 27.36\left(\frac{a}{W}\right)^3\right]$$

$$\approx 1.12\sigma\sqrt{\pi a}$$

9. Single-edge notch bend (SENB), thickness B

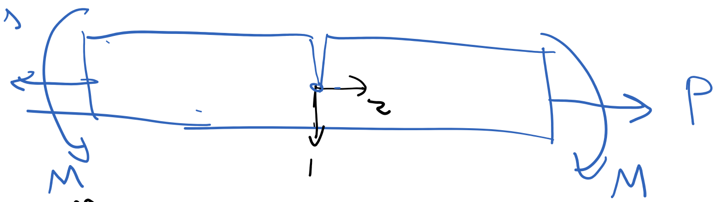


$B = W/2$

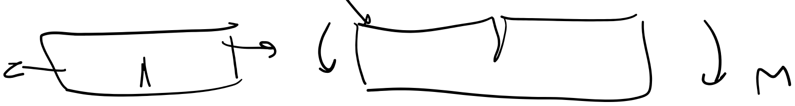
$$K_I = Y\frac{4P\sqrt{\pi}}{B\sqrt{W}}$$

$$Y = 1.63\left(\frac{a}{W}\right)^{1/2} - 2.6\left(\frac{a}{W}\right)^{3/2} + 12.3\left(\frac{a}{W}\right)^{5/2} - 21.3\left(\frac{a}{W}\right)^{7/2} + 21.9\left(\frac{a}{W}\right)^{9/2}$$

Use of tables



$$\sigma_{ij} = \sigma_{ij}^{P} + \sigma_{ij}^{M}$$



$$= K_I^P \frac{1}{\sqrt{2\pi r}} f_{ij}^{I}(\theta) + K_I^M \frac{1}{\sqrt{2\pi r}} f_{ij}^{I}(\theta)$$

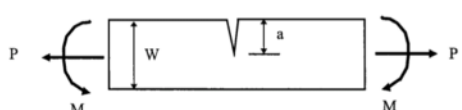
$$= \left(K_I^P + K_I^M\right) \frac{1}{\sqrt{2\pi r}} f_{ij}^{I}(\theta)$$

$$K_I = K_I^P + K_I^M$$

Determine the stress intensity factor for an edge cracked plate subjected to a combined tension and bending.

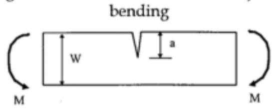
$a/W = 0.2$

B thickness



$$K_I = K_I^{bending} + K_I^P, \quad K_I^{bending} = 1.055 \frac{6M}{BW^2}\sqrt{\pi a}$$

5. Edge crack in a beam of width B subjected to bending



$K_I = f\left(\frac{a}{W}\right)\sigma\sqrt{\pi a}$ where $\sigma = \frac{6M}{BW^2}$

a/W	f(a/W)
0.1	1.044
0.2	1.055
0.3	1.125

look for table

$$(1.12)\left(\frac{P}{WB}\right)\sqrt{\pi a}$$

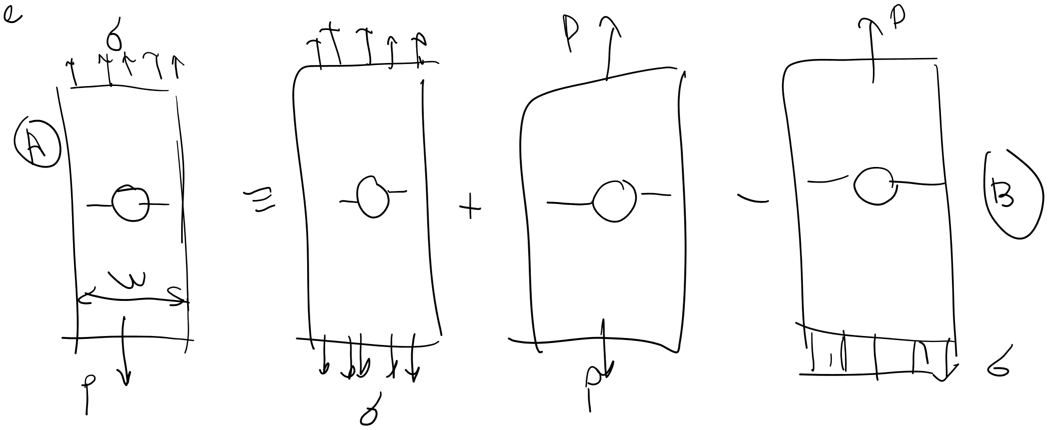
Actual calculation for $\frac{a}{W} = .2$ is higher than 1.12

use (A)

higher than 1.12

Use A

Example



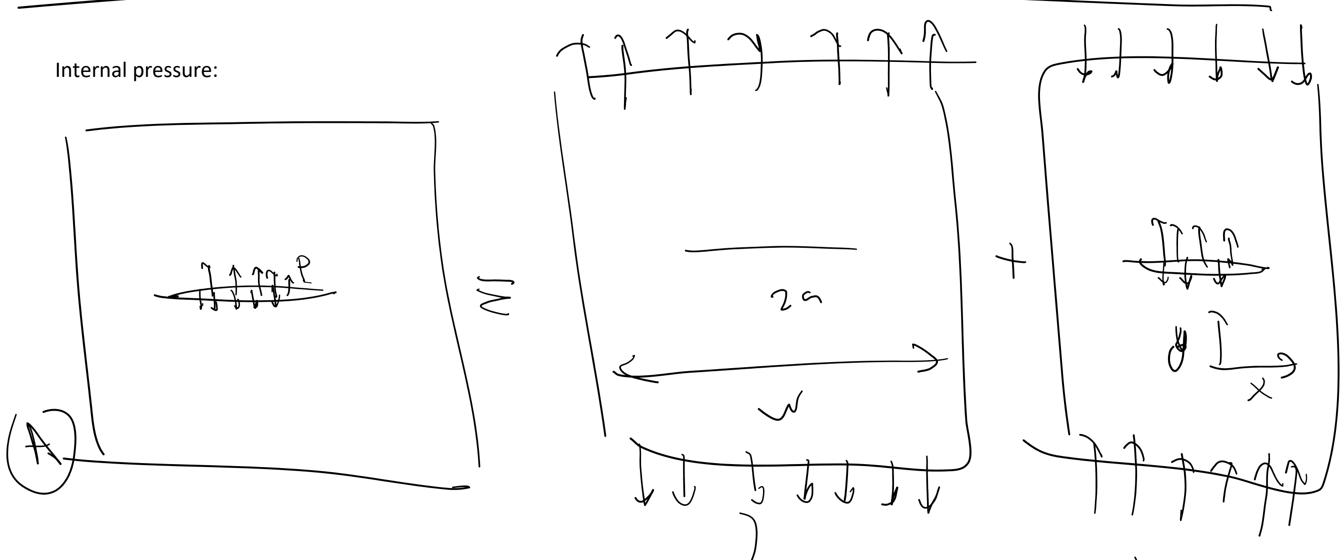
$P = \sigma W$

$$K_I^A = K_I^\sigma + K_I^P - K_I^B \;\rightarrow\; K_I^A = \frac{K_I^\sigma + K_I^P}{2}$$

circle is centered in the domain

$K_I^B = K_I^A$

Internal pressure:



$\sigma_{yy} = -P$

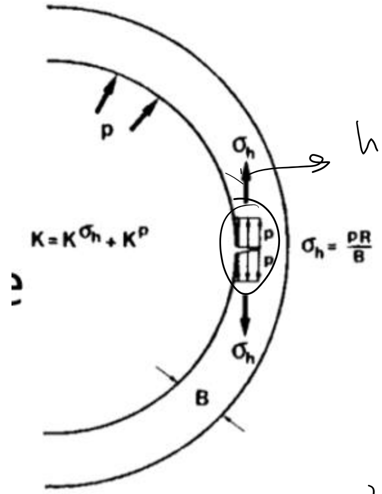
$$K^I = K^I = \sqrt{\frac{1}{\cos\frac{\pi a}{W}}}\,\sqrt{\pi a} \qquad K_I = 0$$

Example of internal pressure

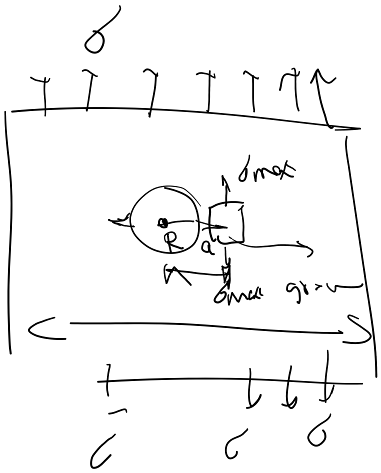
$a \ll t$

$K_I \quad \sigma(1.12)\left(\frac{pR}{B}\right)\sqrt{\pi a}$

correction



hoop stress



3σ

$K_I = (1.12)\sqrt{\pi a}\ \sigma_{max} \approx 3.36\sqrt{\pi a}$

correction factor

a very small

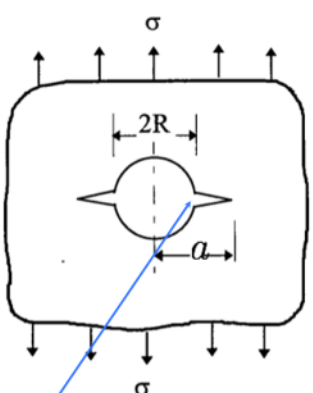
$a \ll R, W$

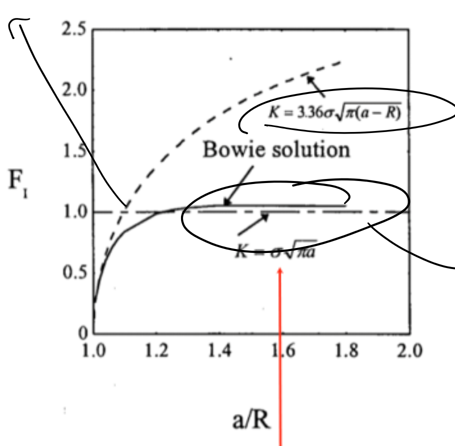
$a \gg R$ but still $a \ll W$

$\sqrt{\pi(R+a)}\ \sigma \approx \left(\sqrt{\pi a}\right)\sigma$

$R \ll a$

very good $a \ll R$





other solution

$R \ll a$

3σ edge crack

hole as a part of the crack

$$K_I = 1.12(3\sigma)\sqrt{\pi(a-R)} = 3.36\sigma\sqrt{\pi a}\sqrt{1-\frac{R}{a}} = 3.36\sqrt{1-\frac{1}{a/R}}\sigma\sqrt{\pi a}$$