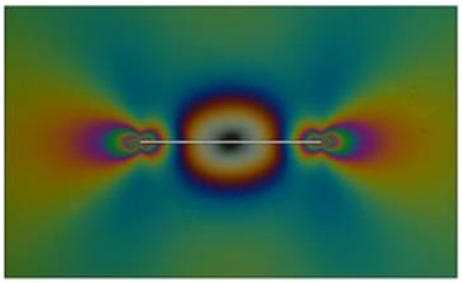
2022/09/27

Tuesday, September 27, 2022 2:17 PM

Photoelasticity

Wikipedia

Photoelasticity is an experimental method to determine the stress distribution in a material. The method is mostly used in cases where mathematical methods become quite cumbersome. Unlike the analytical methods of stress determination, photoelasticity gives a fairly accurate picture of stress distribution, even around abrupt discontinuities in a material. The method is an important tool for determining critical stress points in a material, and is used for determining stress concentration in irregular geometries.

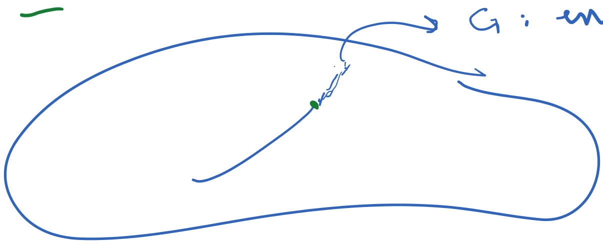


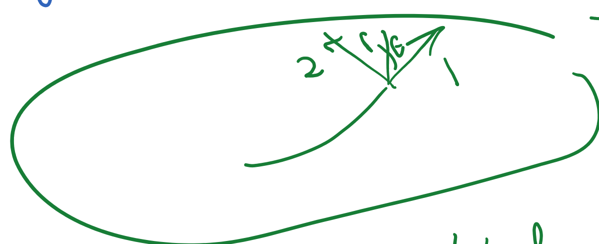
Relation between G and K

G : energy release rate
= energy released per unit area of crack advance

K : stress intensity factor

G : energy / global





K : stress/stress / local

$$\sigma_{ij}(r,\theta) = \frac{K_I}{\sqrt{2\pi r}} f_{ij}^{I}(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} f_{ij}^{II}(\theta) + \frac{K_{III}}{\sqrt{2\pi r}} f_{ij}^{III}(\theta)$$

Relating G and K
Let's look at their dimensions:

$$[G] = \frac{\overbrace{[E]}^{\text{energy}}}{\underbrace{[A]}_{\text{area}}} = \frac{\overbrace{[F]}^{\text{force}}[L]}{[L]^2} \qquad [F] = [\sigma][L]^2$$

$$[G] = [\sigma][L]$$

$[K]$

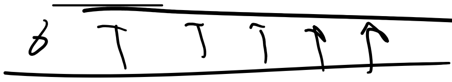
$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} \underbrace{f_{ij}^{I}(\theta)}_{\text{dimensionless}} \rightarrow [\sigma] = \frac{[K]}{[L]^{1/2}} \rightarrow [K] = [\sigma][L]^{1/2}$$

$$[K]^2 = [\sigma]^2[L] \rightarrow \frac{[G]}{[K]^2} = \frac{1}{[\sigma]}$$

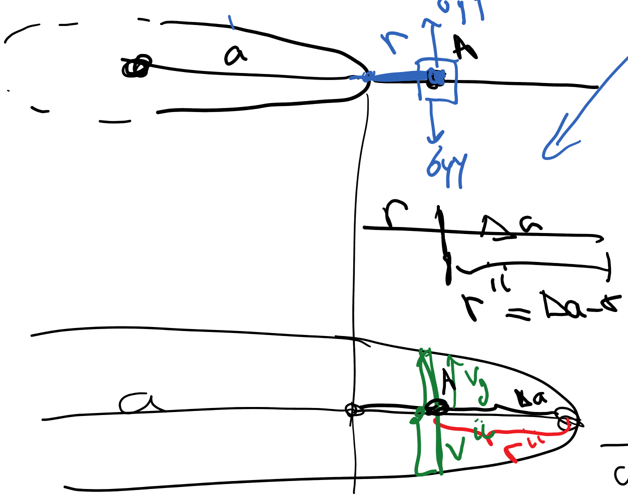
$$[K]^2 = [\sigma]^2[L], \quad [G] = [\sigma][L] \;\rightarrow\; \frac{[G]}{[K]^2} = \frac{1}{[\sigma]}$$

$[\sigma]$ → stress, strength, stiffness

we'll find out $\boxed{G \propto \frac{K^2}{E}}$ → E: elastic modulus

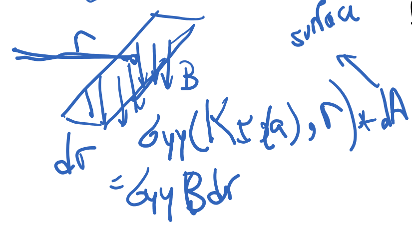


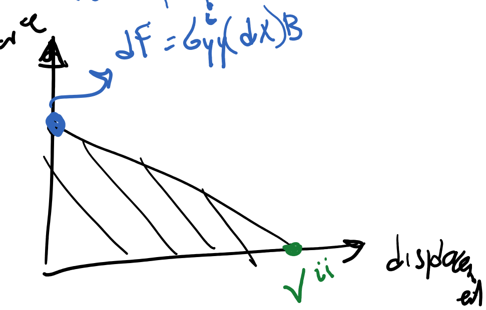
$K_I - G$ relation — Irwin (mode I)



B: width

state i — before crack propagation





state ii — after crack advances by Δa

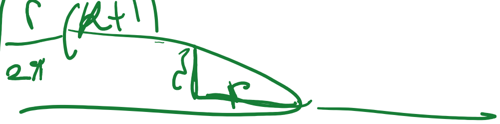
work done = energy released for $dr * B$ segment

$$= \frac{1}{2}(dF_i)(V_{ii})$$

$$(dF)_i = (\sigma_{yy})_i (B\,dr) = \underbrace{\frac{K_I(a)}{\sqrt{\pi r}}}_{(\sigma_{yy})_i} B\,dr$$

$$(V)_{ii} = 2 V_y(A) = 2 \cdot \frac{K_I(a+\Delta a)}{2\mu}\sqrt{\frac{r^{ii}}{2\pi}}\,(\kappa+1), \qquad r^{ii} \rightarrow \Delta a - r$$

$$V_y = \frac{K}{2\mu}\sqrt{\frac{r}{2\pi}}\,(\kappa+1)$$



$$\underbrace{d\mathcal{E}}_{\text{change of energy}} = \frac{1}{2}\,\frac{K_I(a)\,B\,dr}{\sqrt{\pi r}} \cdot 2\,\frac{K_I(a+\Delta a)}{2\mu}\sqrt{\frac{\Delta a - r}{2\pi}}\,(\kappa+1)$$

$$G = \frac{\mathcal{E}}{B\Delta a} = \lim \frac{\int d\mathcal{E}}{B\,\Delta a} = \lim \int^{\Delta a} \frac{K_I(a)\,K_I(a+\Delta a)}{\quad} \sqrt{\frac{\Delta a - r}{\quad}}\, dr$$

$$G = \frac{\mathcal{E}}{B\Delta a} = \lim_{\Delta a \to 0} \frac{J}{B\,\Delta a} = \lim_{\Delta a \to 0} \int_{r=0} \underbrace{\frac{K_I(a)\,K_I(a+\Delta a)}{4\pi\mu\,\Delta a}}_{\text{Const}} \sqrt{\frac{\Delta a - r}{r}}\,dr$$

change of area of crack

Const → goes out of integral

$\Delta a \to 0 \quad K_I(a+\Delta a) \to K_I(a)$

$$G = \frac{(\kappa+1)\,K_I^2}{8\mu}$$

μ → shear modulus $= \frac{E}{2(1+\nu)}$

$$G = \frac{K_I^2}{E'} \qquad E' = \begin{cases} E & \text{plane stress} \\ \frac{E}{1-\nu^2} & \text{" strain} \end{cases}$$

pure mode I

pure mod II

$$G = \frac{K_{II}^2}{E'}$$

in-plane

$$G = \frac{K_I^2 + K_{II}^2}{E'}$$

Mixed mode

$$\boxed{G = \frac{K_I^2}{E'} + \frac{K_{II}^2}{E'} + \frac{K_{III}^2}{2\mu}} \qquad E' = \begin{cases} \frac{E}{1-\nu^2} & \text{for plane strain} \\ E & \text{for plane stress} \end{cases}$$

Pure mode I (-> pure mode II and III)

ERR ← $G = R$ → resistance toughness

K^2 K_I^2 K_I^2 ... D ... T

$$\frac{K_I^2 + K_{II}^2}{E'} + \frac{K_{III}^2}{2\mu} = R$$

mode I

$$K_I^2 / E' = R \rightarrow$$

$K_I = \sqrt{G E'}$ general mode I

when then crack can propagate

$K_{Ic} = \sqrt{R E'}$

fracture toughness $MPa\sqrt{m}$

resistance $MPa\text{-}m$

toughness

Checking if a crack can propagate:

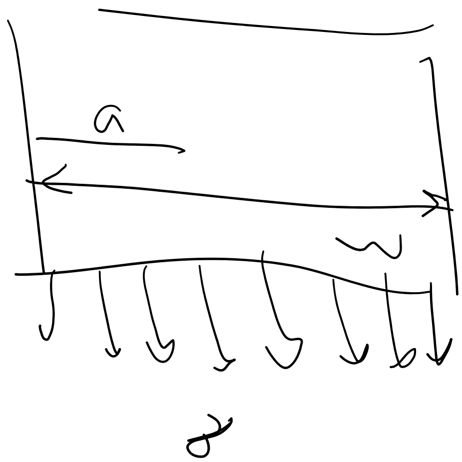
$G = R$ general (mixed mode) energy based

$K_I = K_{Ic}$ only for mode I

Typical fracture mechanics problems

$$K_I = f\left(\frac{a}{W}\right) \sigma \sqrt{\pi a} = K_{Ic}$$

- a crack length
- σ loading
- K_{Ic} material



i) a & K_{Ic} given $\sigma_{max} = \dfrac{K_{Ic}}{f(\frac{a}{W})\sqrt{\pi a}}$

ii) σ given, material given (K_{Ic}) $\rightarrow a_c = ?$

$$f\left(\frac{a}{W}\right)\sqrt{\pi a} = \frac{K_{Ic}}{\sigma}$$

generally nonlinear needs iteration

iii) design σ known, a known $\rightarrow$ K_{IC}

$$K_{IC} = f\left(\frac{a}{W}\right)\sqrt{\pi a}\ \sigma$$

Example

A cylindrical pressure vessel with closed ends has a radius $R = 1$ m and thickness $t = 40$ mm and is subjected to internal pressure p. The vessel must be designed safely against failure by yielding (according to the von Mises yield criterion) and fracture. Three steels with the following values of yield stress σ_Y and fracture toughness K_{Ic} are available for constructing the vessel.

Steel	σ_Y(MPa)	K_{Ic}(MPa $\sqrt{m}$)
A: 4340	860	100
B: 4335	1300	70
C: 350 Maraging	1550	55

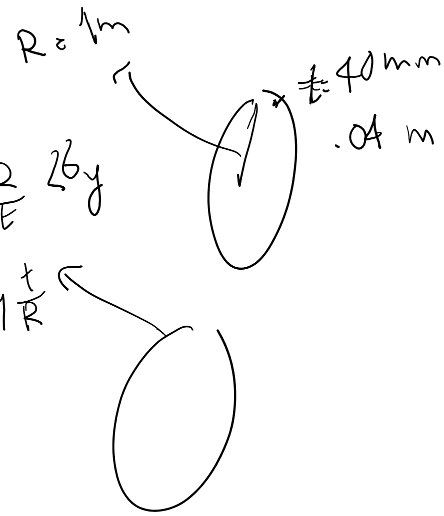
Fracture of the vessel is caused by a long axial surface crack of depth a. The vessel should be designed with a factor of safety $S = 2$ against yielding and fracture. For each steel:

(a) Plot the maximum permissible pressure p_c versus crack depth a_c;

(b) Calculate the maximum permissible crack depth a_c for an operating pressure $p = 12$ MPa;

(c) Calculate the failure pressure p_c for a minimum detectable crack depth $a = 1$ mm.

161



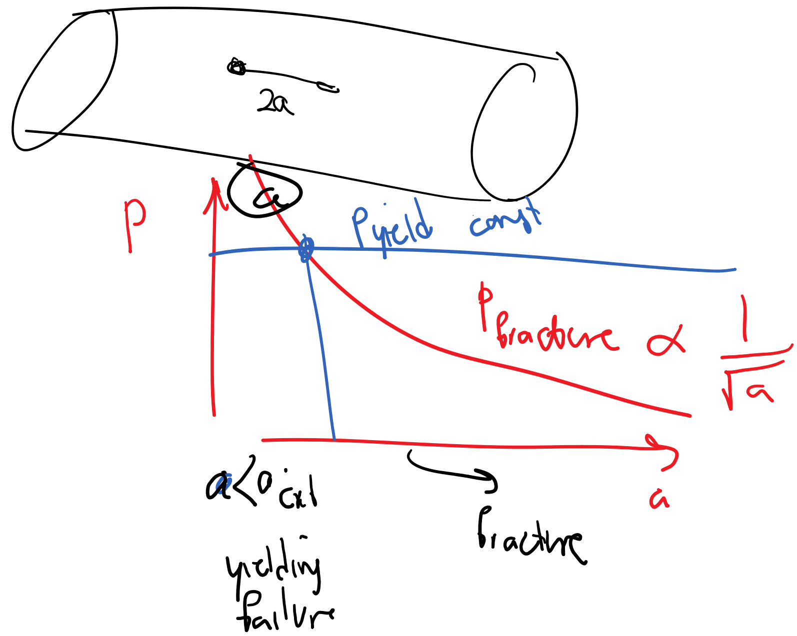
$$\sigma_{\theta\theta} = p\frac{R}{t} \leq \sigma_y$$

$$P_{yield} = \sigma_y \frac{t}{R}$$

$$K_I = \sigma_{\theta\theta}\sqrt{\pi a} = K_{IC}$$

$$p\frac{R}{t}\sqrt{\pi a} = K_{IC}$$

$$p_{fracture} = \frac{K_{IC}\cdot t}{R\sqrt{\pi a}}$$

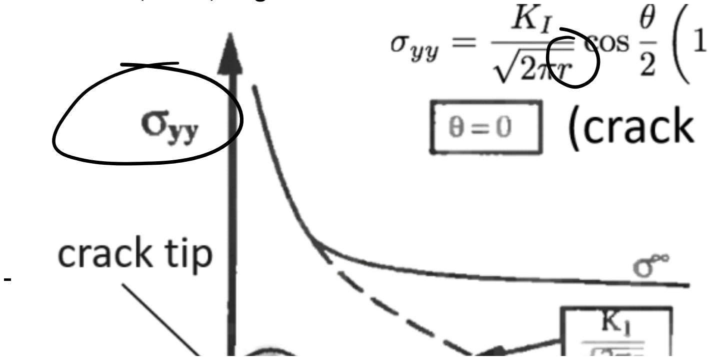


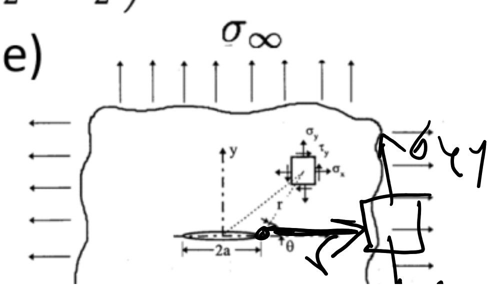
5.2. Plastic zone models

- 1D models, Irwin, Dugdale and Barenbolt models

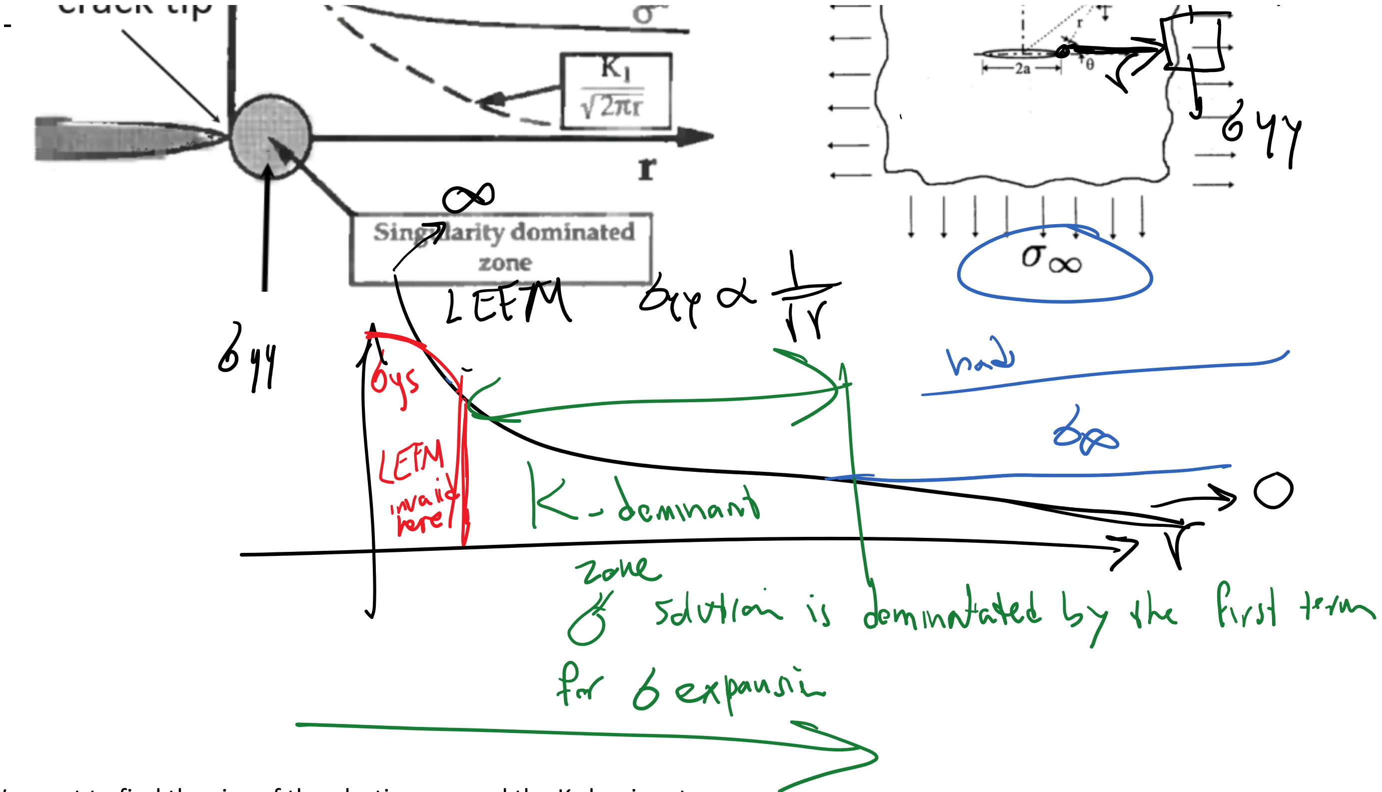
$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$$

$\boxed{\theta = 0}$ (crack plane)



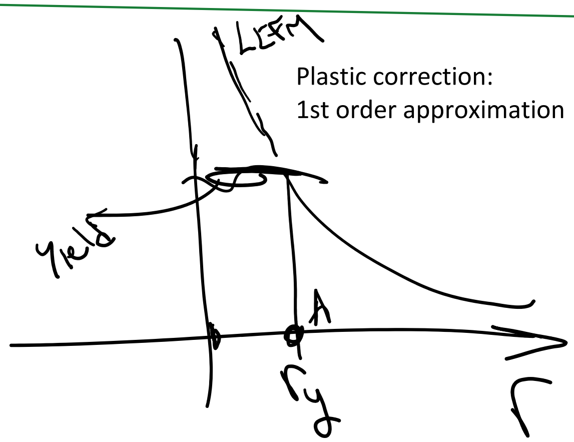


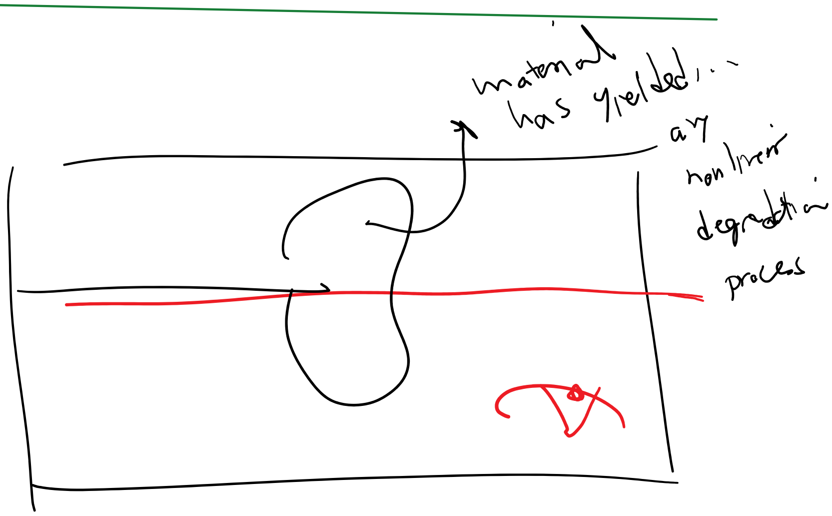
-



We want to find the size of the plastic zone and the K-dominant zone -> as one use of this, we can check the validity of LEFM

Plastic correction:
1st order approximation





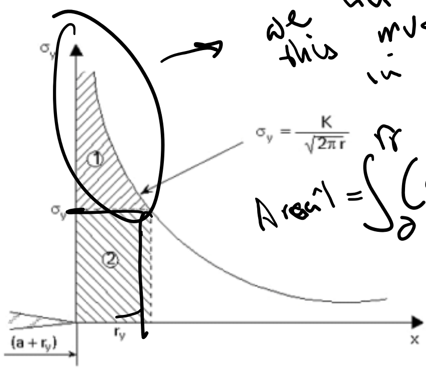
$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$$

$$\sigma_{ys} = \frac{K_I}{\sqrt{2\pi r_y}} \quad @A$$

$$2\pi r_y = \left(\frac{K_I}{\sigma_{ys}}\right)^2 \rightarrow \boxed{r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2}$$

If we want to do better than this, at least we can recover the stress force that is lost in the previous approach

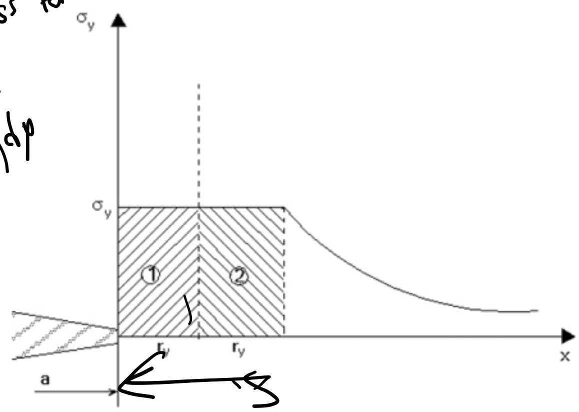
-> Irvin plastic zone size estimate (r_p)



(a) Elastic crack

we have lost this much stress force in this region

$$\text{Area1} = \int_0^{r_y} (\sigma_{yy} - \sigma_{ys})\, dr$$



(b) Real crack with plastic zone

$$r_p = 2r_y = \frac{1}{\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2$$