2022/09/29

Thursday, September 29, 2022 2:24 PM

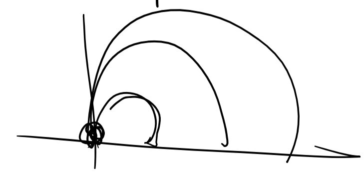
- Find out how large the stress can get on the crack line so we have yielding there.

von - Mises criterion

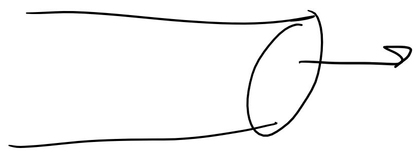
$$\underline{\sigma_e = \sigma_{vm} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}} \leq \sigma_Y$$

σ_Y → yield stress

$\sigma_1, \sigma_2, \sigma_3$ are principal stresses

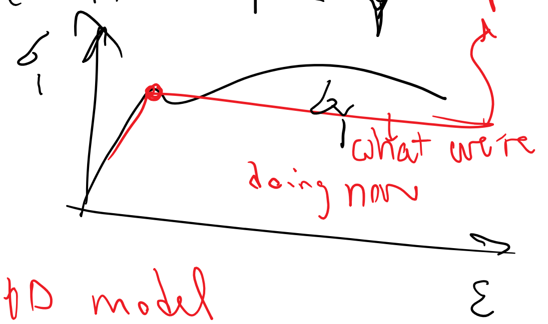


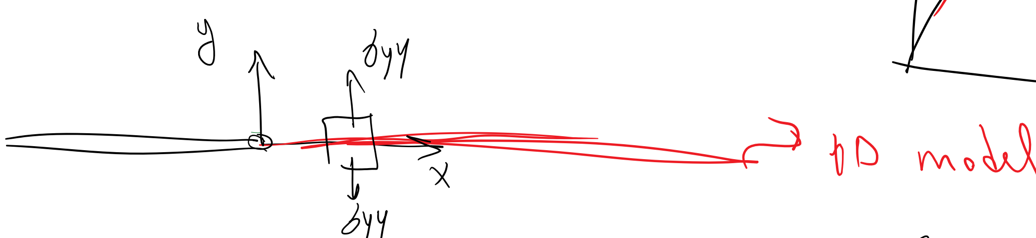
1D) tensile test



$$\sigma_e = \sigma_{vm} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - 0)^2 + (0-0)^2 + (0-\sigma_1)^2} = \sigma_1 < \sigma_Y$$

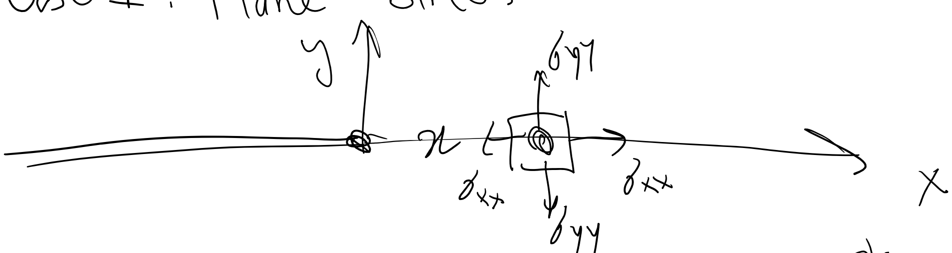
Use of this for 1D yield models





what should σ_{yy} be compared against for yielding?

Case 1: Plane stress



$$\underline{\underset{\sigma_1}{\sigma_{xx}} = \underset{\sigma_2}{\sigma_{yy}} = \frac{K_I}{\sqrt{2\pi r}}}$$

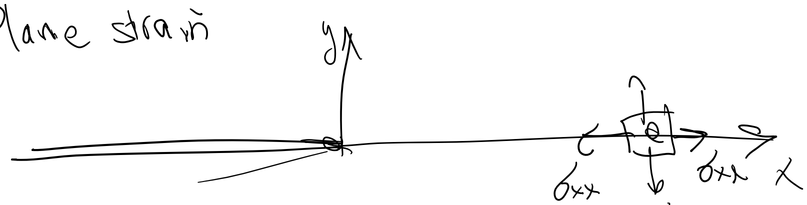
plane stress ← $\underset{\sigma_3}{\sigma_{zz}} = 0$

$$\underline{\sigma_{xy} = \sigma_{yz} = \sigma_{zx} = 0}$$

$$\sigma_{vm} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \underset{\sigma_2}{\sigma_1})^2 + (\underset{\sigma_2}{\sigma_1} - \underset{\sigma_3}{0})^2 + (\underset{\sigma_3}{0} - \sigma_1)^2} = \sigma_1 = \sigma_{yy} \leq \sigma_Y$$

Plane stress $\sigma_{yy} \leq \sigma_{ys}$ $\sigma_{ys} = \sigma_Y$

Plane strain



$\sigma_1 = \sigma_2$

$\sigma_{xx} = \sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$

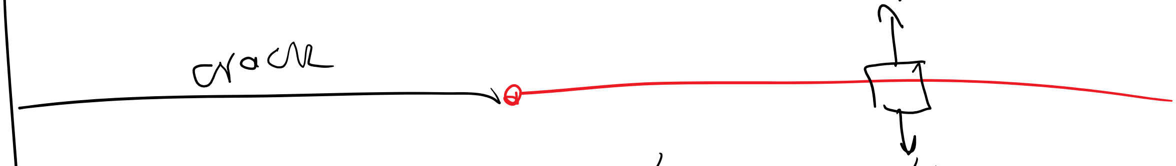
$$\epsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu(\sigma_{xx}+\sigma_{yy})\right) = 0 \longrightarrow \sigma_3 = \sigma_{zz} = \nu(\sigma_{xx}+\sigma_{yy}) = 2\nu\sigma_{yy}$$

$$\sigma_{vm} = \frac{1}{\sqrt{2}}\sqrt{(\underset{\sigma_{yy}}{\sigma_{xx}} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \underset{\sigma_{yy}}{\sigma_{xx}})^2}$$

$$= (1-2\nu)\,\sigma_{yy} \leq \sigma_Y \longrightarrow \sigma_{yy} \leq \frac{\sigma_Y}{1-2\nu} =: \sigma_{ys}$$

$$\sigma_{yy} \leq \sigma_{ys} = \frac{\sigma_Y}{1-2\nu}$$

$0 < \nu < .5$



crack

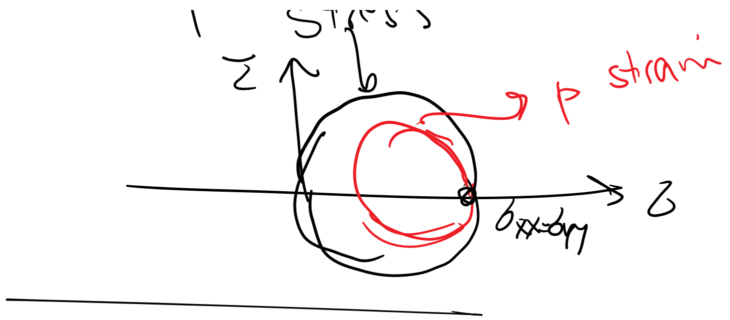
$$\sigma_{yy} \leq \sigma_{ys} = \begin{cases} \sigma_Y & \text{p. stress} \\ \dfrac{\sigma_Y}{1-2\nu} & \text{p strain} \end{cases}$$

Mohr circle

p stress

$\Sigma \uparrow$ σ

p strain $\frac{K_I}{\sqrt{2\pi r}} = \sigma_{yy} > \sigma_{ys}$ w/o correction



$$\frac{K_I}{\sqrt{2\pi r_y}} = \sigma_{yy} \geq \sigma_{ys}$$

$$\rightarrow \boxed{r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2}$$

w correction

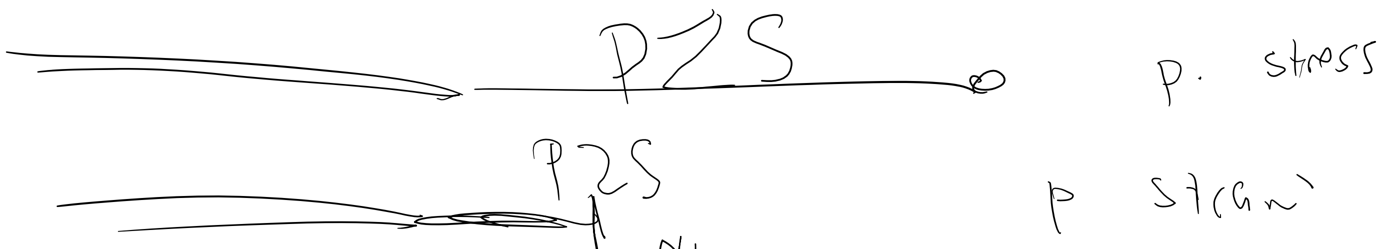
$$r_p = \frac{1}{\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2$$

Comparing Process zone sizes for plane stress & strain

$$r_p = \begin{cases} \frac{1}{\pi}\left(\frac{K_I}{\sigma_y}\right)^2 & \text{p stress} \\ \frac{1}{\pi}\left(\frac{K_I}{\sigma_y/(1-2\nu)}\right)^2 = (1-2\nu)^2\frac{1}{\pi}\left(\frac{K_I}{\sigma_y}\right)^2 & \text{p strain} \end{cases}$$

e.g. for $\nu = 0.2$ $\approx \frac{1}{3\pi}\left(\frac{K_I}{\sigma_y}\right)^2$

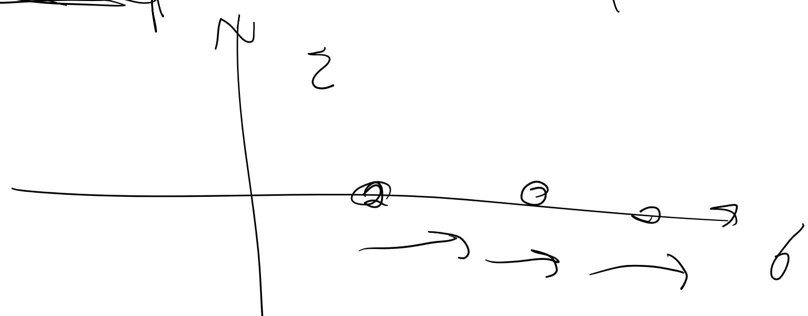
roughly 3x smaller for $\nu = 0.2$



$\sigma_1 = \sigma_2 = \sigma_3$

$\sigma_1 \uparrow$

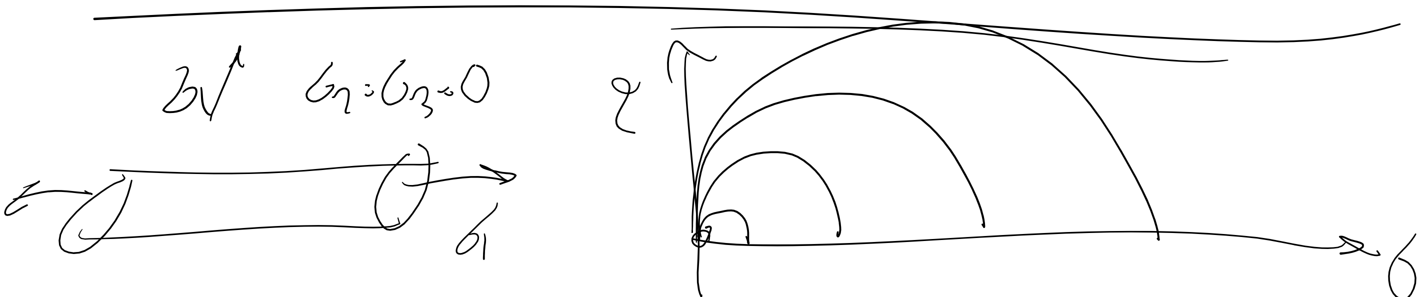
Brittle



$\sigma_m = 0$

$\sigma_{Tresca} = \tau_{max} = 0$

3D $\sigma_2 = \sigma_3 = 0$



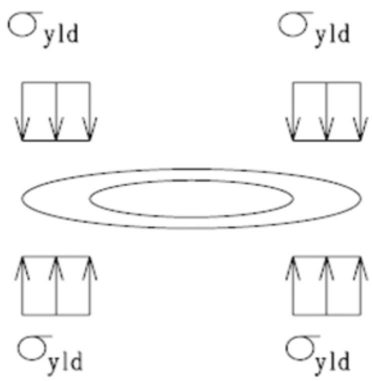
Uniaxial $\longrightarrow$ yielding $\rightarrow$ ductile

Uniaxial → yielding → ductile

3rd 1D plastic zone model

3. Strip Yield Model: Dugdale vs Barenblatt model

Dugdale: Uniform stress

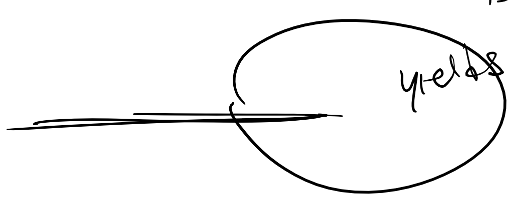


More appropriate for <u>polymers</u>

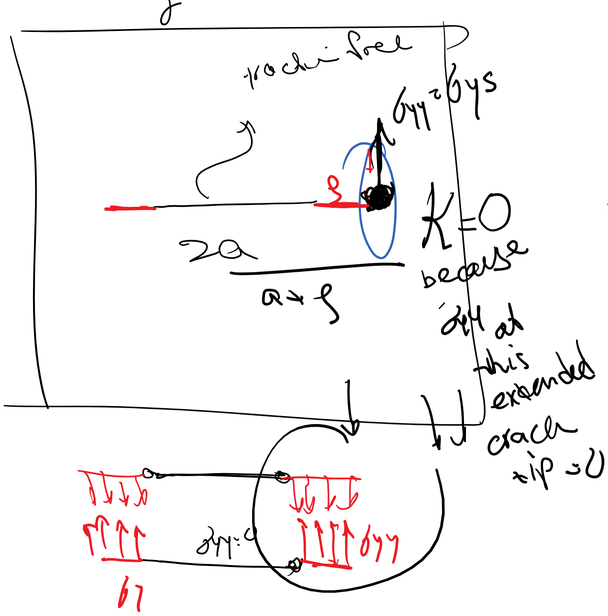
Barenblatt: Linear stress

More appropriate for <u>metals</u>

Difficult to solve this



Dugdale solution



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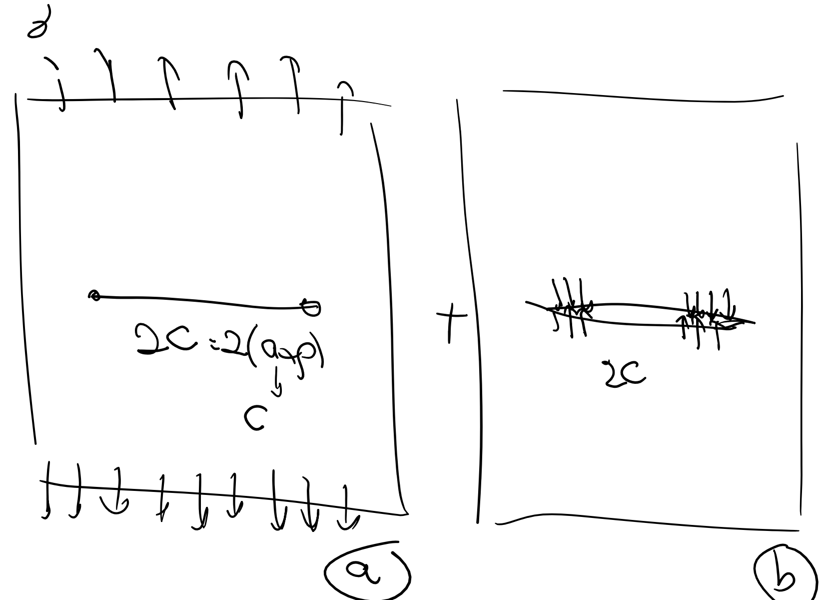
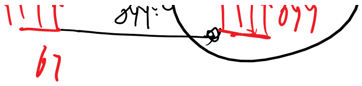
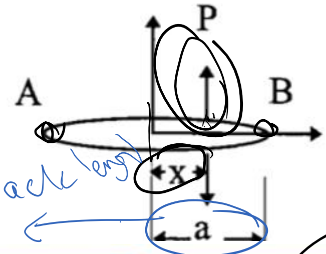


$K_a = (\sqrt{\pi c})\sigma = \sigma\sqrt{\pi(a+\rho)}$ $\quad K_b = ($

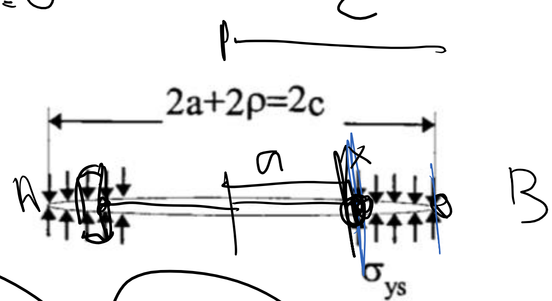


$K_a = (\sqrt{\pi c})\sigma = \sigma\sqrt{\pi(a+\rho)}$ $K_b = 0$

$K = K_a + K_b = 0$



$P = -\sigma_{ys} dx$



$$K_A = \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a+x}{a-x}}$$

$$K_B = \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a-x}{a+x}}$$

Anderson, p64

$$K_B = \int_a^c \left(-\frac{\sigma_{ys}\, dx}{\sqrt{\pi c}}\sqrt{\frac{c-x}{c+x}} - \frac{\sigma_{ys}\, dx}{\sqrt{\pi c}}\sqrt{\frac{c+x}{c-x}} \right) dx$$

$P = \sigma_{ys} dx$

$$K_I^{b} = -\frac{\sigma_{ys}}{\sqrt{\pi c}}\int_a^c \left(\sqrt{\frac{c-x}{c+x}} + \sqrt{\frac{c+x}{c-x}}\right) dx$$

$$K_I^{b} = -2\sigma_{ys}\sqrt{\frac{a+\rho}{\pi}}\cos^{-1}\left(\frac{a}{a+\rho}\right)$$

$$K_I = K_I^a + K_I^b = 0$$

$$(\sqrt{\pi c})\,\sigma - 2\sigma_{ys}\sqrt{\frac{c}{\pi}}\cos^{-1}\left(\frac{a}{c}\right) = 0$$

$a+\rho$

$$2\sigma_{ys}\sqrt{\frac{a+\rho}{\pi}}\cos^{-1}\left(\frac{a}{a+\rho}\right) = \sigma\sqrt{\pi(a+\rho)}$$

$$\cos\left(\frac{\pi\sigma}{2\sigma_{ys}}\right) = \frac{a}{a+\rho} \longrightarrow \text{Find } \rho?$$

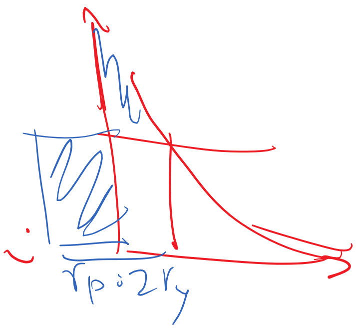
$$\cos\left(\frac{\pi\sigma}{2\sigma_{ys}}\right) = \frac{a}{a+\rho} \longrightarrow \text{Find } \rho?$$

assume $\sigma \ll \sigma_{ys}$ — as will see : Small scale Yielding (SSY) is satisfied

$\rightarrow x \lesssim 1$

$$\cos x \simeq 1 - \frac{x^2}{2}$$

$$\rho = \frac{\pi^2 \sigma^2 a}{8\sigma_{ys}^2}$$



$$\boxed{\rho = \frac{\pi^2\sigma^2 a}{8\sigma_{ys}^2} = \frac{\pi}{8}\left(\frac{K_I}{\sigma_{ys}}\right)^2}$$

Exact 1D result

close to **0.392**

$$\boxed{r_p = \underbrace{\frac{1}{\pi}}_{0.318}\left(\frac{K_I}{\sigma_{ys}}\right)^2}$$

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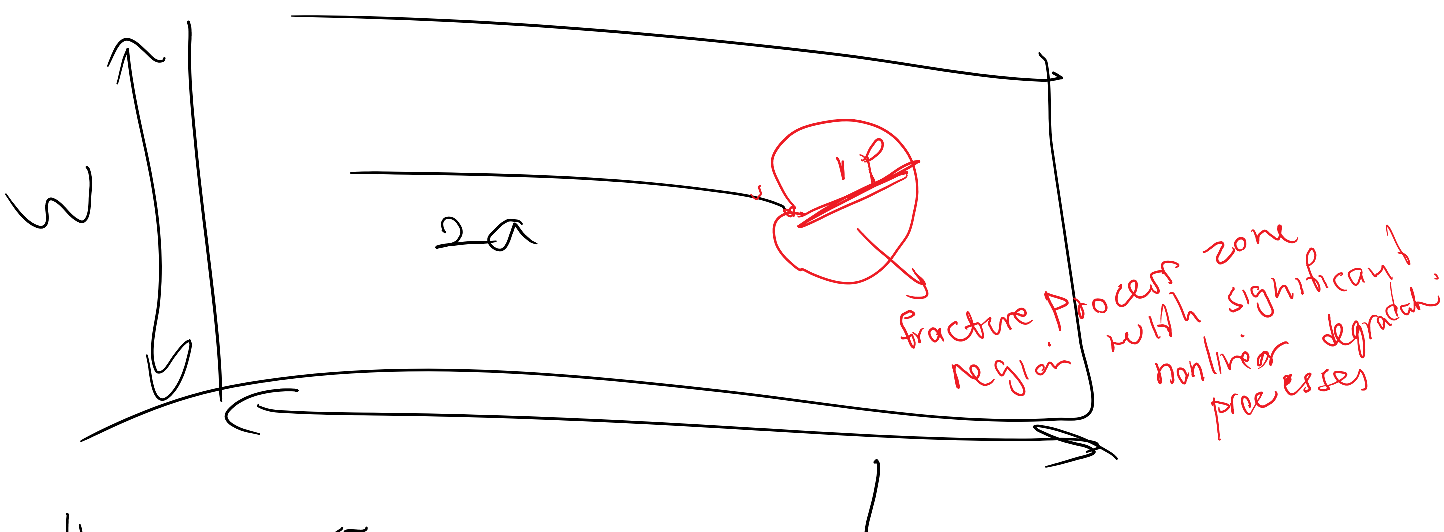
Small Scale Yielding



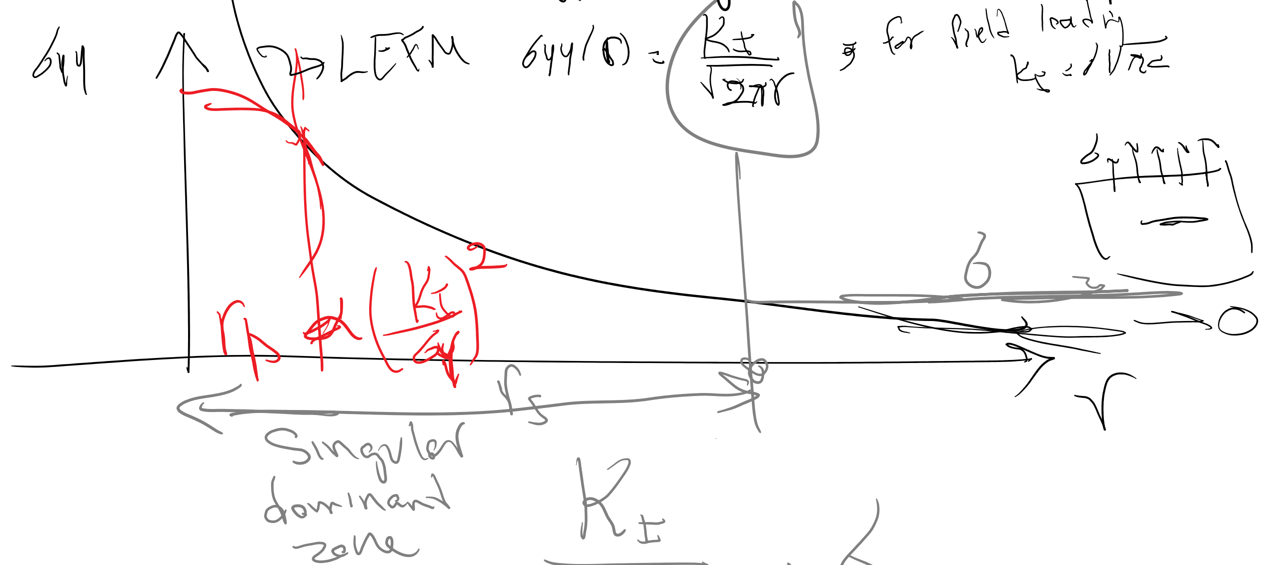
When is LEFM a good model

$$r_p \ll L,\ a,\ W,\ \ldots,\ r_s$$

singular dominant zone radius

$$r_p \ll \underbrace{L, a, W, \dots, r_s}_{\text{all relevant length scale of the problem}}$$

zone radius



$$\frac{K_I}{\sqrt{2\pi r_s}} = \sigma$$

$$r_s = \frac{1}{2\pi}\left(\frac{K_I}{\sigma}\right)^2$$

$$\frac{r_p}{r_s} \propto \frac{\left(\frac{K_I}{\sigma_Y}\right)^2}{\left(\frac{K_I}{\sigma}\right)^2} = \left(\frac{\sigma}{\sigma_Y}\right)^2$$

$$\left(\frac{\sigma}{\sigma_Y}\right)^2 \ll 1 \Rightarrow \frac{r_p}{r_s} \ll 1$$

$\sigma/\sigma_Y \lesssim 0.3 \longrightarrow \frac{r_p}{r_s} < 0.09$

may be this is good for us