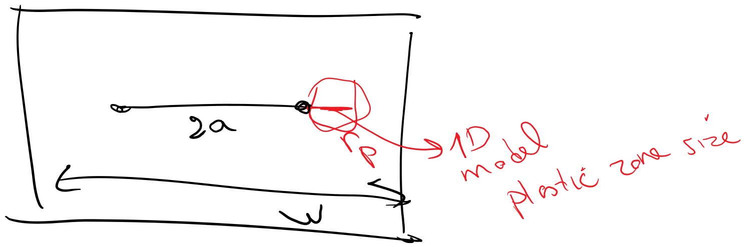
2022/10/04

Tuesday, October 4, 2022 2:28 PM

As SSY is about to be violated, for example, as higher far field load is being applied, we want to still use LEFM as much as possible (we want to extend the applicability of LEFM)



$$a \longrightarrow a + r_p$$

or other estimates

we basically use a longer length

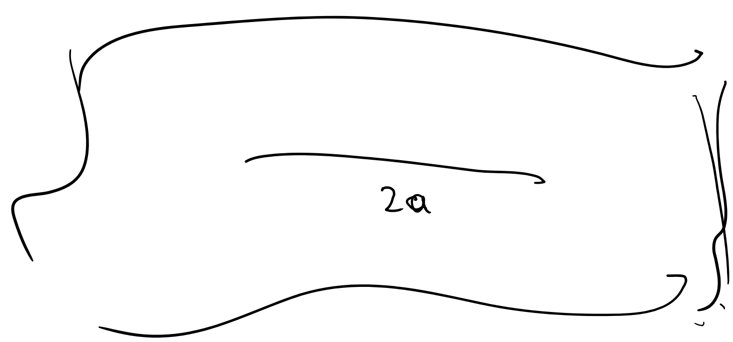
$$\begin{cases} r_p \approx \dfrac{1}{2\pi}\left(\dfrac{K_{eff}}{\sigma_{ys}}\right)^2 \qquad a_{eff} = a + r_p \\ K_{eff} = f(a_{eff}, W, \theta, \ldots)\, \sigma \sqrt{\pi a_{eff}} \end{cases}$$

θ: crack angle,

- This is a coupled system for a_eff and K_ff.
- In general, we need to use an iterative process. For example, start with a_eff = a -> K -> update a_eff = a + rp(K_eff), …, continue the process until a_eff is not changing much
- For an infinite domain, we can solve this.

$$r_p = \frac{1}{2\pi}\left(\frac{K_{eff}}{\sigma_{ys}}\right)^2$$

$$K_{eff} = \sigma\sqrt{\pi a_{eff}} \qquad (f = 1)$$



$$K_{eff}^2 = \sigma^2 \pi a_{eff} = \sigma^2 \pi (a + r_p)$$

$$= \sigma^2 \pi \left(a + \frac{1}{2\pi}\left(\frac{K_{eff}}{\sigma_{ys}}\right)^2\right)$$

$$\left[1 - \frac{1}{2}\left(\frac{\sigma}{\sigma_{ys}}\right)^2\right] K_{eff}^2 = \pi a \sigma^2$$

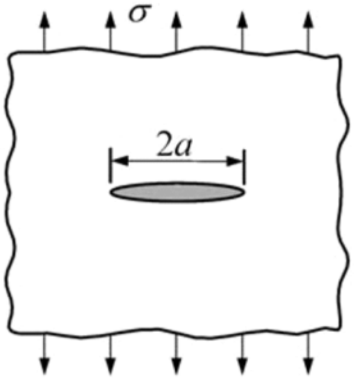
$$\boxed{K_{eff} = \frac{\sigma\sqrt{\pi a}}{\sqrt{1 - \frac{1}{2}\left(\frac{\sigma}{\sigma_{ys}}\right)^2}}}$$

effective K by extending the crack length using $a_{eff} = a + r_p$

$$\sqrt{1-\frac{1}{2}\left(\frac{\sigma}{\sigma_{ys}}\right)^2}$$ length using $a_{eff} = a + r_p$

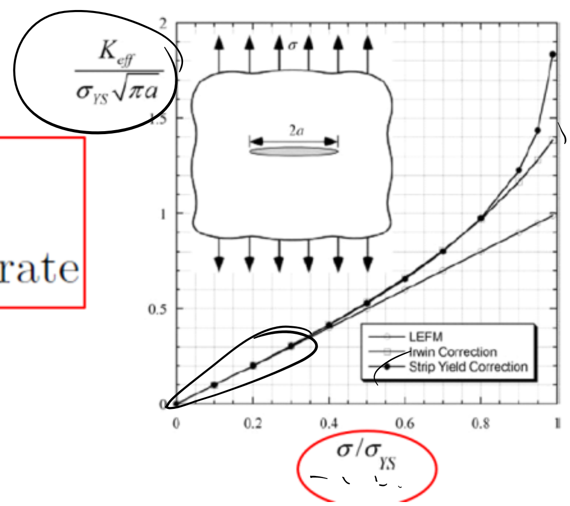
Recall from last time $\frac{r_s}{r_p} \propto \left(\frac{\sigma}{\sigma_{ys}}\right)^2$

Effective crack length



$$K_{eff} = \frac{\sigma\sqrt{\pi a}}{\sqrt{1-\frac{1}{2}\left(\frac{\sigma}{\sigma_{YS}}\right)^2}}$$

As $\frac{\sigma}{\sigma_{ys}}$ increases $\Rightarrow$
LEFM becomes less accurate



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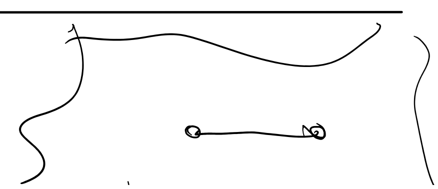
$\left(\frac{\sigma}{\sigma_{ys}}\right)^2 \ll 1$ can use LEFM

$\left(\frac{\sigma}{\sigma_{ys}}\right)^2 \lesssim 1$ a_{eff} idea extends LEFM

$\left(\frac{\sigma}{\sigma_{ys}}\right)^2 \sim O(1)$ Plastic Fracture Mechanics (PFM)

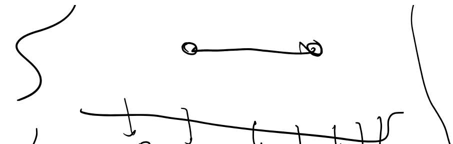
Example:

Consider a large central cracked plate subjected to a uniform stress of 130 MPa. The fracture toughness K_c=50MPa√m, the yield strength σ_{ys}=420MPa.



Consider a large central cracked plate subjected to a uniform stress of 130 MPa. The fracture toughness K_c=50MPa√m, the yield strength σ_{ys}=420MPa.

(a) What is the maximum allowable crack length?
(b) What is the maximum crack length if plastic correction is taken into account. Plane stress and Irwin's correction.



$\sigma = 130\,MPa$

$K_c = 50\,MPa\sqrt{m}$

$\sigma_{ys} = 420\,MPa$

check : $\left(\frac{\sigma}{\sigma_{ys}}\right) = \frac{130}{420} \approx .2 - .3$ relatively large → good case to use the correction

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(a) $K_I = K_c$

$\sigma\sqrt{\pi a} = K_c$

$130\,(MPa)\sqrt{\pi a} = 50\,MPa\sqrt{m}$ → w/o correction $\boxed{a = 94.2\ mm}$

(b) Using the correction

$$\frac{\sigma\sqrt{\pi a}}{\sqrt{1 + \frac{1}{2}\left(\frac{\sigma}{\sigma_{ys}}\right)^2}} = K_c \rightarrow \frac{130\sqrt{\pi a}}{\sqrt{1 + \frac{1}{2}\left(\frac{130}{420}\right)^2}} = 50 \rightarrow$$

$\boxed{a = 89.7\ mm}$ with the correction

LEFM crack length is longer (not conservative), so we must do the correction or use more advanced models as SSY is about to be violated.

2D models for plastic zone around the crack tip

Plastic yield criteria

von-Mises criterion

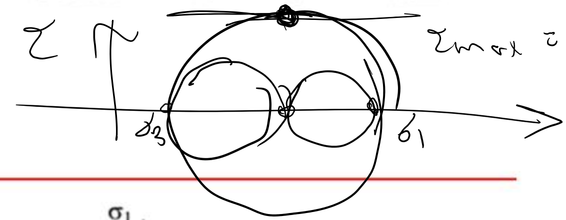
$$\tau_v = \sqrt{3J_2}$$
$$= \sqrt{\frac{(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2 + 6(\sigma_{12}^2+\sigma_{23}^2+\sigma_{31}^2)}{2}}$$
$$= \sqrt{\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_1-\sigma_3)^2}{2}}$$
$$= \sqrt{\tfrac{3}{2}\, s_{ij}s_{ij}}$$

s is stress deviator tensor

$$\boldsymbol{\sigma}^{dev} = \boldsymbol{\sigma} - \frac{1}{3}(\mathrm{tr}\,\boldsymbol{\sigma})\,\mathbf{I}$$

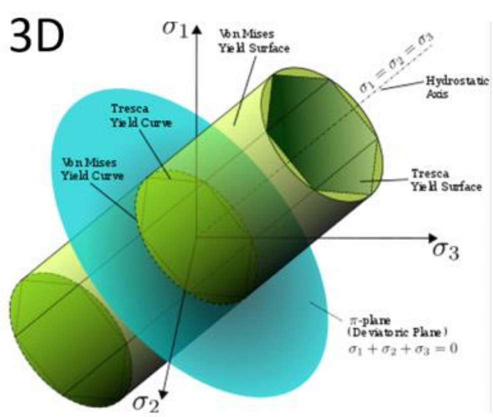
Tresca criterion
Maximum shear stress

$$\sigma_{tresca} = \sigma_1 - \sigma_3 > \sigma_{max}$$

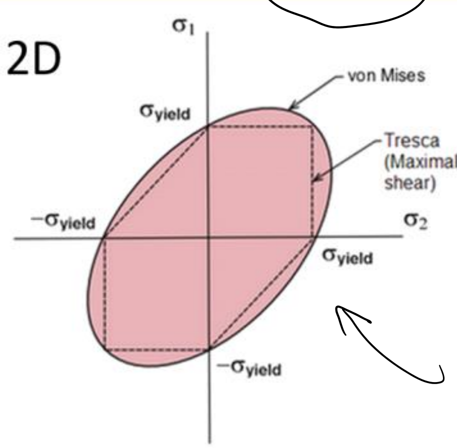


$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$

3D



2D



$\sigma_3 = 0$
plane stress

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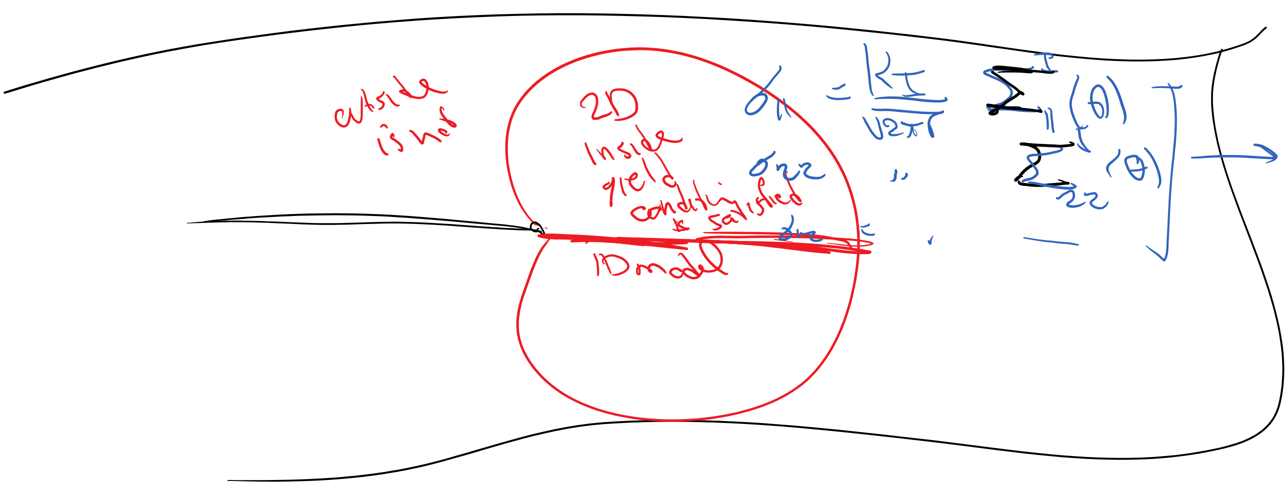


$\sigma_{11} = \frac{K_I}{\sqrt{2\pi r}} \Sigma_{11}(\theta)$

σ_{22} " $\Sigma_{22}(\theta)$

σ_{12} = " —

→ σ_1, σ_2 check v.M or Tresca

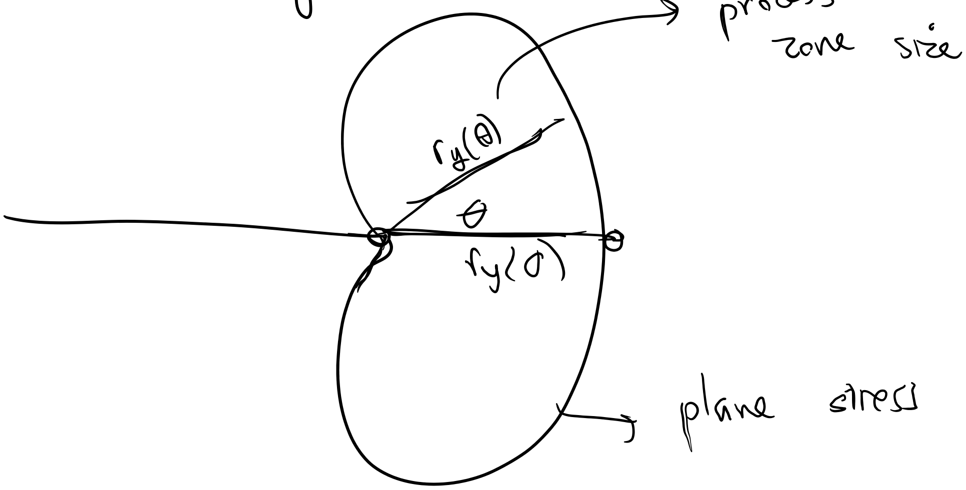
Mode I, principal stresses

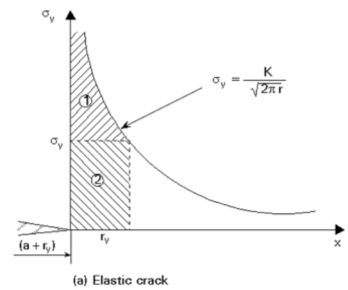
$$\sigma_1 = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\right)$$
$$\sigma_2 = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\right)$$
$$\sigma_3 = \begin{cases} 0 & \text{plane stress} \\ \dfrac{2\nu K_I}{\sqrt{2\pi r}} \cos\dfrac{\theta}{2} & \text{plane strain} \end{cases}$$

$\nu(\sigma_1+\sigma_2)$ $(\epsilon_3 = 0)$

$$\boxed{r_y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2\left[1+\cos\theta+\frac{3}{2}\sin^2\theta\right]}$$ plane stress

$$r_y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2\left[(1-2\mu)^2(1+\cos\theta)+\frac{3}{2}\sin^2\theta\right]$$ plane strain



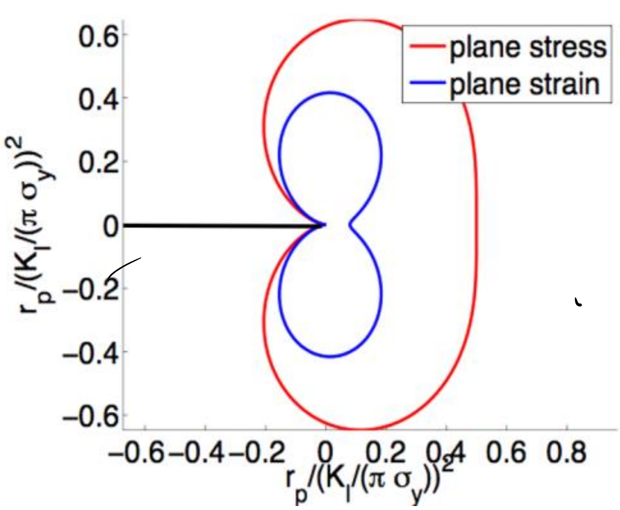


Plastic zone shape

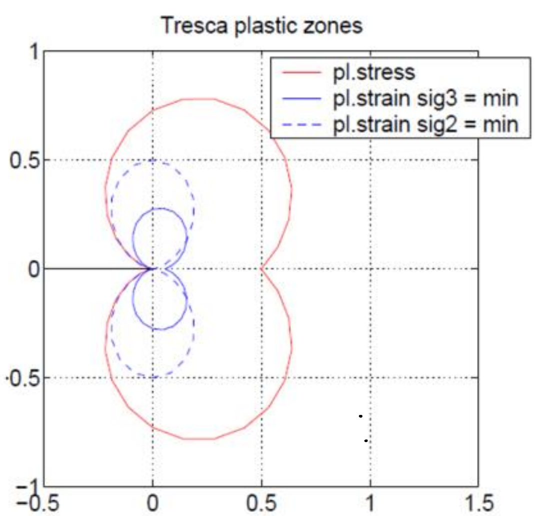
von-Mises criterion

$$\boxed{r_y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2\left[1+\cos\theta+\frac{3}{2}\sin^2\theta\right]}$$

plane stress

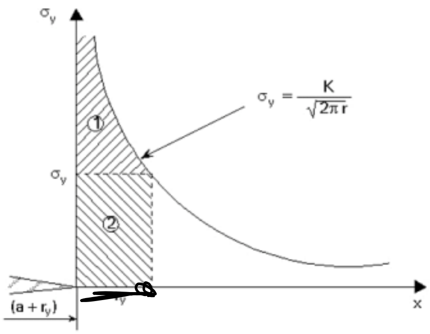


Tresca criterion



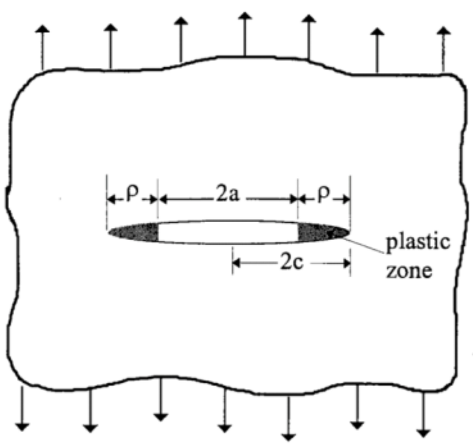
This approach does not have the stress redistribution needed from material yielding and it suffers from the same problem we had here

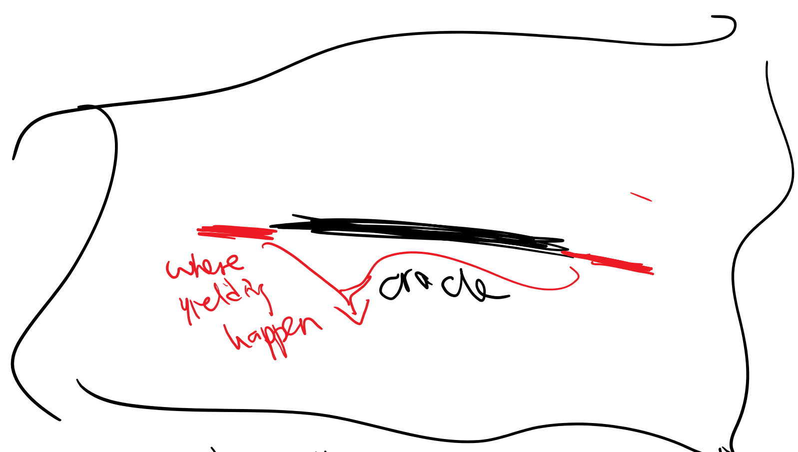
we did 2D version of this



(a) Elastic crack

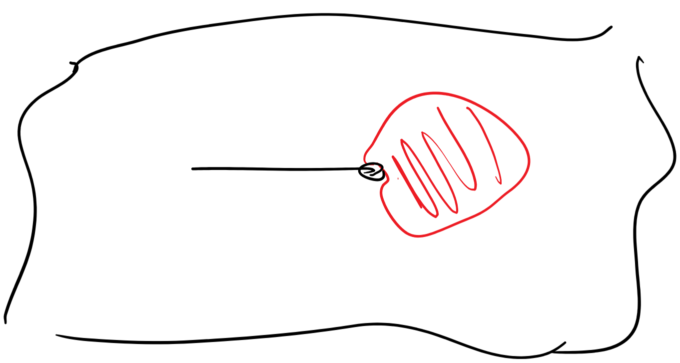
To get a better solution, we need to solve the problem with plastic limit from the very beginning. We did this with strip yield model





extending this to 2D

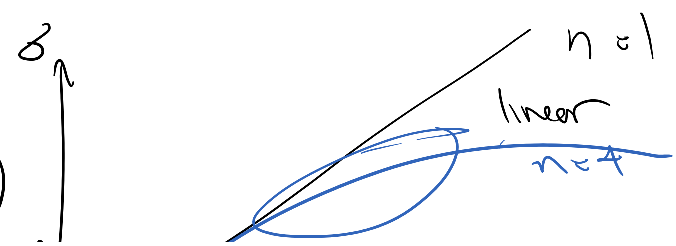
allow material to yield anywhere



we need to do this numerically

Dodds, 1991, FEM solution
Ramberg-Osgood material model

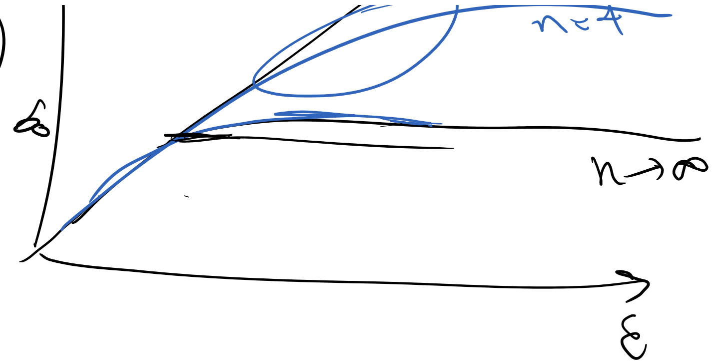
$\varepsilon = \frac{\sigma}{} + \alpha\left(\frac{\sigma}{}\right)^{n}$



Ramberg-Osgood material model

$$\frac{\varepsilon}{\varepsilon_o} = \frac{\sigma}{\sigma_o} + \alpha\left(\frac{\sigma}{\sigma_o}\right)^n$$

Nonlinear elastic



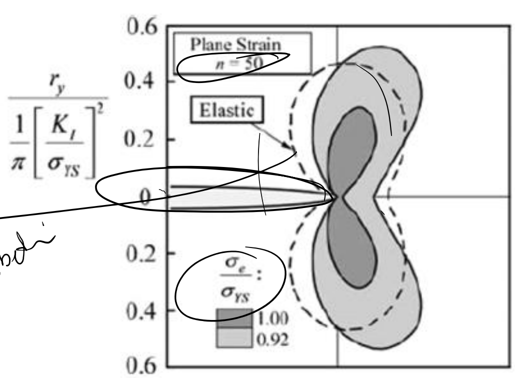
$n \nearrow$ hardening

$n \rightarrow \infty$ this approximates elastic perfectly plastic

Dodds, 1991, FEM solutions
Ramberg-Osgood material model

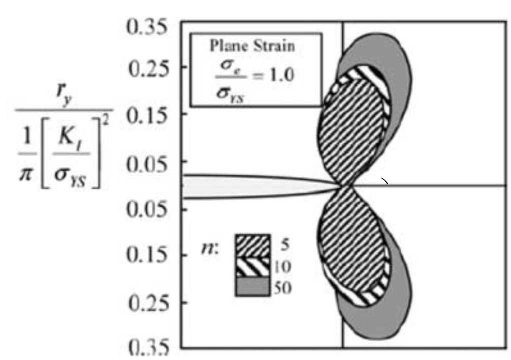
$$\frac{\varepsilon}{\varepsilon_o} = \frac{\sigma}{\sigma_o} + \alpha\left(\frac{\sigma}{\sigma_o}\right)^n$$

- Low n: High strain-hardening.
- $n \rightarrow \infty$: Similar to elastic perfectly plastic.



what I showed you without redistribution

Effect of definition of yield (some level of ambiguity)



Effect of strain-hardening: Higher hardening (lower n) => smaller zone

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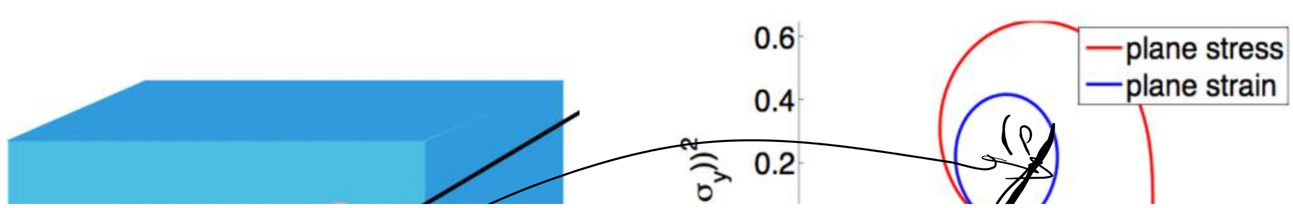
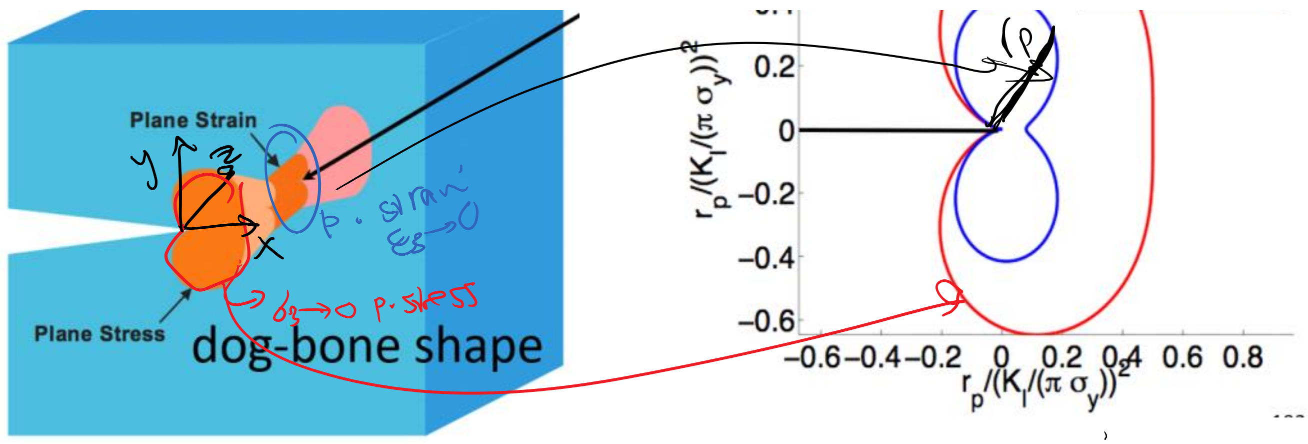
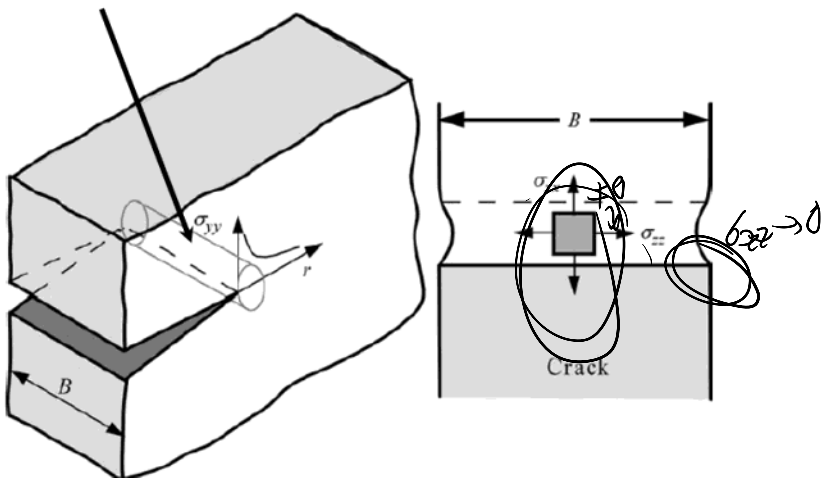
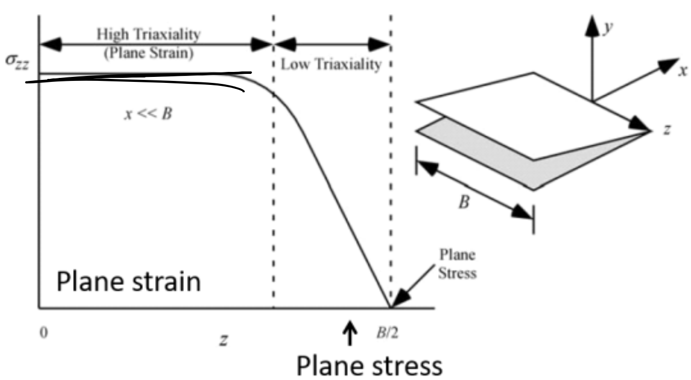
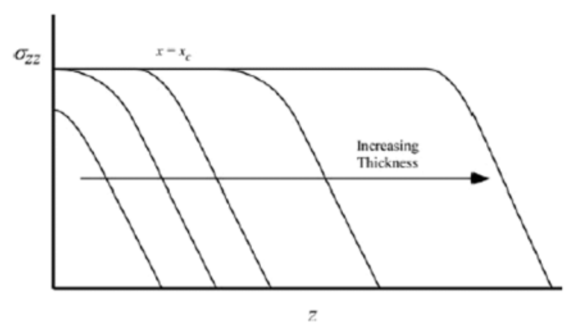
Plane stress vs plane strain









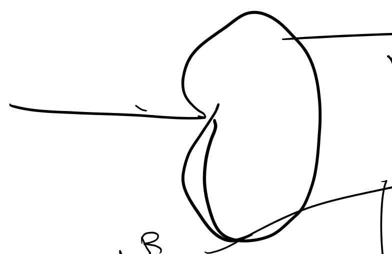


As the thickness increases more through the thickness bahaves as plane strain

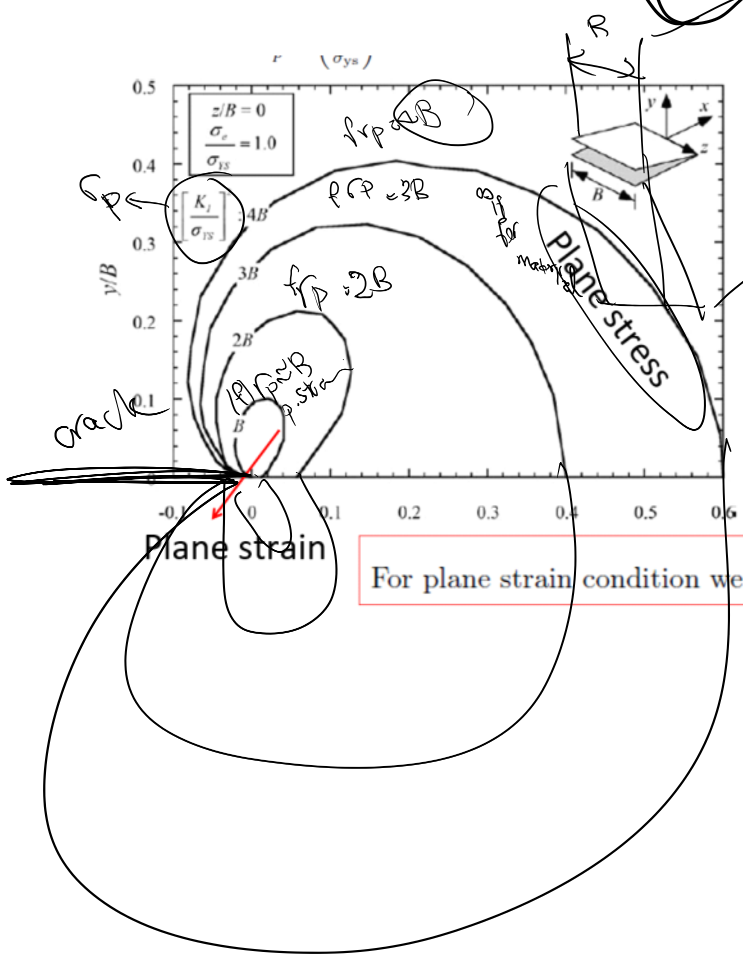
To determine plane strain / stress condition for FRACTURE we need to compare B with rp:

$$r_p \propto \left(\frac{K_I}{\sigma_y}\right)^2$$

$\frac{r_p}{B}$ very small	p. strain
$\frac{r_p}{B}$ large	p stress



$\frac{r_p}{B}$ large | P stress

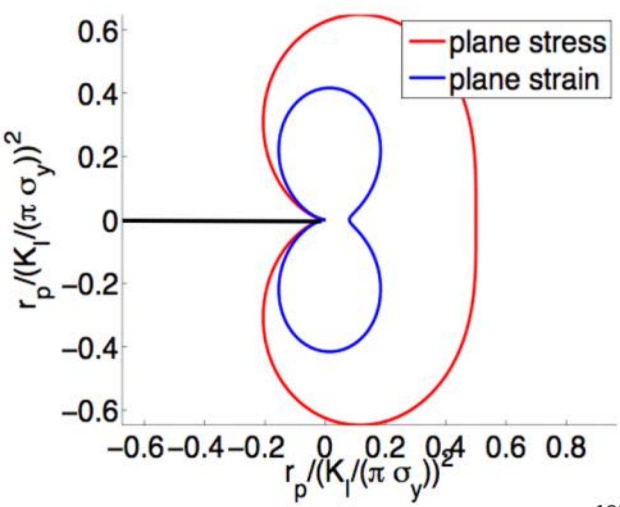


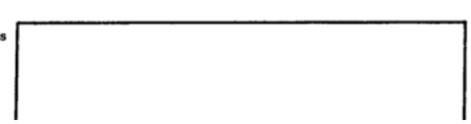
$$\frac{\left(\frac{K}{\sigma_{ys}}\right)^2}{B} \propto \frac{r_p}{B} : \begin{cases} \text{low (high } B) & \text{plane strain} \\ \text{high (low } B) & \text{plane stress} \end{cases}$$

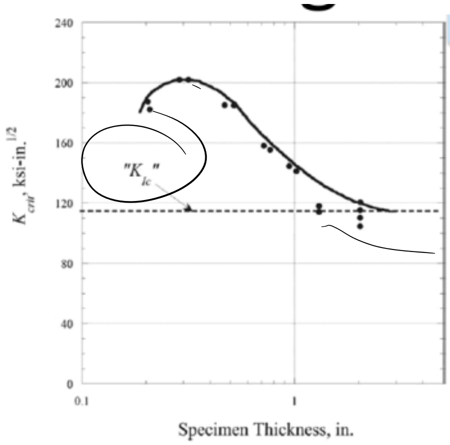
Change of plastic loci to plane stress mode as “relative B decreases”. Nakamura & Park, ASME 1988

$\boxed{\text{For plane strain condition we must have } B > \left(\frac{K}{\sigma_{ys}}\right)^2 \propto r_p}$

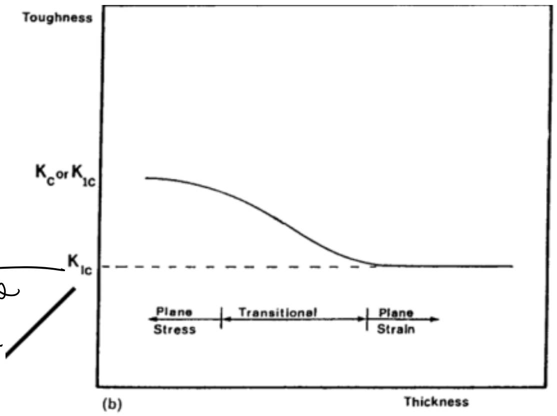
Plane stress has a larger process zone size -> should have higher toughness (measured in terms of R or Kc)







fracture toughness for plane strain



Plane strain fracture toughness **lowest K**
(safe)

(Irwin) $$K_c = K_{Ic}\left(1 + \frac{1.4}{B^2}\left[\frac{K_{Ic}}{\sigma_{ys}}\right]^4\right)^{1/2}$$ Note that $\frac{1}{B^2}\left[\frac{K}{\sigma_{ys}}\right]^4 \propto \left(\frac{r_p}{B}\right)^2$

p. strain

$$\left(\frac{K_{Ic}}{\sigma_{ys}}\right)^2 \propto r_p$$

$$\left(\frac{r_p}{B}\right)^2$$

To get thickness independent measurement of K we want to be well into plane strain regime

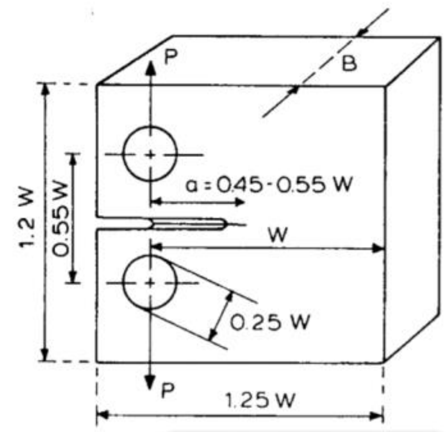
$$B > 2.5\left(\frac{K_I}{\sigma_y}\right)^2$$

- Prediction of failure in real-world applications: need the value of fracture toughness
- Tests on cracked samples: PLANE STRAIN condition!!!

Compact Tension Test

$$K_I = \frac{P}{B\sqrt{W}} \frac{\left(2+\frac{a}{W}\right)\left[0.886 + 4.64\frac{a}{W} - 13.32\left(\frac{a}{W}\right)^2 + 14.72(\left(\frac{a}{W}\right)^3 - 5.6\left(\frac{a}{W}\right)^4\right]}{\left(1-\frac{a}{W}\right)^{3/2}}$$

$$a, B, (W-a) \geq 2.5\left(\frac{K_{Ic}}{\sigma_{YS}}\right)^2$$



ASTM (based on Irwin's model) for plane strain condition:

$$\boxed{a, B, (W-a) \geq 2.5\left(\frac{K_{Ic}}{\sigma_{Y}}\right)^2}$$

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basically small scale yielding (SSY) is satisfied