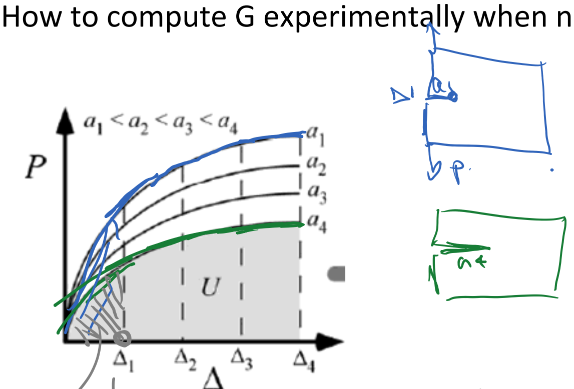
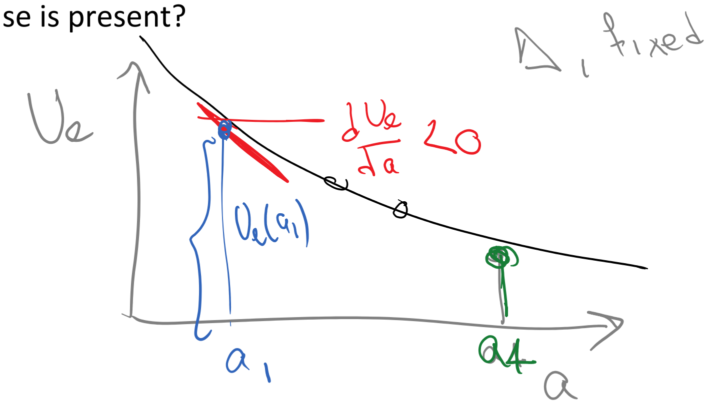
2022/10/18

Tuesday, October 18, 2022 2:29 PM

How to compute G experimentally when nonlinear response is present?





generate fixed grip data

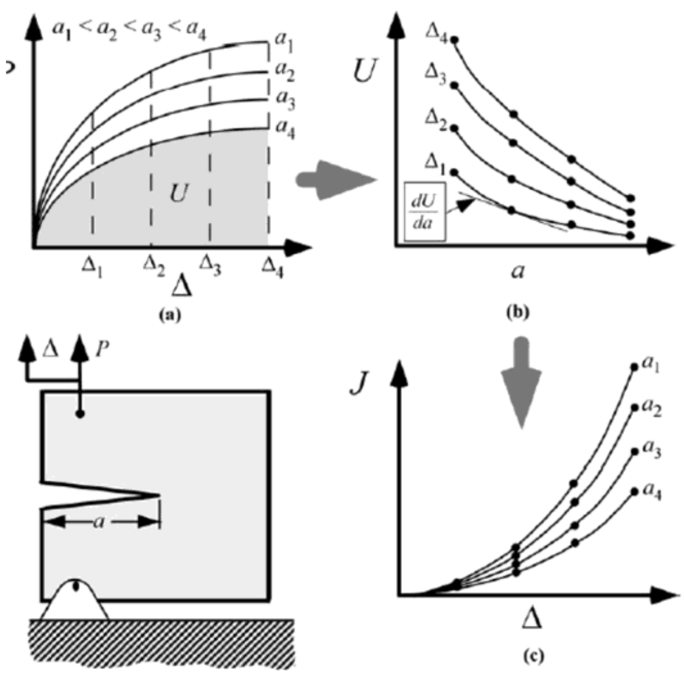
$\Delta = \Delta_1$

$V_e(a_4)$

$$G(a_1) = -\frac{1}{B}\frac{dU_e}{da} \quad \text{fixed grip}$$

from the plot

Landes and Begley, ASTM 1972



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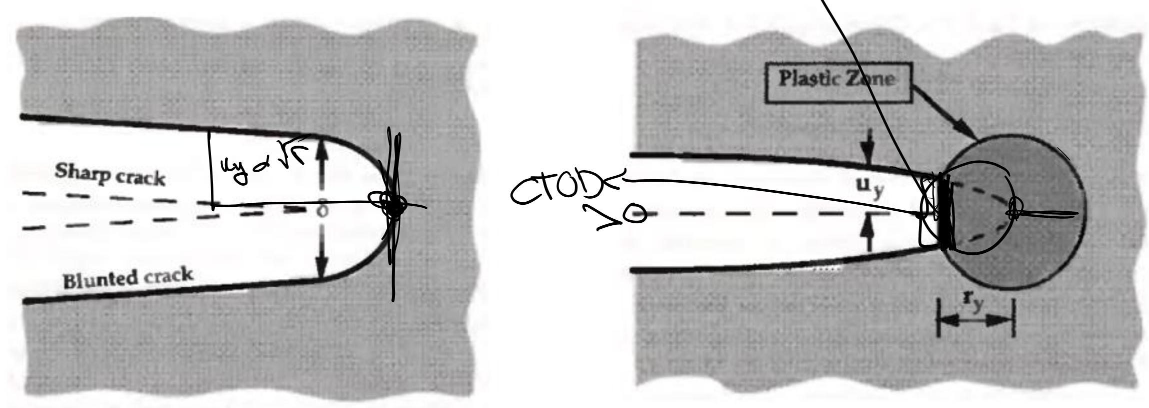
. Rice proposes a method to obtain J = G, with only one test for certain geometries.

Crack tip opening displacement (CTOD) versus J-integral

5.4. Crack tip opening displacement (CTOD), relations with J and G

crack blunting

LEFM PFM



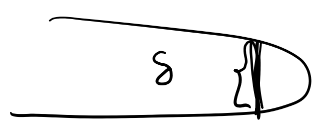
LEFM $u_y \propto \sqrt{r}$

$\sigma \propto \frac{1}{\sqrt{r}}$

$\sigma \to$ bounded by σ_y ...

σ_0

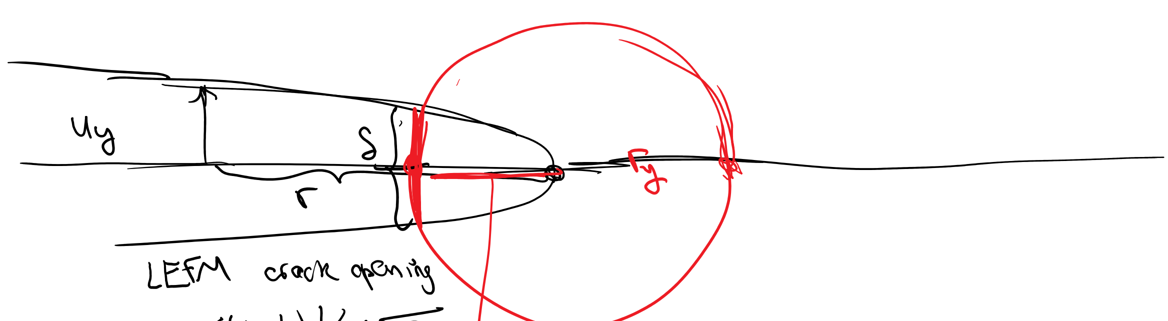
Parallel to Rice's work in the US, Wells in the UK looked at the CTOD as a measure of nonlinear material response in the Facture Process Zone (FPZ)



$\delta \nearrow \longrightarrow$ more nonlinear response

we want to relate δ to J

Estimates for CTOD (δ)



LEFM crack opening

$$u_y = \frac{(\kappa+1)K_I}{2\mu}\sqrt{\frac{r}{2\pi}}$$

we go back by r_y

$$r \to r_y \Longrightarrow u_y(r_y) = \frac{\delta}{2}$$

$\to$

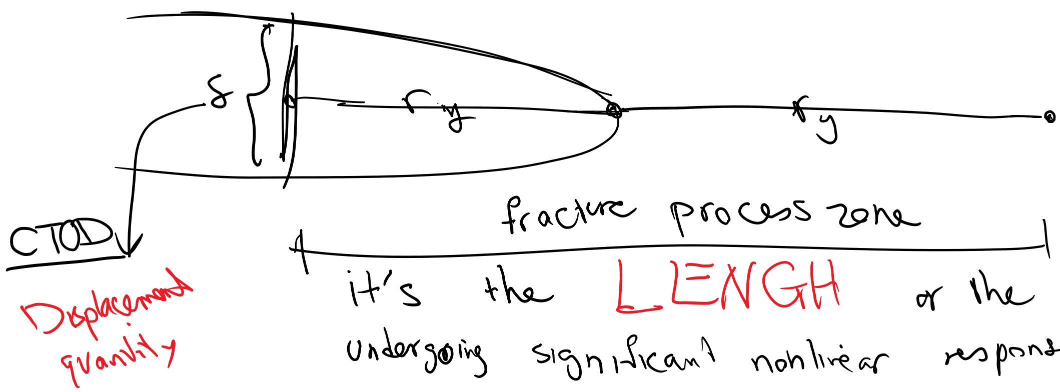
$r \longrightarrow r_y \longrightarrow u_y(r_y) = \frac{\delta}{2}$ $\longrightarrow$ $\boxed{\delta \simeq \frac{4}{\pi}\frac{K_I^2}{\sigma_{ys}E}}$

estimate for CTOD

$r_y = \left(\frac{K_I}{\sigma_{ys}}\right)^2$

$\kappa = \frac{3-\nu}{1+\nu}$ p. strain

$\mu = \frac{E}{2(1+\nu)}$



it's the LENGH or the region that is undergoing significant nonlinear response

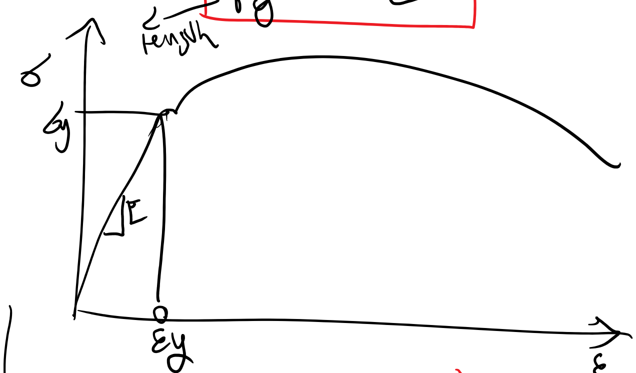
$r_y = (\,)\left(\frac{K_I}{\sigma_{ys}}\right)^2$ length FPZS

$\delta = (\,)\frac{K_I}{\sigma_{ys}E}$ displacement scale

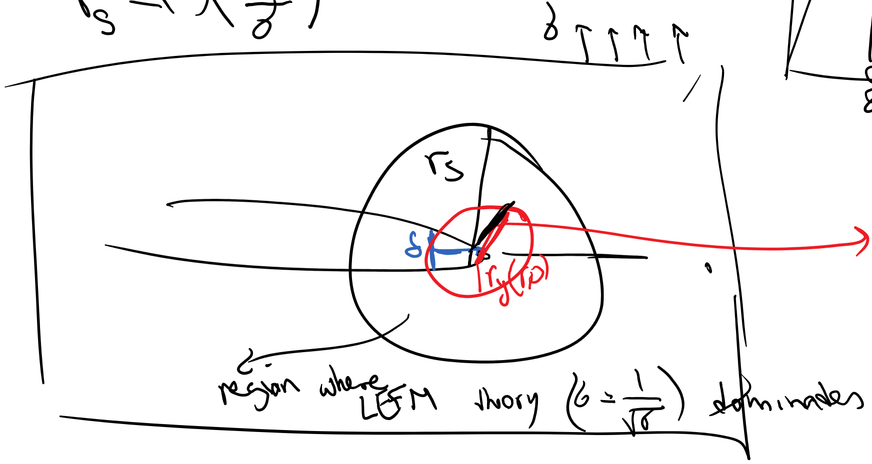
displacement

$\boxed{\frac{\delta}{r_y} \propto \frac{\sigma_y}{E}}$ strain scale

length



$r_s = (\,)\left(\frac{K_I}{\bar{\sigma}}\right)^2$



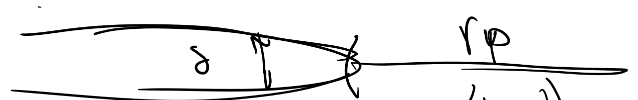
(length scale) size where material is significantly yielding, ...

$\delta \ll r_y \ll r_s$ SSY holds $\left(\frac{\bar{\sigma}}{\sigma_Y}\right)^2 \ll 1$

$\frac{r_y}{r_s} \propto \left(\frac{\bar{\sigma}}{\sigma_Y}\right)^2$

In many instances, LEFM, PFM, Traction separation relations, frictional laws

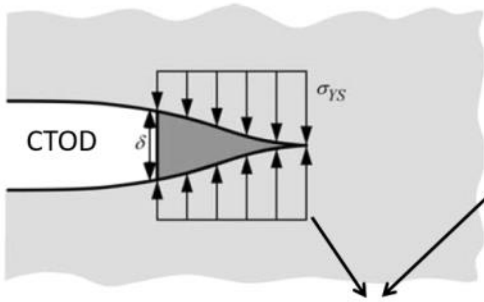
In many instances, LEFM, PFM, Traction separation relations, frictional laws, …
FPZ size (**length** scale) is much larger than the **displacement** scale of the model and typically proportional to **E/strength**

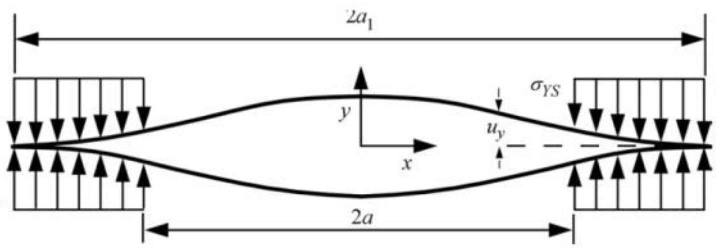


$\delta / r_p \propto \frac{\delta}{\text{strength}}$

The above estimate for CTOD was very crude and can be improved with better models

Crack Tip Opening Displacement: Strip yield model





K = 0 $\rightarrow$ $Z = \frac{2\sigma_{YS}}{\pi}\left[\frac{k}{z}\sqrt{\frac{z^2-a_1^2}{1-k^2}}\right] \rightarrow \tilde{Z} = \frac{2\sigma_{YS}}{\pi}[z\omega_1 - a\omega_2]$

Stresses that yielded K = 0

$$\rightarrow \; u_y = \frac{2}{E}\mathrm{Im}\tilde{Z} = \frac{4\sigma_{YS}}{\pi E}\left[a\coth^{-1}\left(\frac{1}{a_1}\sqrt{\frac{a_1^2-z^2}{1-k^2}}\right) - z\coth^{-1}\left(\frac{k}{z}\sqrt{\frac{a_1^2-z^2}{1-k^2}}\right)\right]$$

$$z = a \rightarrow \quad \delta = 2u_y = \frac{8\sigma_{YS}a}{\pi E}\ln\left(\frac{1}{k}\right) = \frac{8\sigma_{YS}a}{\pi E}\left[\frac{1}{2}\left(\frac{\pi}{2}\frac{\sigma}{\sigma_{YS}}\right)^2 + \frac{1}{12}\left(\frac{\pi}{2}\frac{\sigma}{\sigma_{YS}}\right)^4 + \cdots\right] \rightarrow$$

For $\sigma/\sigma_{YS} \rightarrow 0$ $\quad \boxed{\delta = \frac{K_I^2}{\sigma_{YS}E}}$

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or crude approach earlier had

$$\delta = \frac{4}{\pi}\frac{K_I^2}{\sigma_{ys}E}$$

Is there a relation between J and CTOD?
Both measure the extend of material nonlinear response.

$$\left.\begin{array}{l}\delta = \frac{K_I^2}{E\sigma_{ys}} \\ J = \ldots K^2\end{array}\right\} \longrightarrow \quad \delta \propto \frac{J}{\sigma_y} \qquad (1)$$

$E\delta_{ys}$

$J = G = \frac{K_I^2}{E'}$ $\Big\} \longrightarrow J \propto \frac{\sigma}{\sigma_y}$ (1)

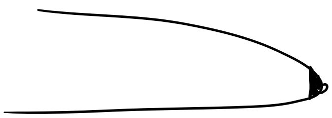
$E \propto$

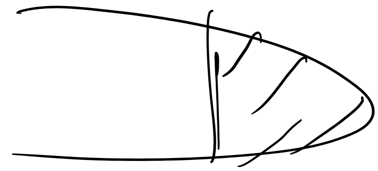
$$\boxed{J = m \delta \sigma_y}$$

$J \nearrow$ (higher loads)

or

$\sigma_y \searrow$ (more ductile response) $\Big\} \longrightarrow \delta \nearrow$





$\delta \nearrow$

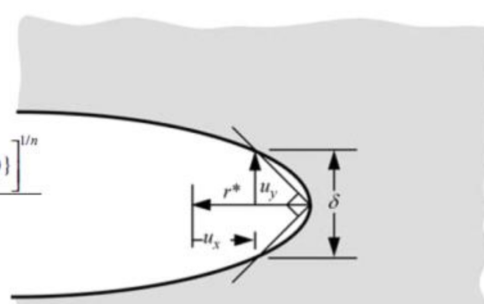
CTOD-J relation

- When SSY is satisfied G = J so we expect:

$$G = m\sigma_y\delta \quad \Rightarrow \quad \boxed{J = m\sigma_y\delta}$$

- In fact this equation is valid well beyond validity of LEFM and SSY
- E.g. for HRR solution Shih showed that:

$$u_i = \frac{\alpha\sigma_o}{E}\left(\frac{EJ}{\alpha\sigma_o^2 I_n r}\right)^{\frac{n}{n+1}} r\tilde{u}_i(\theta, n) \qquad d_n = \frac{2\tilde{u}_y(\pi,n)\left[\frac{\alpha\sigma_o}{E}\{\tilde{u}_x(\pi,n)+\tilde{u}_y(\pi,n)\}\right]^{1/n}}{I_n}$$



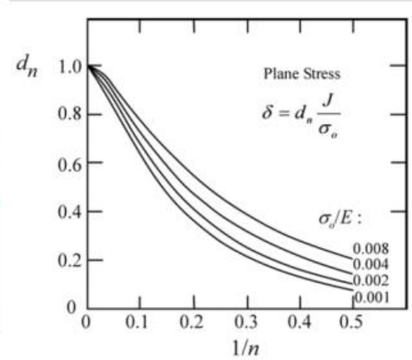
- δ is obtained by 90 degree method:
  Deformed position corresponding to $r^* = r$ and
  $\varphi = -\pi$ forms 45 degree w.r.t crack tip)

$$\frac{\delta}{2} = u_y(r^*, \pi) = r^* - u_x(r^*, \pi)$$

$$\boxed{r^* = \left(\frac{\alpha\sigma_o}{E}\right)^{1/n}\{\tilde{u}_x(\pi,n)+\tilde{u}_y(\pi,n)\}^{\frac{n+1}{n}}\frac{J}{\sigma_o I_n}} \Rightarrow J = m\sigma_0\delta$$

$\approx \sigma_y$

for
$$\boxed{m = \frac{1}{d_n},\ d_n = \frac{2\tilde{u}_y(\pi,n)\left[\frac{\alpha\sigma_o}{E}\{\tilde{u}_x(\pi,n)+\tilde{u}_y(\pi,n)\}\right]^{1/n}}{I_n}}$$



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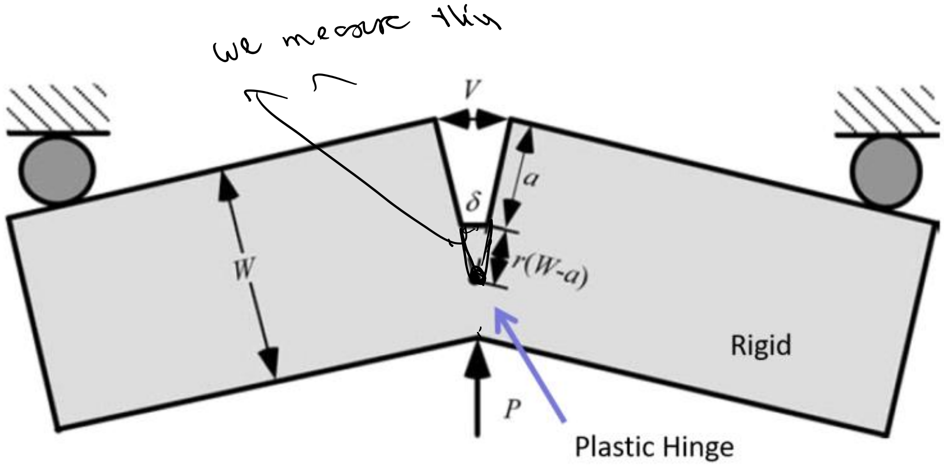
We can experimentally measure CTOD (and needed obtain J yet from another way)
Anderson

CTOD experimental determination



$$\frac{\delta_r}{CMOD} = \frac{r(W-a)}{r(W-a)+a}$$ similarity of triangles

r : rotational factor [-], between 0 and 1

For high elastic deformation contribution, elastic corrections should be added

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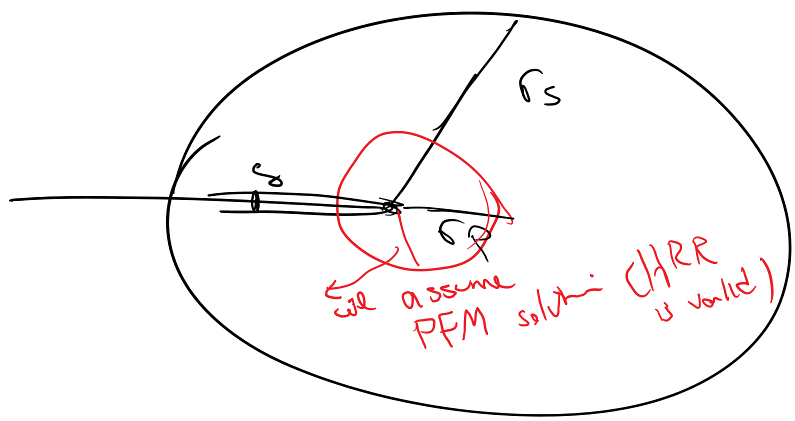
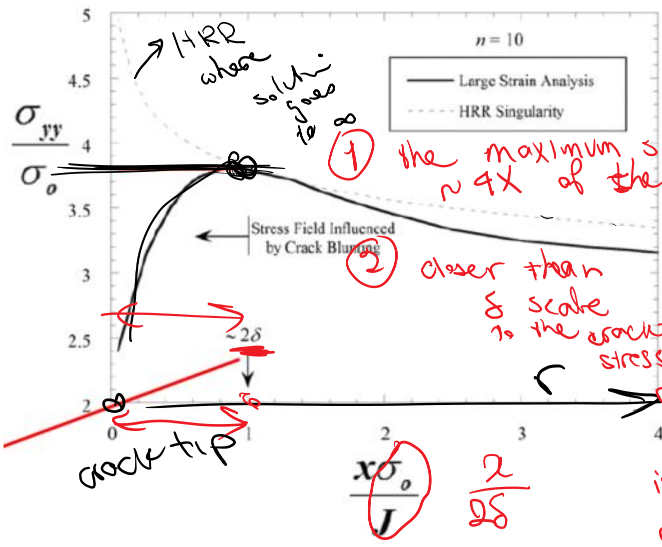
When to use LEFM, PFM, …



SSY: rp is significantly smaller than all relevant length scale of the problem.
As a necessary condition (but not sufficient)

$$r_p \ll r_s \qquad \left(\frac{\sigma}{\sigma_Y}\right)^2 \ll 1$$



↗ HRR where solution goes to ∞

① the maximum stress is only ~4X of the strength scale

② closer than δ scale to the crack tip stress decays & HRR solution is not accurate

crack tip

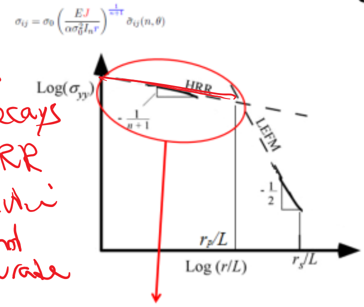
r

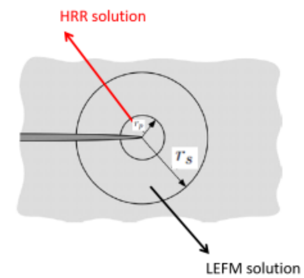
$\frac{2}{2\delta}$

McMeeking and Parks, ASTM STP 668, ASTM 1979

HRR solution: Stress singularity

$\sigma_{ij} = \sigma_0 \left(\frac{EJ}{\alpha\sigma_0^2 I_n r}\right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}(n,\theta)$





Stress is still singular but with a weaker power of singularity!

$\delta = \frac{1}{m}\frac{J}{\sigma_y}$

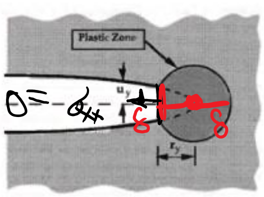
Limitations of HRR analysis

- Small strain: $\epsilon = \frac{1}{2}(\nabla \mathbf{u} + \nabla^T \mathbf{u})$ (accurate for $\epsilon \lesssim 0.1$)
- Small deformation theory (*e.g.*, not using PK stresses, etc)
- Elastic HRR model instead of plastic model
- Crack tip blunting: $\Rightarrow \sigma_{xx} = 0$

prev. HRR solution used infinitesimal deformation theory

$G = \frac{1}{2}(C - I)$, $C = \nabla u^T \nabla u$

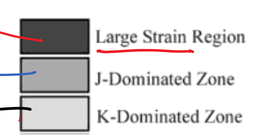
HRR solution did not allow plastic unloading



δ size ∝ to CTOD

PFM $r_p/\sigma_y \propto \left(\frac{K_I}{\sigma_y}\right)^2$

LEFM $r_s \propto \left(\frac{K_I}{\bar{\sigma}}\right)^2$

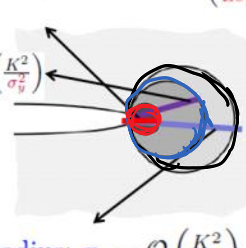


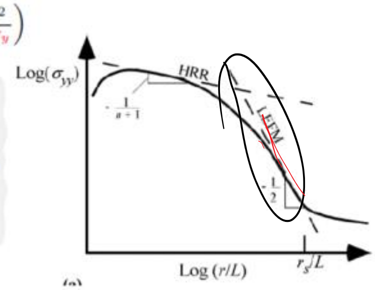
From SSY to LSY

Large strain radius $r_n \propto \delta$ (CTOD): $\delta = \mathcal{O}\left(\frac{K^2}{E\sigma_y}\right)$

plastic radius: $r_p = \mathcal{O}\left(\frac{K^2}{\sigma_y^2}\right)$

K-dominant radius: $r_s = \mathcal{O}\left(\frac{K^2}{\bar{\sigma}^2}\right)$

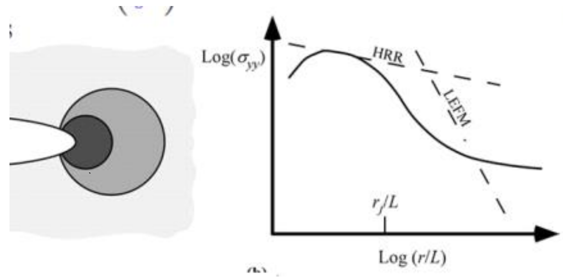




SSY (Small Scale Yielding)

$r_n \ll r_p \ll r_s \quad \Rightarrow$

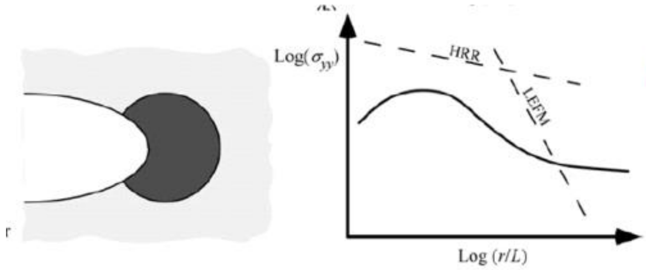
$$\boxed{\frac{r_p}{r_s} \propto \left(\frac{\bar{\sigma}}{\sigma_y}\right)^2 \ll 1}$$



Elastic plastic condition

$r_n \ll r_p \approx r_s \quad \Rightarrow$

$$\boxed{\frac{r_n}{r_p} \ll 1, \quad \frac{r_p}{r_s} \propto \left(\frac{\bar{\sigma}}{\sigma_y}\right)^2 \approx 1}$$



LSY(Large Scale Yielding)

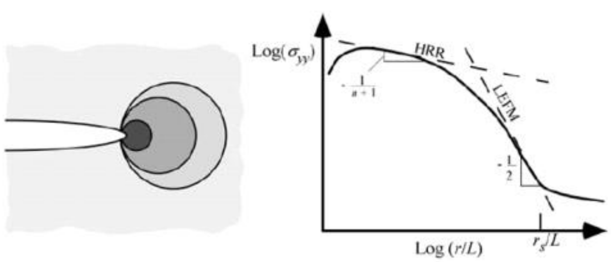
$r_n \approx r_p$

Note that $\frac{\delta}{r_p} \propto (\frac{\sigma_y}{E})$

finite strain
and/or plastic unloading zone

can become significant comparable to r_y

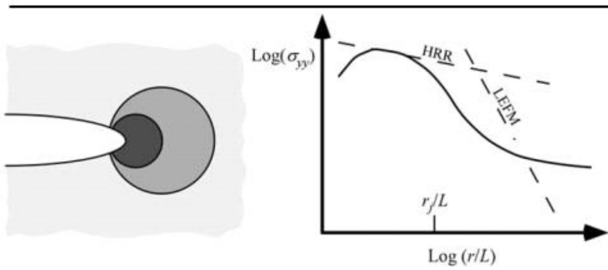
From SSY to LSY



LEFM: SSY satisfied and gener have Generally have

$$\bar{\sigma} \ll \sigma_y$$

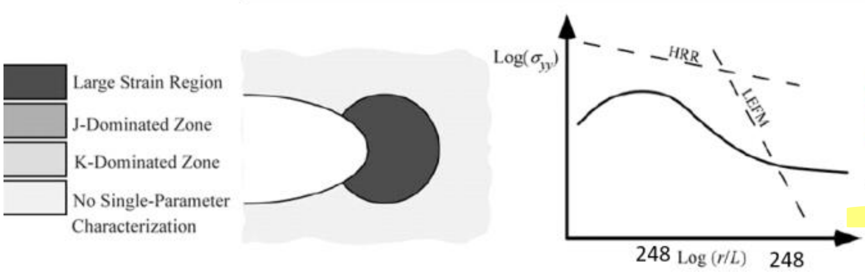
Relevant parameters:
G (energy) K (stress)



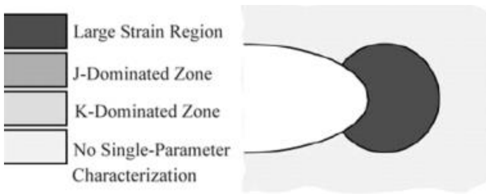
PFM (or NFM): SSY is gradually violated and

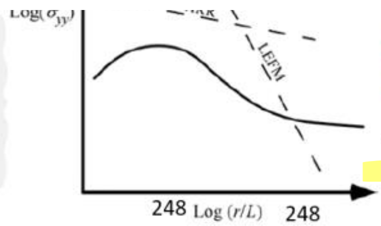
$$\bar{\sigma} \approx \sigma_y$$

Relevant parameters:
J (energy & used for stress)



LSY condition:
No single parameter can characterize fracutre!
J + other parameters (e.g. T stress, Q-J, etc)





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