2022/10/20

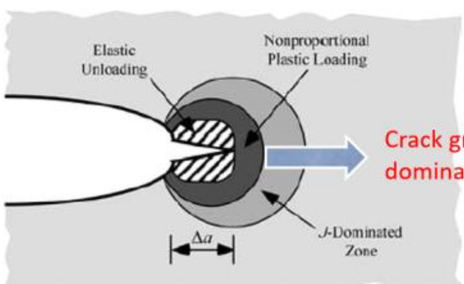
Thursday, October 20, 2022 2:23 PM

When PFM models will fail?

- When we must use finite deformation theory OR there is a significant plastic unloading

LSY: When a single parameter (G, K, J, CTOD) is not enough?

- Under considerable plastic deformation and crack propagation when unloading and non-proportional zones grow out of J dominant zone with crack propagation. Reasons are:
  - Unloading: In J integral analysis plastic model was replaced by a nonlinear solid
  - Single-parameter identification not valid since various stress components increase at different rates



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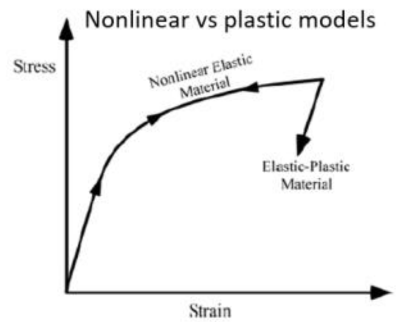


What should be done when LY condition is encountered?

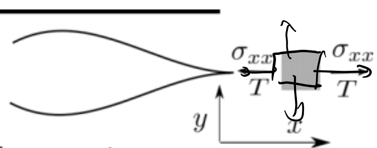
- Advanced computational method with appropriate model assumptions (recommended)
- Extend previous models by adding more parameters (K + T stress) or (J + Q)

LSY: When a single parameter (G, K, J, CTOD) is not enough? T stress

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) + \begin{bmatrix} T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \nu T \end{bmatrix}$$

plane strain

$\frac{T\sqrt{\pi a}}{K_I}$



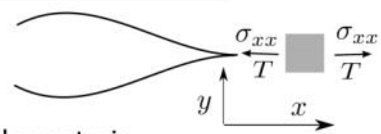
$$\sigma_{x-x} = \frac{K_I}{\sqrt{2\pi r}} f_{xx}(\theta) + T$$

T stress

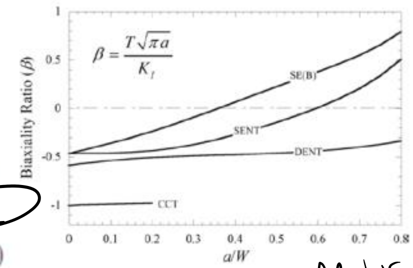
- Higher order terms in stress expansion:
  - T stress (linear analysis)

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) + \begin{bmatrix} T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \nu T \end{bmatrix}$$

plane strain



    - Constant σ_{xx} in LEFM expansion
    - Nondimensional biaxiality ratio: $\beta = \frac{T\sqrt{\pi a}}{K_I}$
    - Example $\beta = -1$ for mode-I crack in infinite domain.
    - T stress redistributes plastic stress
    - $\beta(T)$ depend on particular geometry/loading configuration
    - Effect of $T(\beta)$ on toughness:



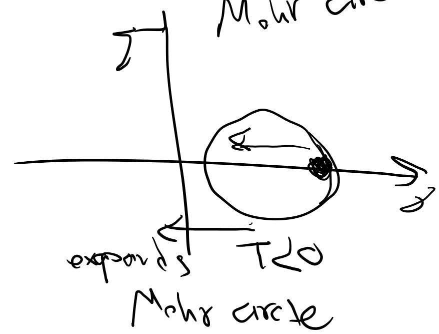
High (+) $T \Rightarrow$ Constrained (triaxial) stress $\Rightarrow$ Toughness ↘ Ductility ↘

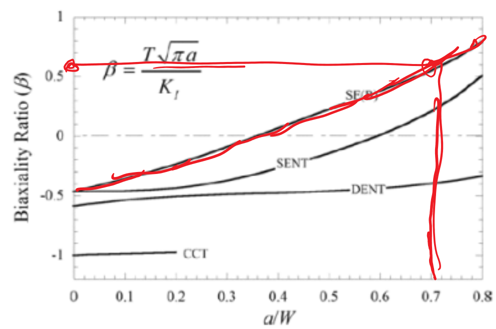
T ↘ Low (-) $T \Rightarrow$ Lose constraint $\Rightarrow$ Toughness ↗ Ductility ↗

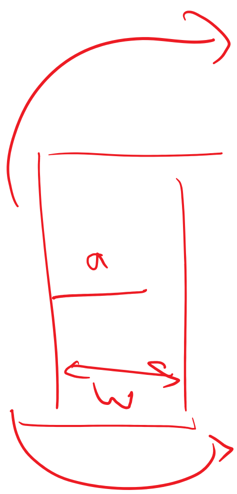
even compress

    - T stress also influnces crack path stability (particulary in dynamic fracture)

$$\beta = \frac{T\sqrt{\pi a}}{K_I}$$ biaxiality ratio







$\frac{a}{W} = .7$

$\beta = -1$

max compressive T we can get

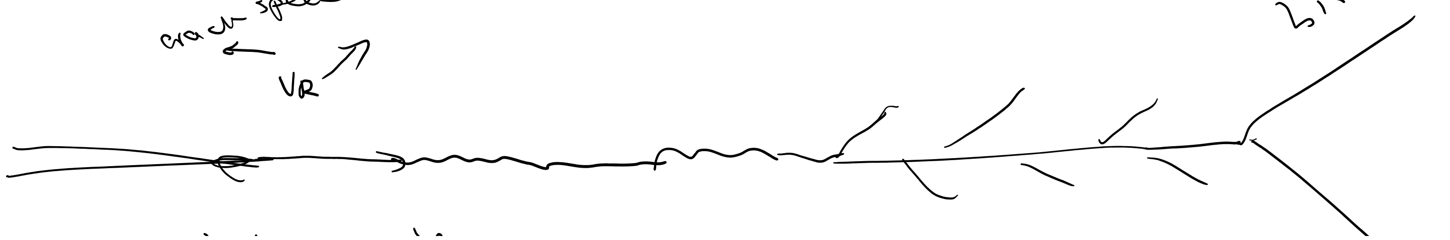
maximum ductility

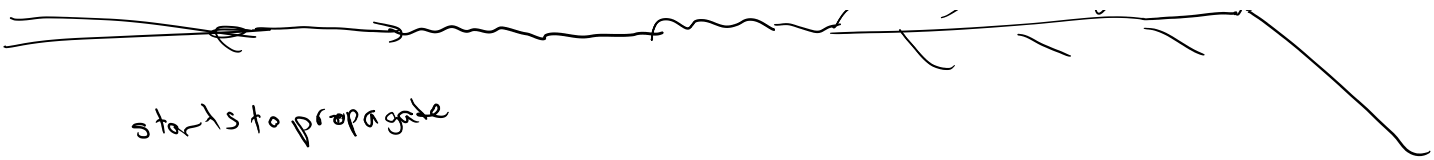
$\beta \cong .6$

$$T = .6 \frac{K_I}{\sqrt{\pi a}} > 0$$

more brittle fracture than in crack infinite domain

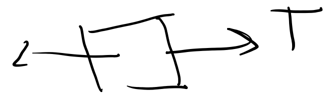
The effect of T stress in dynamic fracture





starts to propagate

T stress > 0   stabilizes the crack path



T stress

- T < 0 -> more ductile fracture / higher toughness
- Higher tensile T stabilizes dynamic crack path

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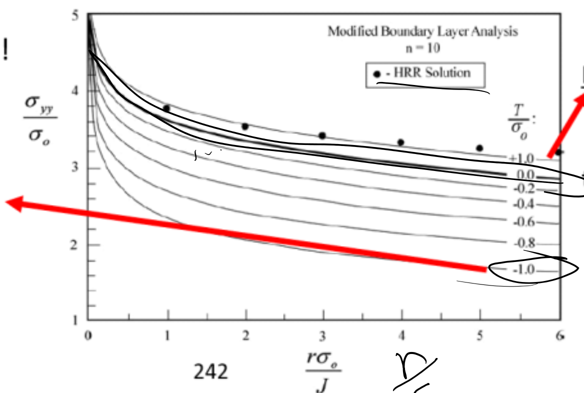
* T stress also influnces crack path stability (particulary in dynamic fracture) a/W

Plastic analysis: σ_{yy} is redistributed!

Kirk, Dodds, Anderson

<u>High negative T stress</u>:

- Decreases σ_{yy}
- Decreases triaxiality



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$\frac{T}{\sigma_o} = -1$

$r/\delta \rightarrow$ CTOD

<u>Positive T stress</u>:

- Slightly Increases σ_{yy} and increase triaxiality

LSY: When a single parameter (G, K, J, CTOD) is not enough? J-Q theory

– Q parameter (J-Q theory) Valid for nonlinear analysis

 * Added as a hydrostatic shift in front of crack to (HRR) stress fields

$$\sigma_{ij} \approx (\sigma_{ij})_{T=0} + Q\sigma_0\delta_{ij} \quad \left(|\theta| \le \frac{\pi}{2}\right)$$

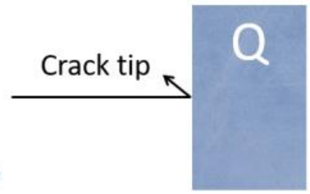
 * Similar to T positive Q increases triaxiality and reduces fracture resistance

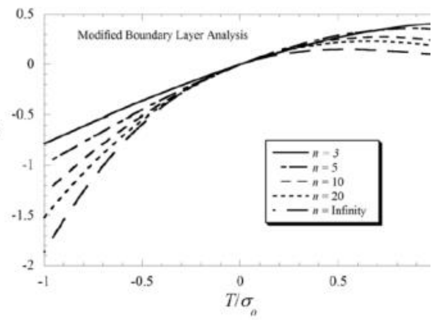
$$J_c = J_c(Q)$$

High (+) $Q \Rightarrow$ Constrained (triaxial) stress $\Rightarrow$ Toughness ↘ Ductility ↘

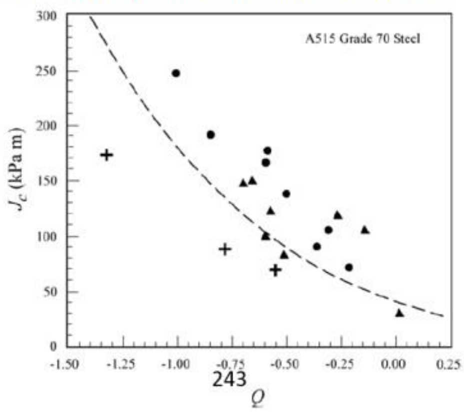
Low (-) $Q \Rightarrow$ Lose constraint $\Rightarrow$ Toughness ↗ Ductility ↗

– More number of parameters: With extensive deformation two-parameter models such as K, T or J, Q evnetully break.





n: strain hardening in HRR analysis



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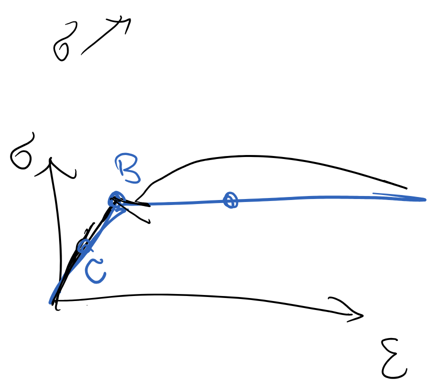
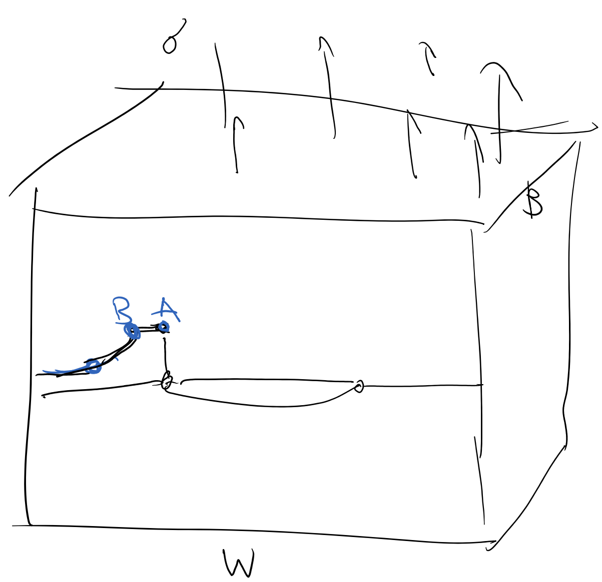
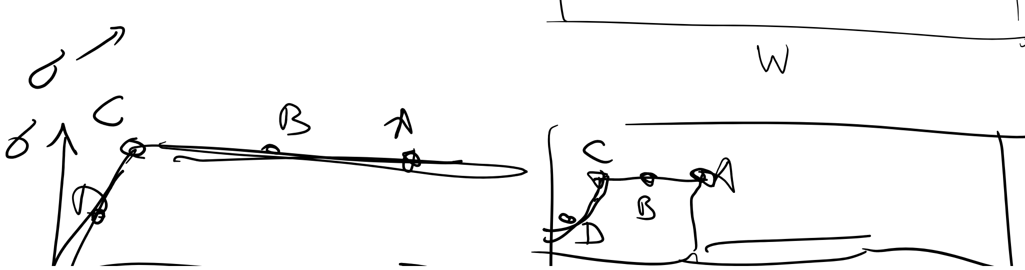
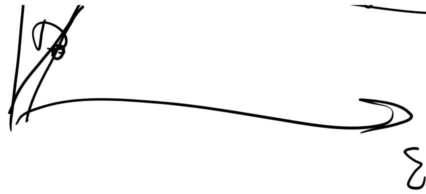
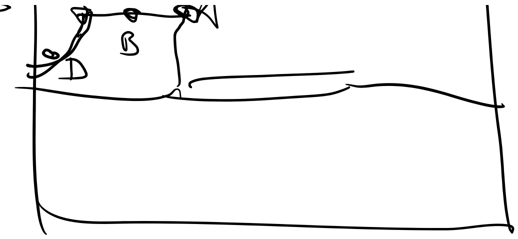
5.3. 7. Fracture mechanics versus material (plastic strength



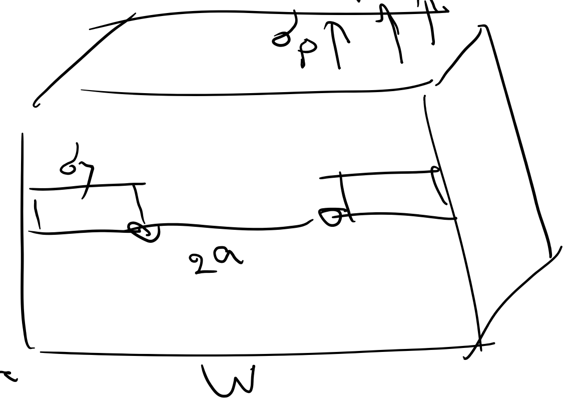








Full plastic load capacity



$$\sigma_p BW = \sigma_y (W-2a) B$$

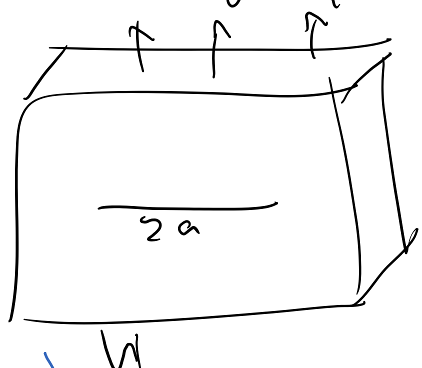
↓ maximum load for plastic failure

$$\boxed{\sigma_p = \left(\frac{W-2a}{W}\right)\sigma_y} \quad ①$$

plastic failure

What about fracture limit of this

$\sigma \to \sigma_f$ when fracture criterion is satisfied



$$K_I = f\left(\frac{a}{W}\right)\sigma\sqrt{\pi a}$$

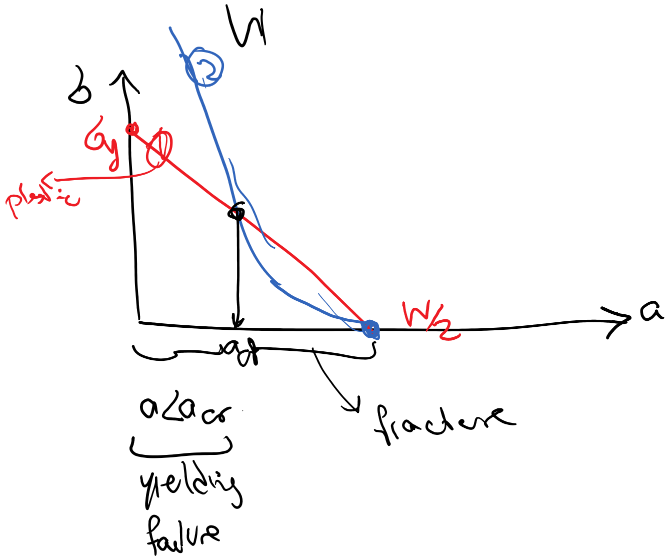
$$K_{IC} = f\left(\frac{a}{W}\right)\sigma_f\sqrt{\pi a}$$

$\sigma \to \sigma_f$

$K_I \to K_{IC}$

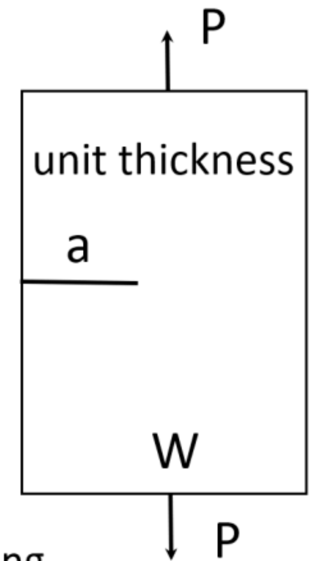
$$\boxed{\sigma_f = \frac{K_{IC}}{f\left(\frac{a}{W}\right)\sqrt{\pi a}}} \quad ②$$

fracture

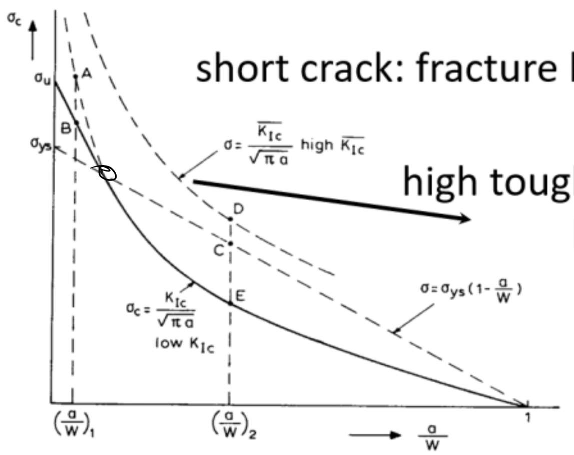


$$\sigma_{\text{net}} = \frac{P}{W-a} = \sigma\frac{W}{W-a} \qquad \sigma = \frac{P}{W}$$

(cracked section)



Yield: $\sigma\dfrac{W}{W-a} = \sigma_{ys} \longrightarrow \sigma = \sigma_{ys}\left(1-\dfrac{a}{W}\right)$



short crack: fracture by plastic collapse!!!

high toughness materials:yielding before fracture

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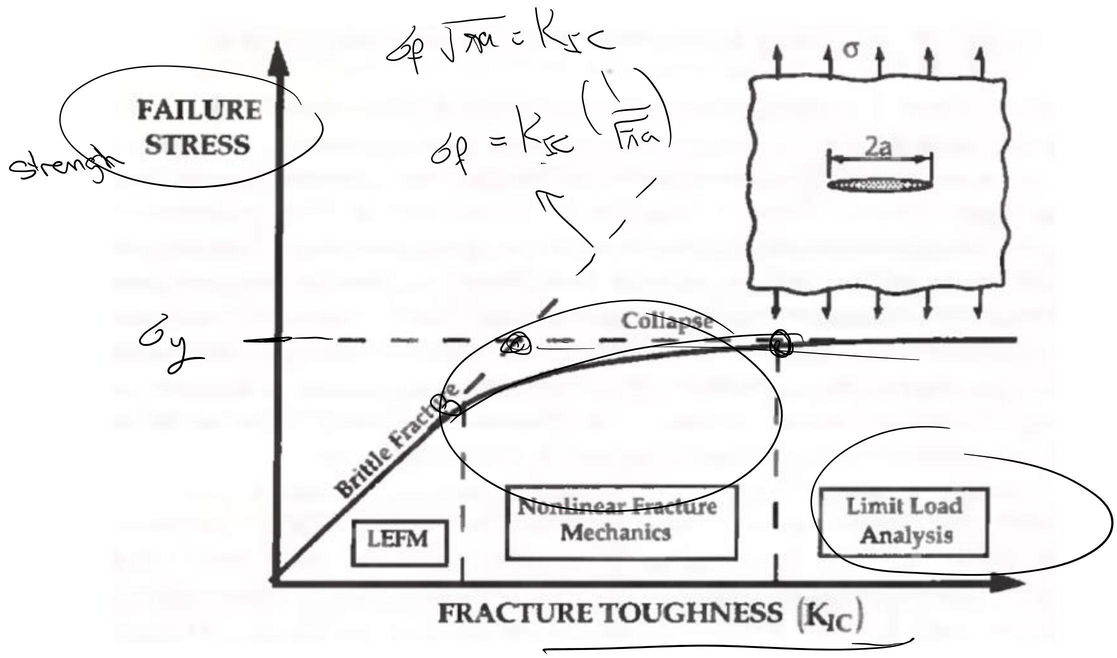
Governing fracture mechanism and fracture toughness



for infinite domain

Example

Example 4.11 Estimate the failure load under uniaxial tension for a centre-cracked panel of aluminium alloy of width W=500 mm, and thickness B=4 mm, for the following values of crack length $2a$ = 20 mm and $2a$ = 100 mm. Yield stress σ_y=350 MPa and fracture toughness K_{Ic}=70 MPa$\sqrt{m}$

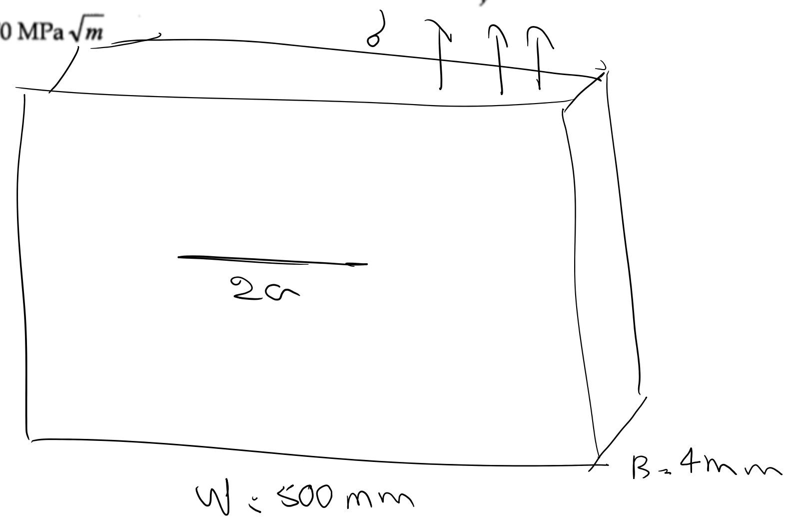
$\sigma_y = 350\,MPa$

$K_{Ic} = 70\,MPa\sqrt{m}$

1) $2a = 20\,mm$

2) $2a = 200\,mm$

what is the ultimate load?



① $2a = 20\,mm \quad a = 10\,mm$

yielding $\quad \sigma_y (W - 2a) B = \sigma_p WB$

$$\sigma_p = \sigma_y \frac{(W - 2a)}{W} = 350 \times \frac{500-20}{500} \Longrightarrow \boxed{F_p = \sigma_p BW = 672\,kN}$$

plastic failure

fracture $\quad \sigma_f = \frac{K_{Ic}}{f\left(\frac{a}{W}\right)\sqrt{\pi a}} = \frac{70\,MPa\sqrt{m}}{\sqrt{\sec\left(\frac{\pi a}{W}\right)}\sqrt{\pi \times (10e{-3}\,m)}} = 394.6\,MPa$

$a \rightarrow 10\,mm$, $W \rightarrow 500\,mm$

$$\boxed{F_f = \sigma_f BW = 790\,kN}$$

② $F_p = 560\,kN \qquad F_f = 172.2\,kN$

fractures first

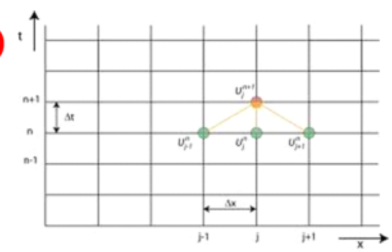
6.1Fracture mechanics in Finite Element Methods (FEM)

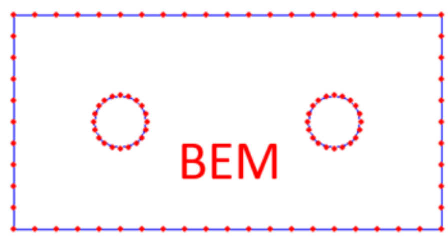
6.1.1. Introduction to Finite Element method
6.1.2. Singular stress finite elements
6.1.3. Extraction of K (SIF), G
6.1.4. J integral
6.1.5. Finite Element mesh design for fracture mechanics
6.1.6. Computational crack growth
6.1.7. Extended Finite Element Method (XFEM)

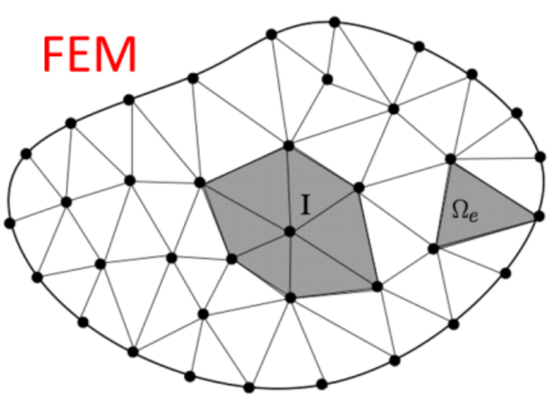
Numerical methods to solve PDEs

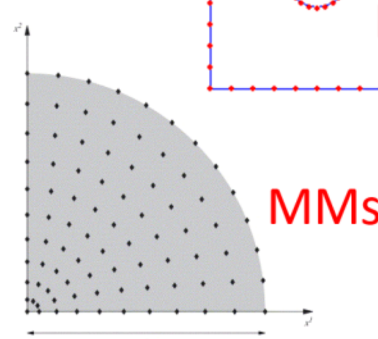
- Finite Difference (FD) & Finite Volume (FV) methods
- FEM (Finite Element Method)
- BEM (Boundary Element Method)
- MMs (Meshless/Meshfree methods)

FD









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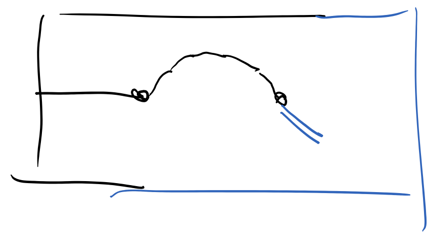
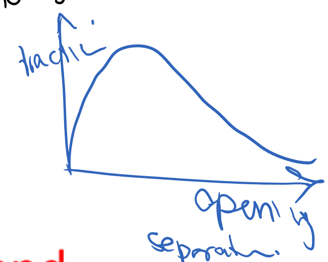
Fracture models

- Discrete crack models (discontinuous models): Cracks are explicitly modeled
  - LEFM
  - EPFM
  - Cohesive zone models

failure is restricted to surfaces



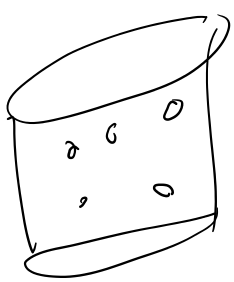


- Continuous models: Effect of (micro)cracks and voids are incorporated in bulk damage
  - Continuum damage models
  - Phase field models

- Peridynamic models: Material is modeled as a set of particles

Bulk models

Continuum / bulk damage



$$\underbrace{\sigma =}_{(1-D)} C\varepsilon$$

$$\sigma = (1-D)\, C\varepsilon$$

$D = 1$ fully debonded

Phase field model: Regularizes LEFM (very different starting point than bulk damage)

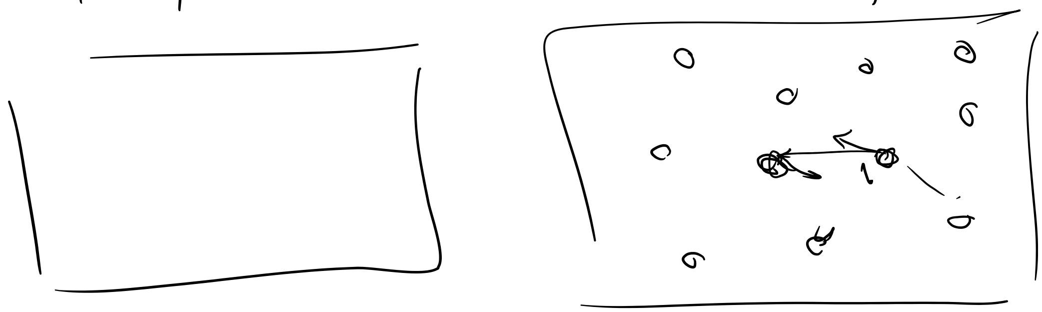
$$\sigma = \underbrace{\omega(d)}_{(1-D)} C\varepsilon$$

or other forms

Peridynamics similar to Molecular dynamic

new models

Peridynamics similar to Molecular dynamic

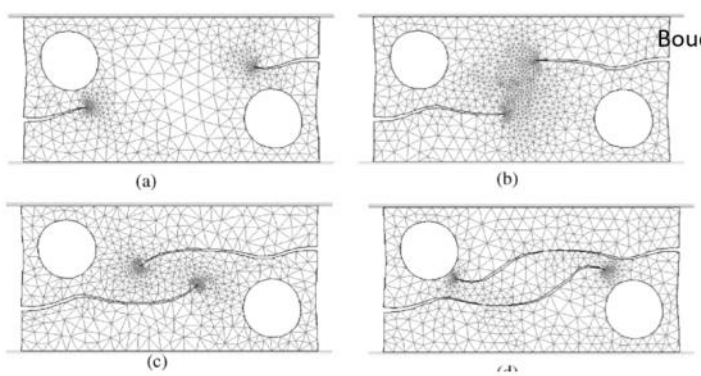


new models don't have the limitation of original models

Advantages and challenges of different models

What's wrong with FEM for crack problems

- Element edges must conform to the crack geometry: make such a mesh is time-consuming, especially for 3D problems.
- Remeshing as crack advances: difficult. Example:



Bouchard et al. CMAME 2003

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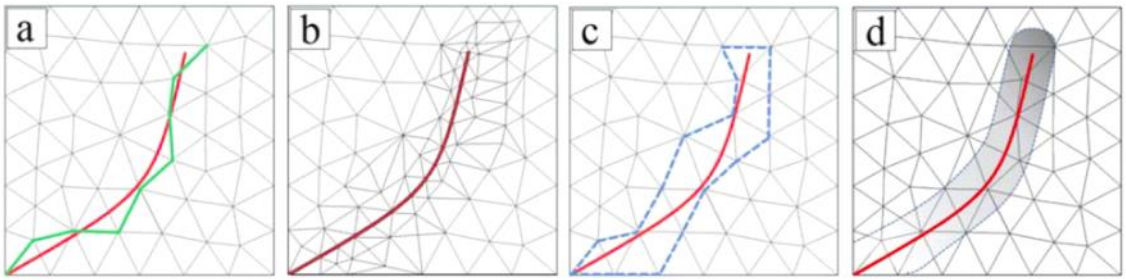
Capturing/tracking cracks



Fixed mesh

Crack tracking

XFEM enriched elements

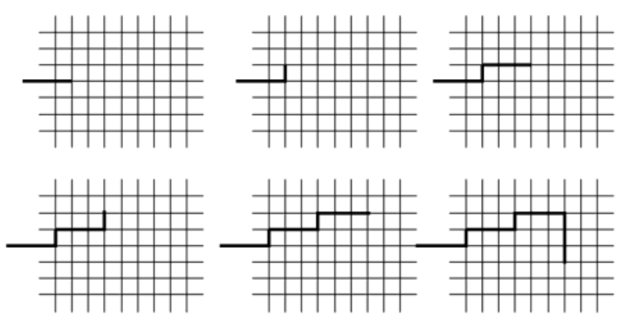
Brief overview in the next section

Crack/void capturing by bulk damage models

Brief overview in continuum damage models

Fixed meshes

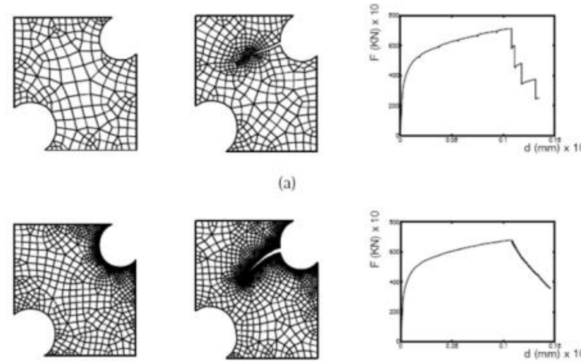
- Nodal release method (typically done on fixed meshes)
  - Crack advances one element edge at a time by <u>releasing FEM nodes</u>
  - Crack path is restricted by discrete geometry



- Also for cohesive elements they can be used for both extrinsic and intrinsic schemes. For intrinsic ones, cohesive surfaces between all elements induces an artificial compliance (will be explained later)

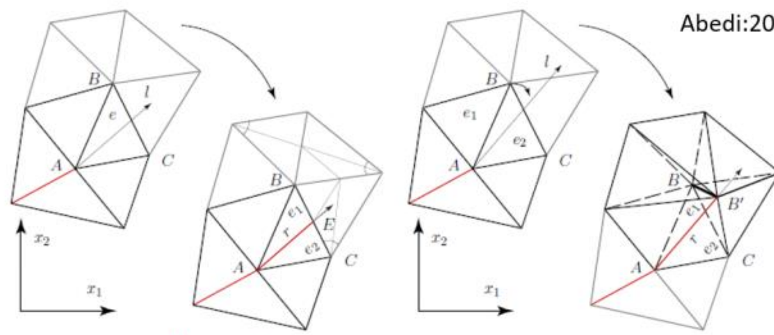
Adaptive meshes

- Adaptive operations align element boundaries with crack direction



Element splitting:
Smoother crack path by element splitting: cracks split through and propagate between newly generated elements

Abedi:2010



Cracks generated by refinement options

Element edges move to desired direction

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