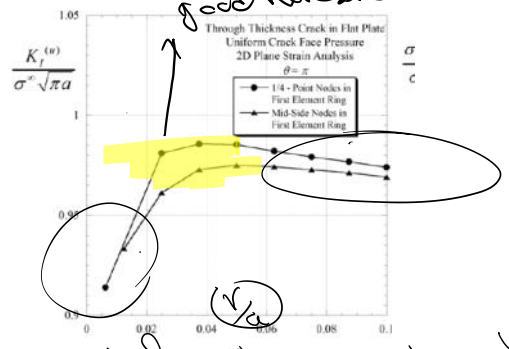
2022/11/01

Tuesday, November 1, 2022 2:31 PM

Different elements/methods to compute K

Displacement method



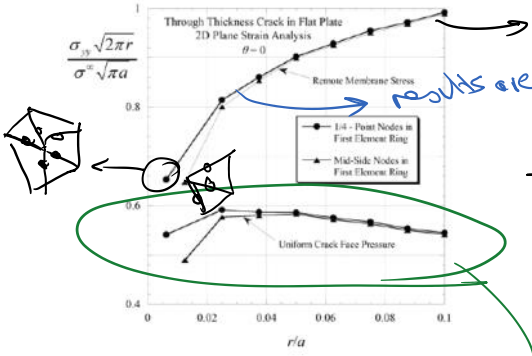
good here 2.5 y.

getting farther from singular dominant zone

Numerical results are not good (cannot capture the singularity well)

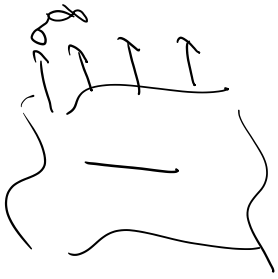
K from displacement *u*

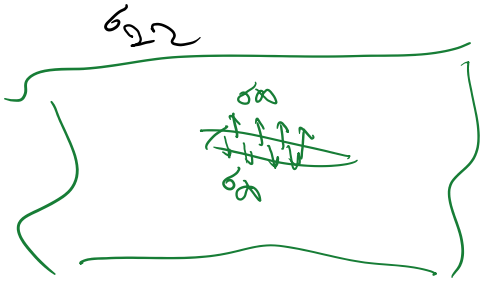
$$K_I = \lim_{r\to 0}\left[\frac{E' u_y}{4}\sqrt{\frac{2\pi}{r}}\right] \quad (\theta = \pi)$$



for field loading

results are bad 30+% error



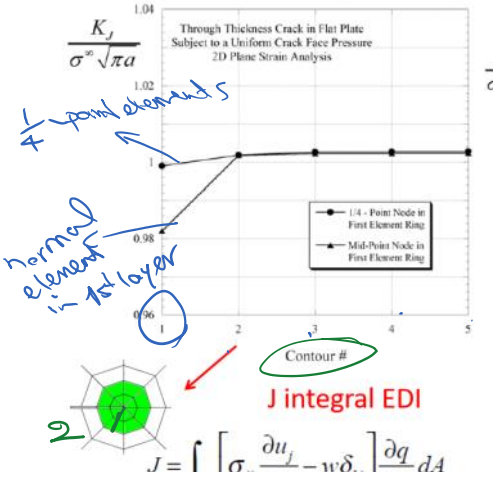


K from stress σ

$$K_I = \lim_{r\to 0}\left(\sqrt{2\pi r}\ \sigma_{22}|_{\theta=0}\right)$$

stress-based approach is sensitive to crack surface tractions

EDI that we covered last time



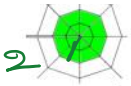
1/4 point elements

normal element in 1st layer

J integral EDI

$$J = \int \left[\sigma_{ij}\frac{\partial u_j}{\partial x} - w\delta_{1j}\right]\frac{\partial q}{\partial x}\,dA$$

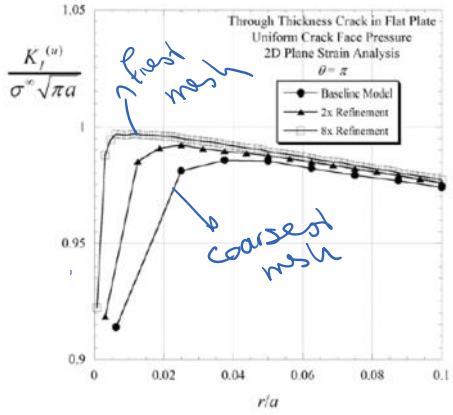
Use two layers + and even with normal elements we get very good results for J and K



J integral EDI

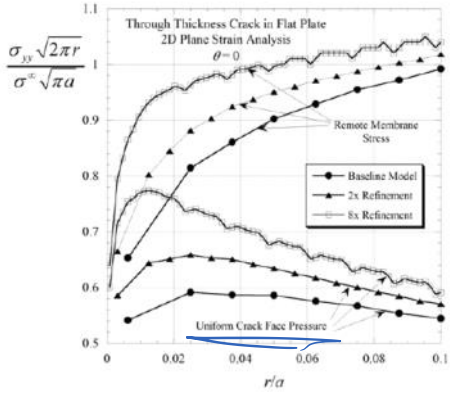
$$J = \int_{A^*} \left[\sigma_{ij} \frac{\partial u_j}{\partial x_1} - w\delta_{1j} \right] \frac{\partial q}{\partial x_j} dA$$

Is using finer finite element method going to address the other two method's inaccuracies



K from displacement ***u***

$$K_I = \lim_{r \to 0} \left[\frac{E' u_y}{4} \sqrt{\frac{2\pi}{r}} \right] \quad (\theta = \pi)$$

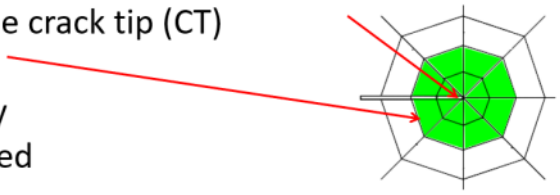


K from stress σ

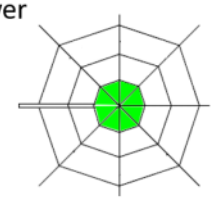
$$K_I = \lim_{r \to 0} \left(\sqrt{2\pi r}\ \sigma_{22}|_{\theta=0} \right)$$

J integral: 2. EDI FEM Aspects

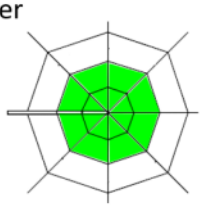
- Since J_0 ~~→0~~ the inner J_0 collapses to the crack tip (CT)
- J_1 will be formed by element edges
- By using spider web (rozet) meshes any reasonable number of layers can be used to compute J:



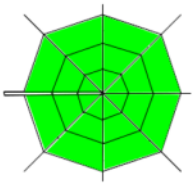
1 layer



2 layer

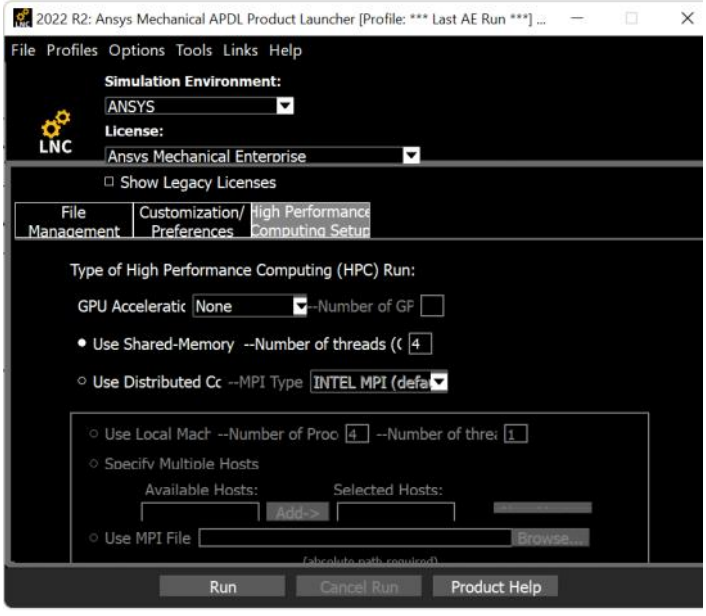


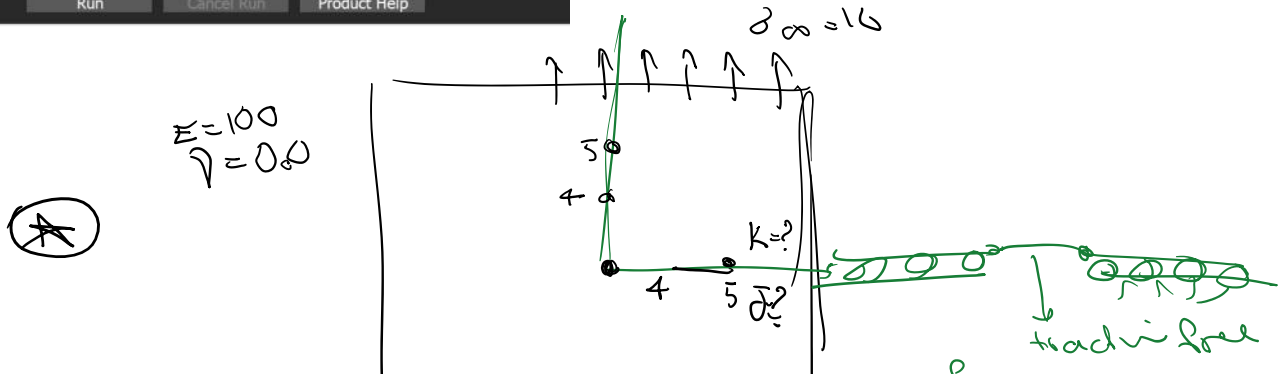
3 layer

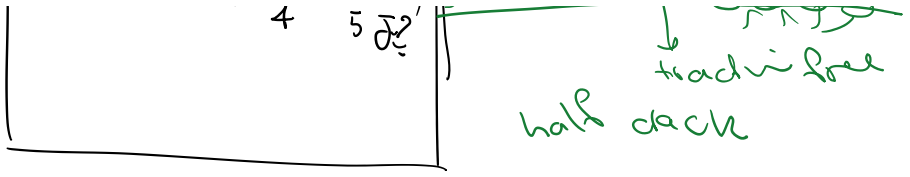


Computational project
Launch Ansys mechanical APDL

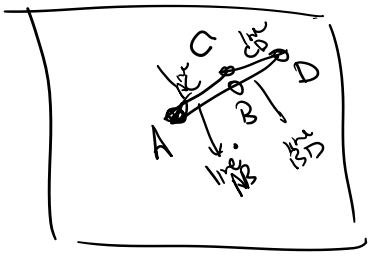
Make sure to use shared memory option on your computer after launching the program:







4C

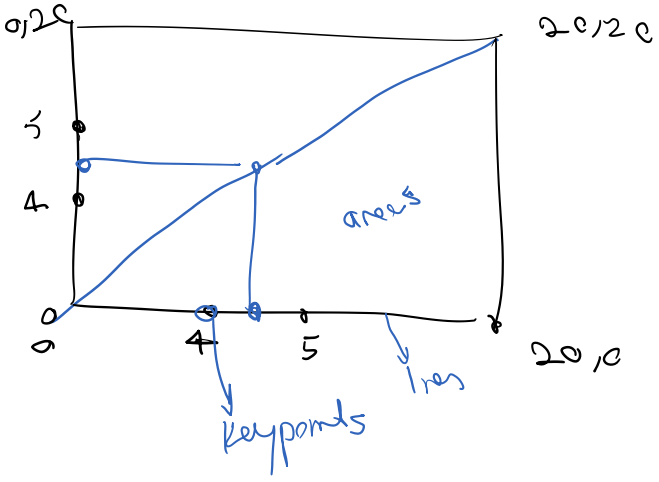


Geometry 0D Keypoint Vertex 1D line 2D surface area 3D volume

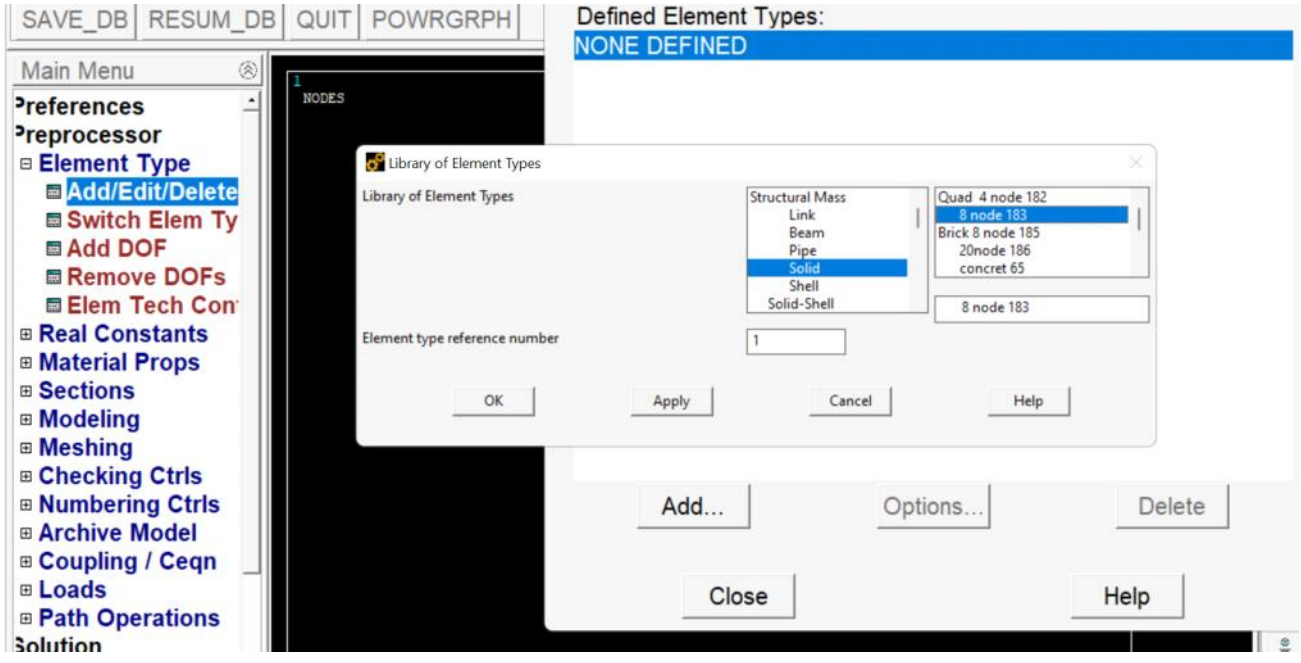
to create internal crack AD
we need at least two internal keypoints
on two crack surfaces with identical coordinate

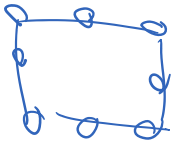
Problem ⭐

our subsequent calculations need at most 1 crack tip in each area

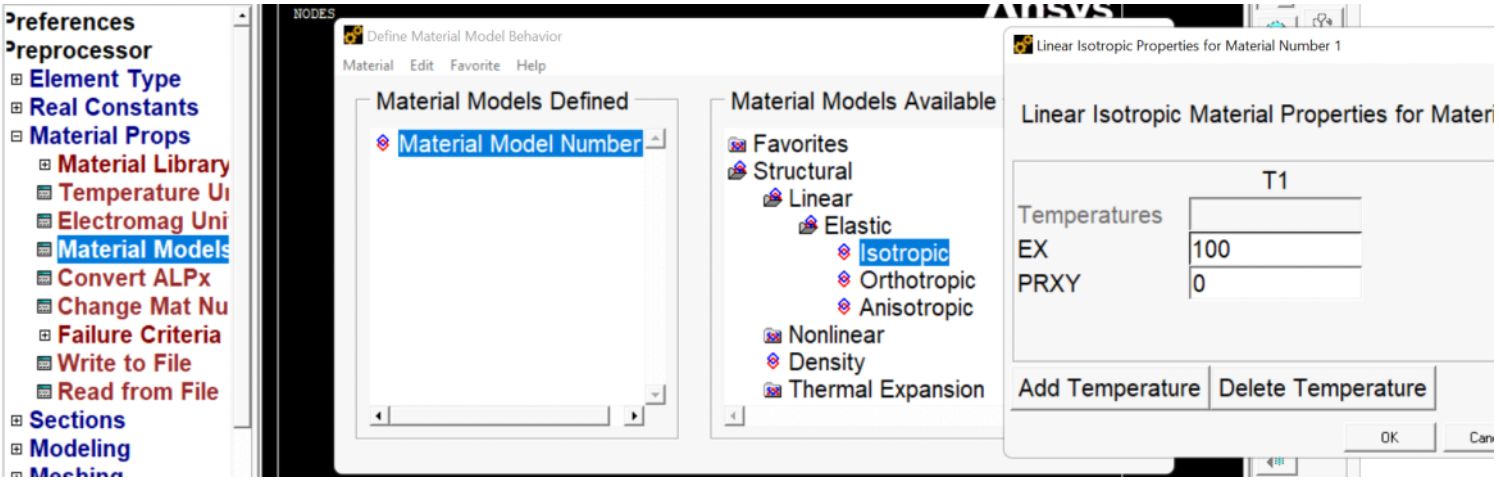


Steps:

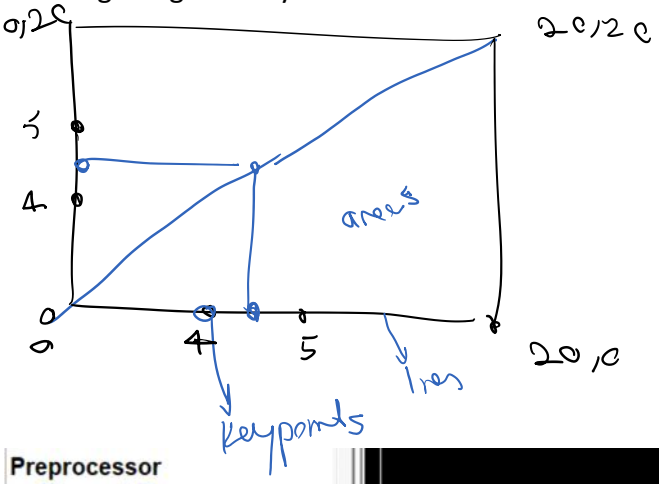


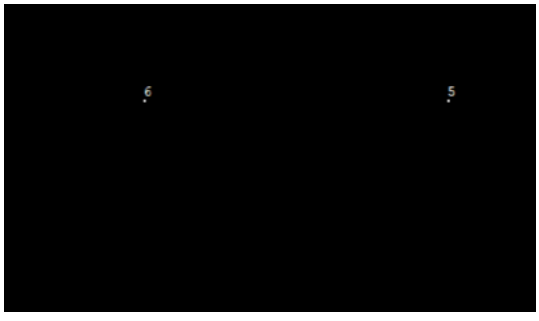


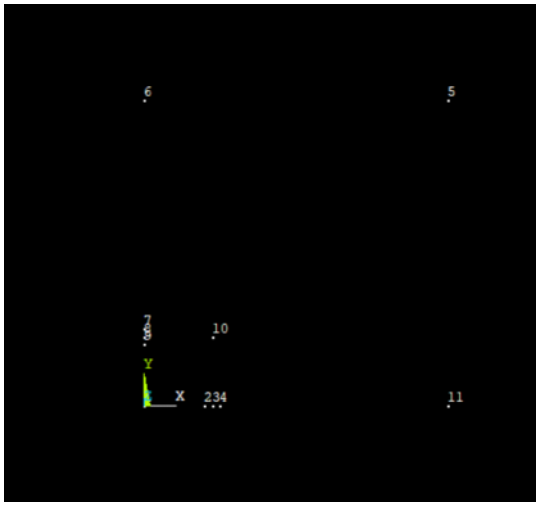
Use plane strain
Add the material



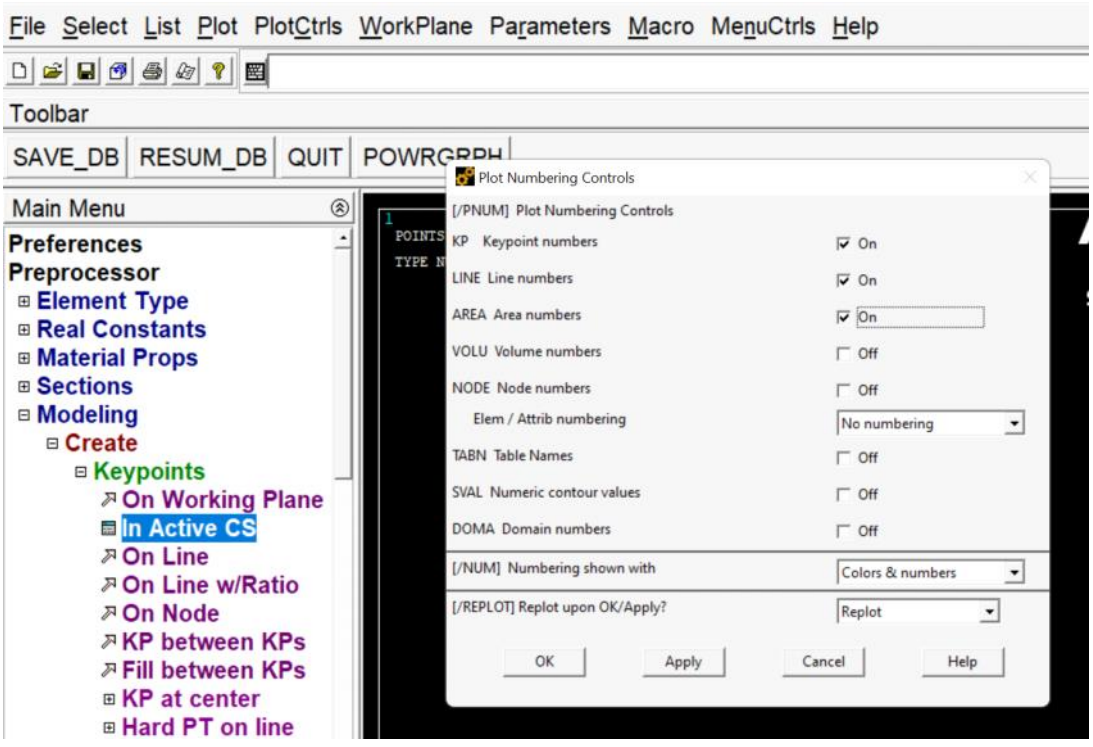
Creating the geometry:



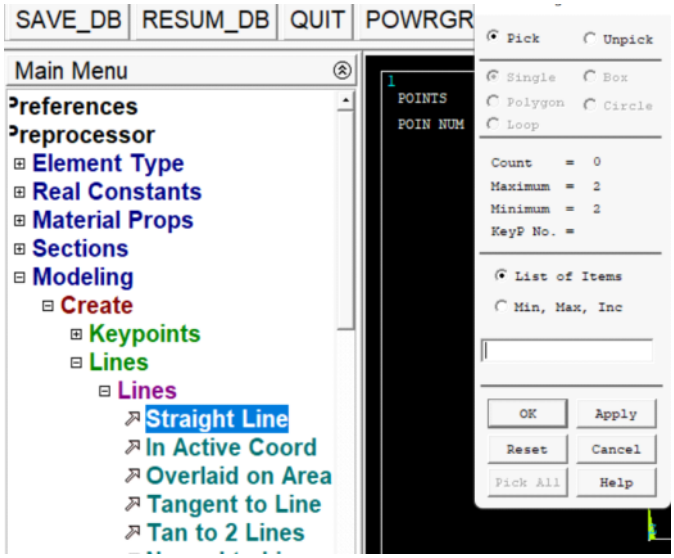




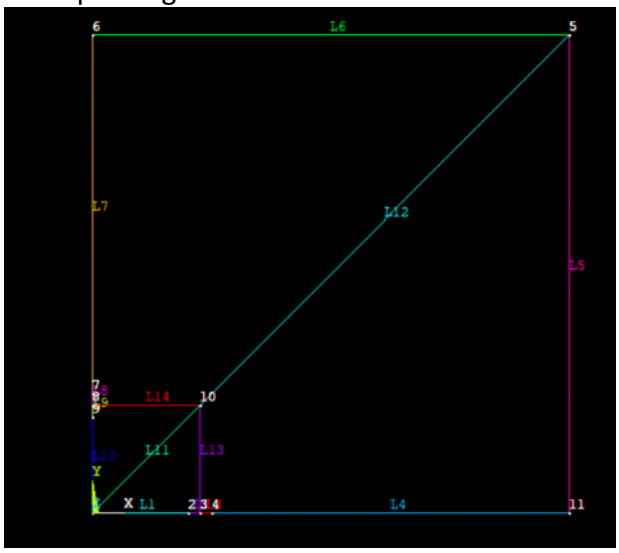
Use plot cntrls to show keypoint, line, area numbers



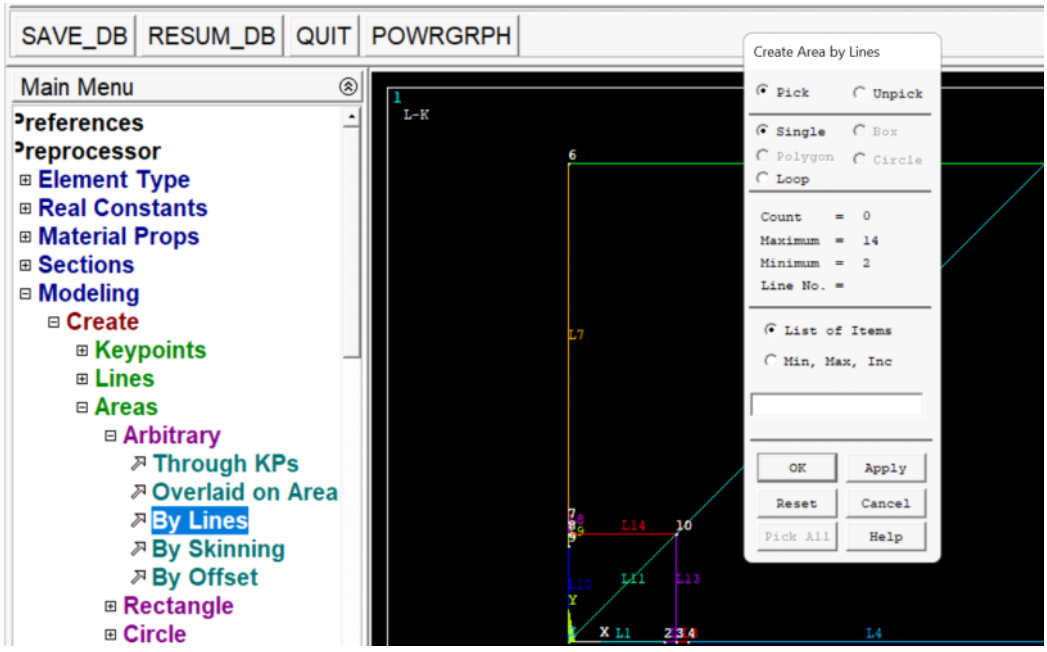
Creating straight lines between keypoints

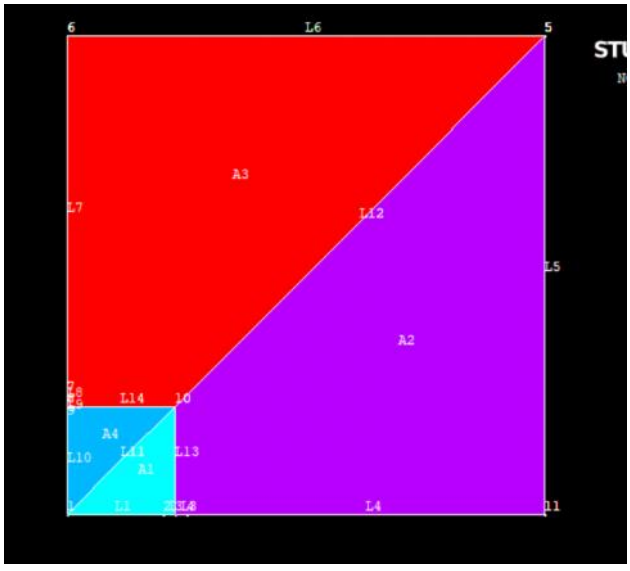


After plotting the lines

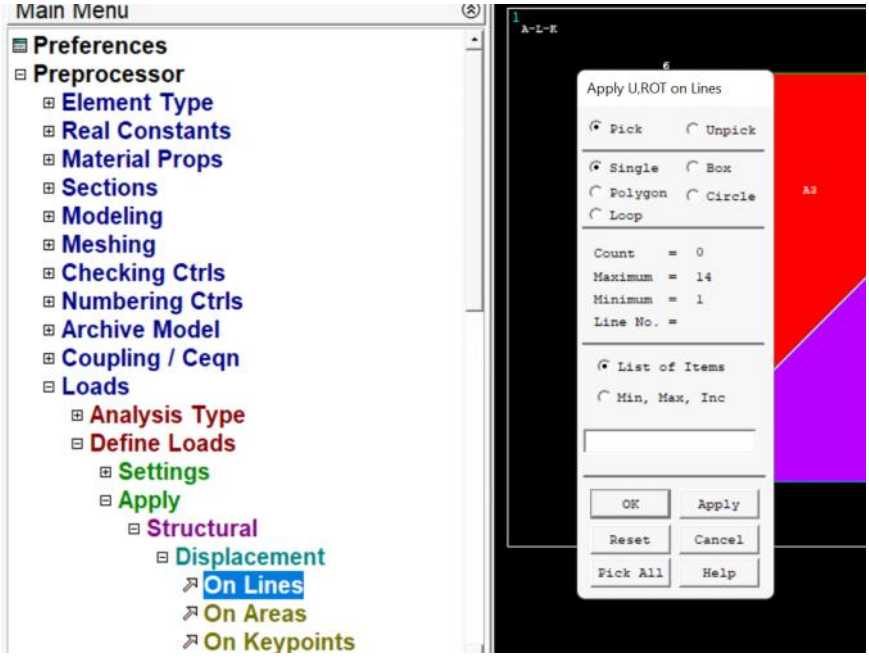


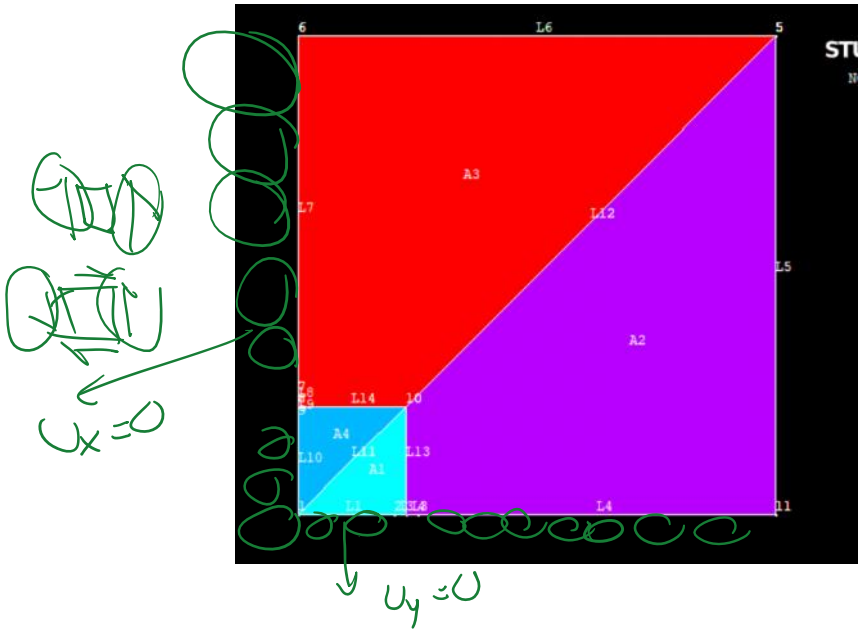
Next, create the areas:


SAVE_DB
RESUM_DB
QUIT
POWRGRPH
Main Menu
Preferences
Preprocessor
Element Type
Real Constants
Material Props
Sections
Modeling
Create
Keypoints
Lines
Areas
Arbitrary
Through KPs
Overlaid on Area
By Lines
By Skinning
By Offset
Rectangle
Circle
Create Area by Lines
Pick
Unpick
Single
Box
Polygon
Circle
Loop
Count = 0
Maximum = 14
Minimum = 2
Line No. =
List of Items
Min, Max, Inc
OK
Apply
Reset
Cancel
Pick All
Help
L-K
L7
L14
L10
L11
L13
L1
L4

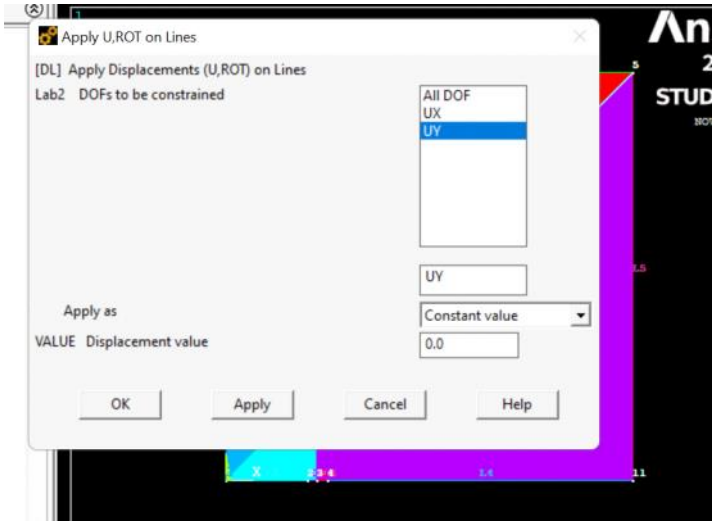

L6
L7
A3
L12
L5
A2
L14
A4
L10
L11
L13
A1
L4

Apply BCs



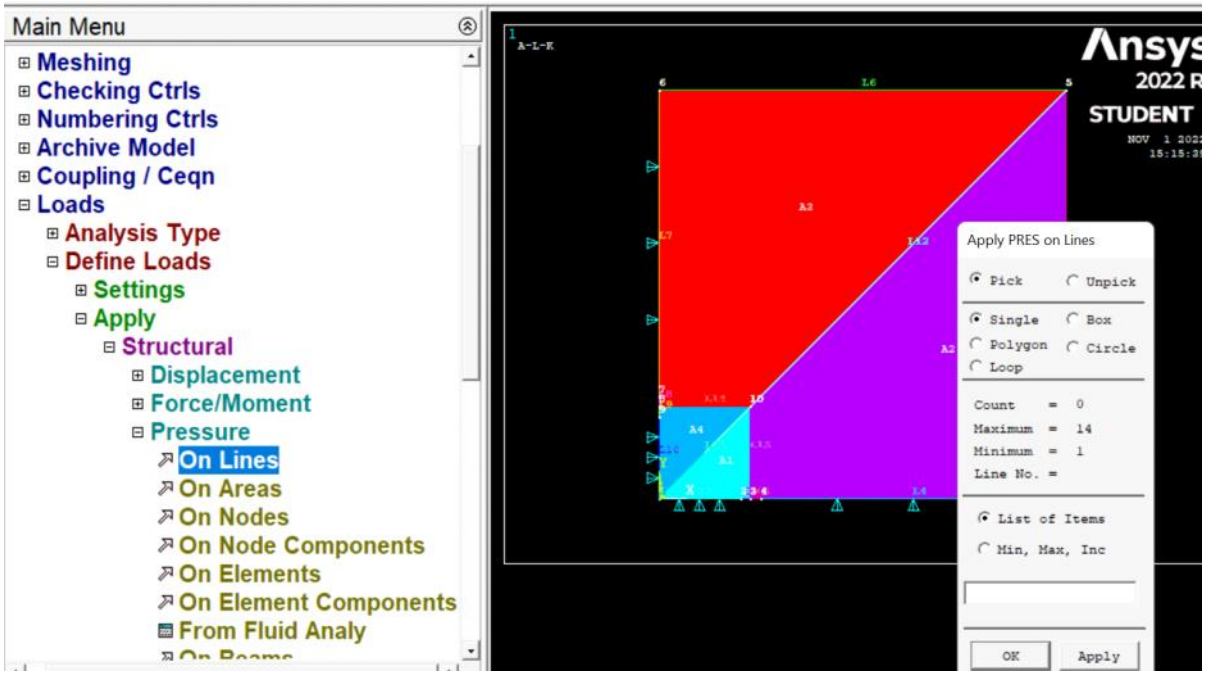


Bottom lines except the crack segment selected



Similar thing for the left side but Ux = 0

Apply the load on the top

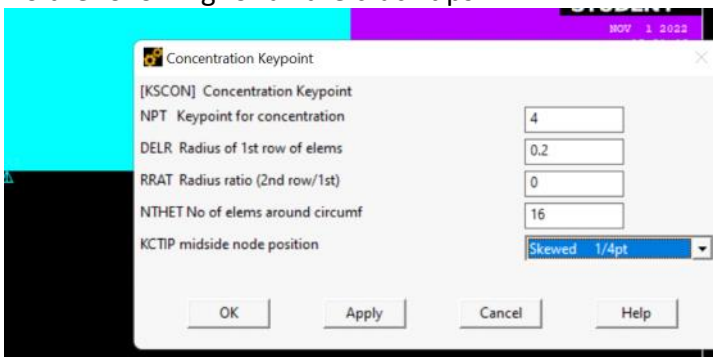


Enter -10

Next is meshing
We first specify crack tips so a spider web mesh is created around it

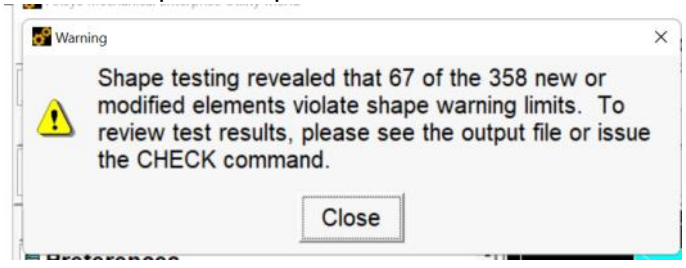
Choose concentrated KP

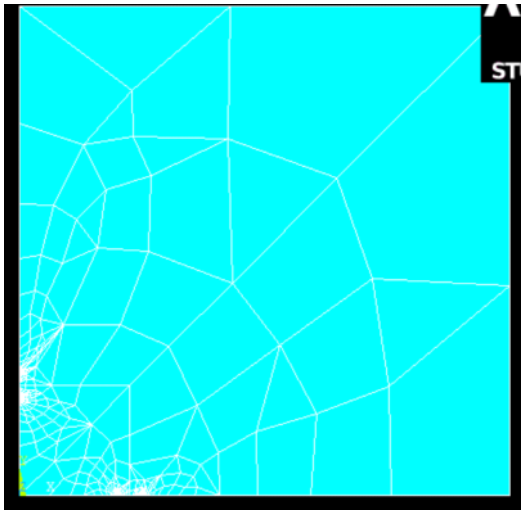
Do the following for all the crash tips



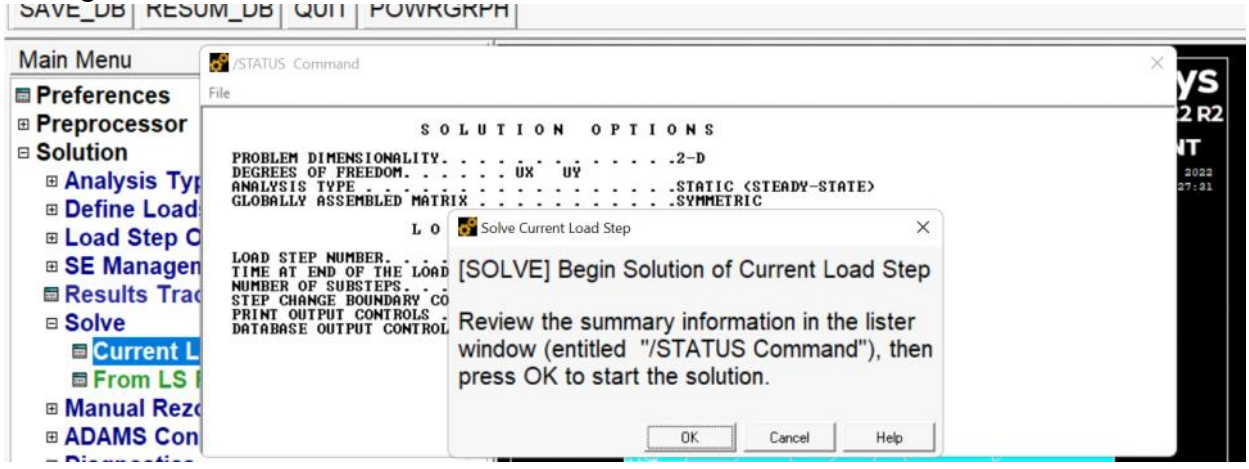
Ready to mesh it now

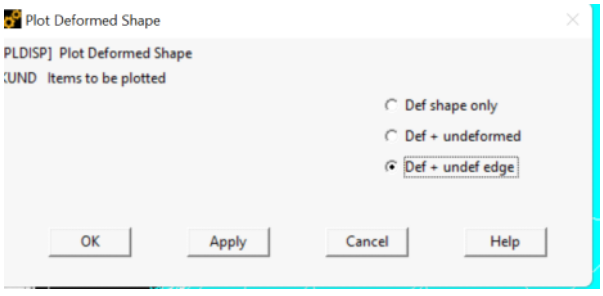
Can choose pick All option

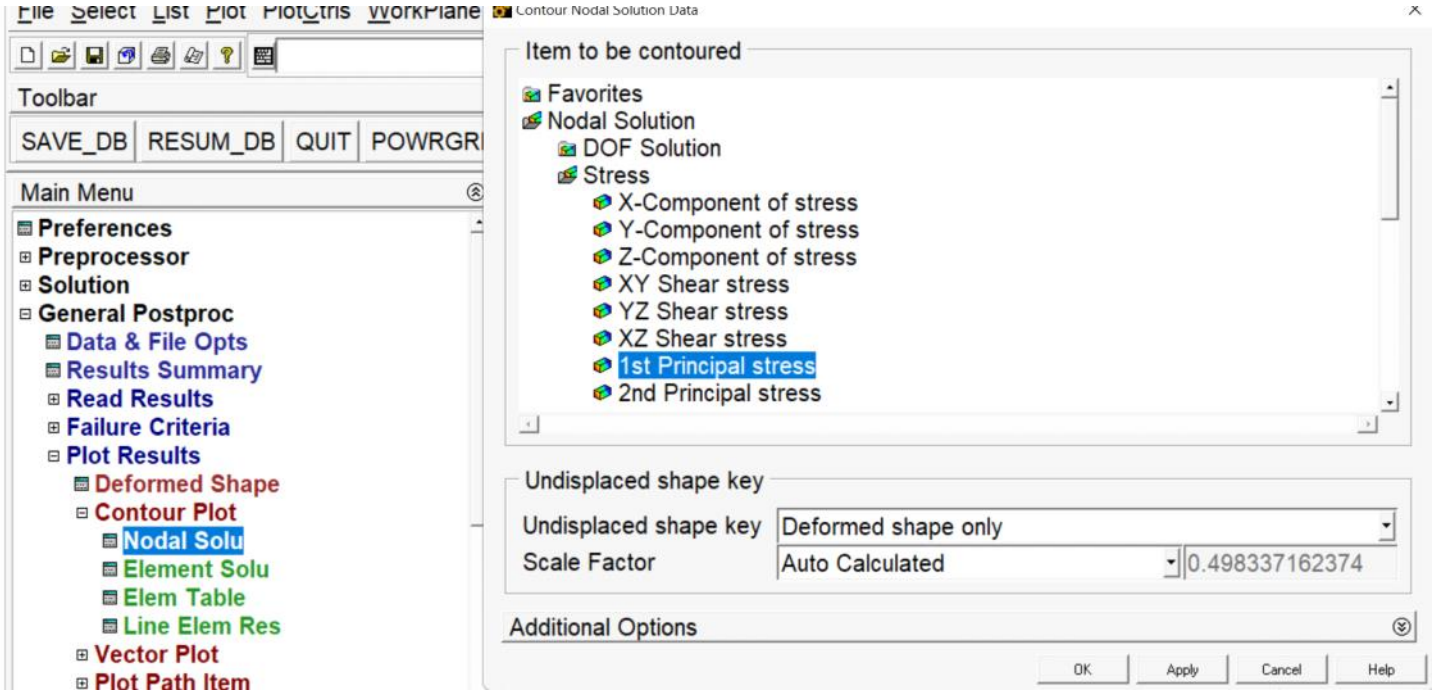


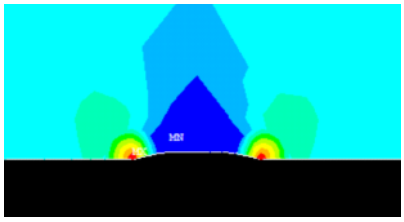


Solving this







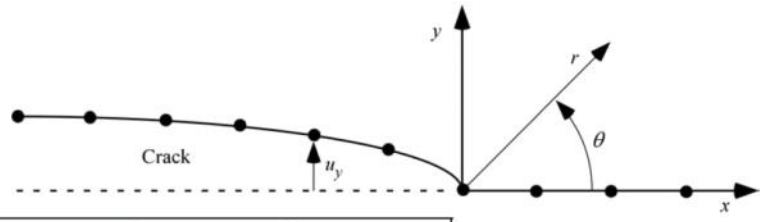


We want to calculate K and J for the right crack tip on the bottom

Recall displacement-based formula:

1. K from local fields

1. Displacement



$$u_y(r,\theta=\pi)=\frac{4K_I\sqrt{r}}{\sqrt{2\pi}E'} \Rightarrow \boxed{K_I=\lim_{r\to 0}\left[\frac{E'u_y}{4}\sqrt{\frac{2\pi}{r}}\right] \quad (\theta=\pi)}$$

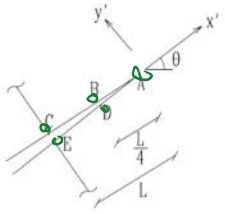
$$E'=\begin{cases}E & \text{plane stress}\\ \frac{E}{1-\nu^2} & \text{plane strain}\end{cases}$$

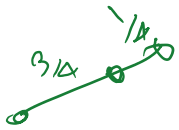
or alternatively from the first quarter point element:

$$v=K_1\frac{\kappa+1}{2G}\sqrt{\frac{r}{2\pi}}$$

$$u' = \overline{u}'_A+\left(-3\overline{u}'_A+4\overline{u}'_B-\overline{u}'_C\right)\sqrt{\frac{r}{L}}+\left(2\overline{u}'_A+2\overline{u}'_C-4\overline{u}'_B\right)\frac{r}{L}$$

$$v' = \overline{v}'_A+\left(-3\overline{v}'_A+4\overline{v}'_B-\overline{v}'_C\right)\sqrt{\frac{r}{L}}+\left(2\overline{v}'_A+2\overline{v}'_C-4\overline{v}'_B\right)\frac{r}{L}$$





Recall for 1D

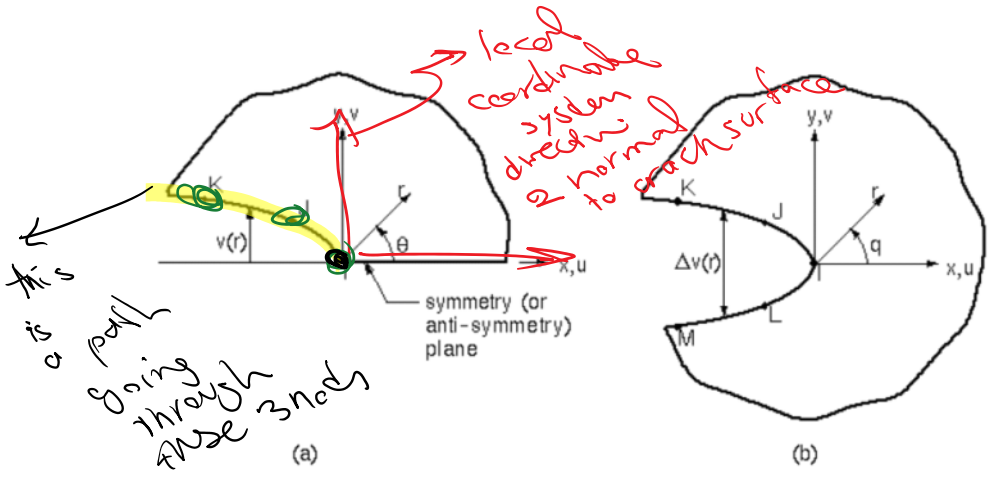
$$u=u_1+\frac{\sqrt{x}}{\sqrt{L}}\left(-3u_1-u_2+4u_3\right)+\frac{2x}{L}\left(u_1+u_2-2u_3\right)$$

$$\boxed{K_I=\frac{2G}{\kappa+1}\sqrt{\frac{2\pi}{L}}\left(-3\overline{v}'_A+4\overline{v}'_B-\overline{v}'_C\right)}$$

$$\begin{Bmatrix}K_I\\ K_{II}\end{Bmatrix}=\frac{1}{2}\frac{2G}{\kappa+1}\sqrt{\frac{2\pi}{L}}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}-3\overline{u}'_A+4\left(\overline{u}'_B-\overline{u}'_D\right)-\left(\overline{u}'_C-\overline{u}'_E\right)\\ -3\overline{v}'_A+4\left(\overline{v}'_B-\overline{v}'_D\right)-\left(\overline{v}'_C-\overline{v}'_E\right)\end{bmatrix}$$

275

Mixed mode generalization:



local coordinate system direction normal to crack surface

this is a path going through these 3 nods

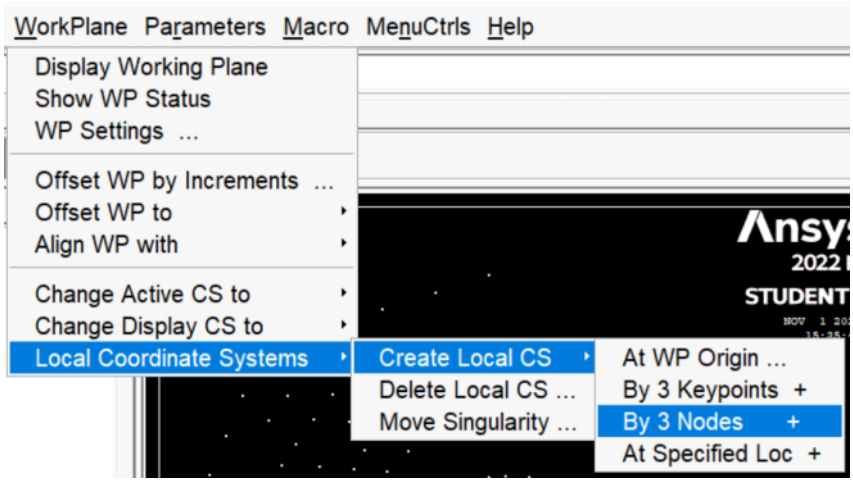
Figure 4) Nodes used for the approximate crack-tip displacements in (a) Half model and (b) Full model

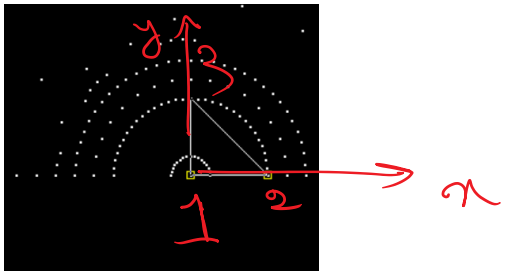
Do 2 things

1. Define a local coordinate system
2. Define path going through the 3 pts above (sym ax)

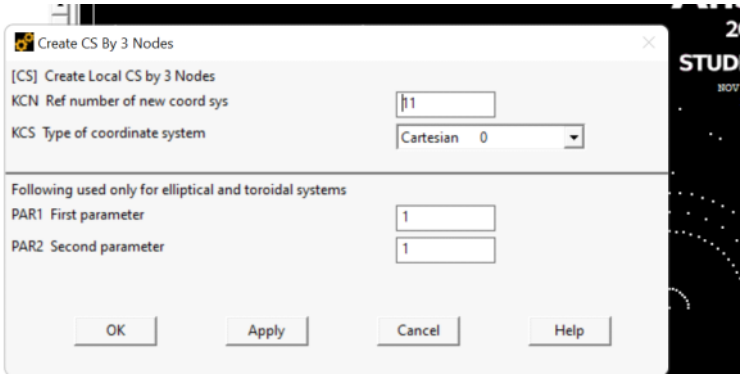
1. Local CS

Plot nodes

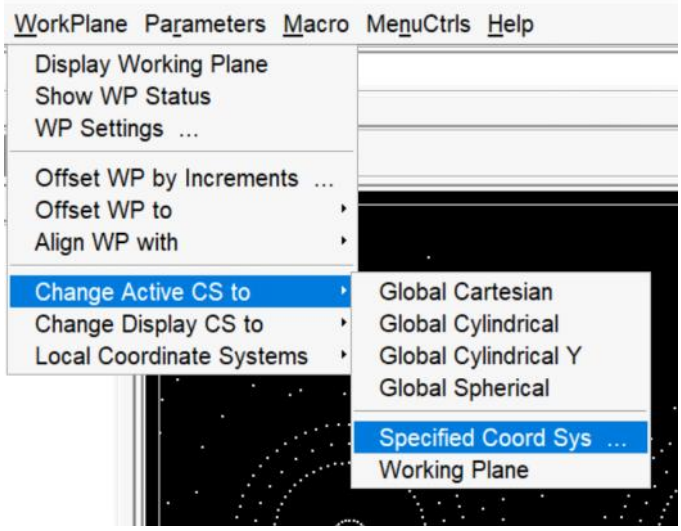


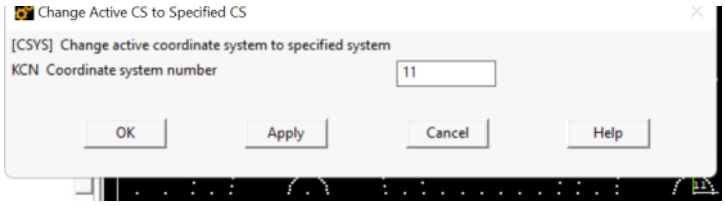


Number 11 is fine



Call this coordinate system





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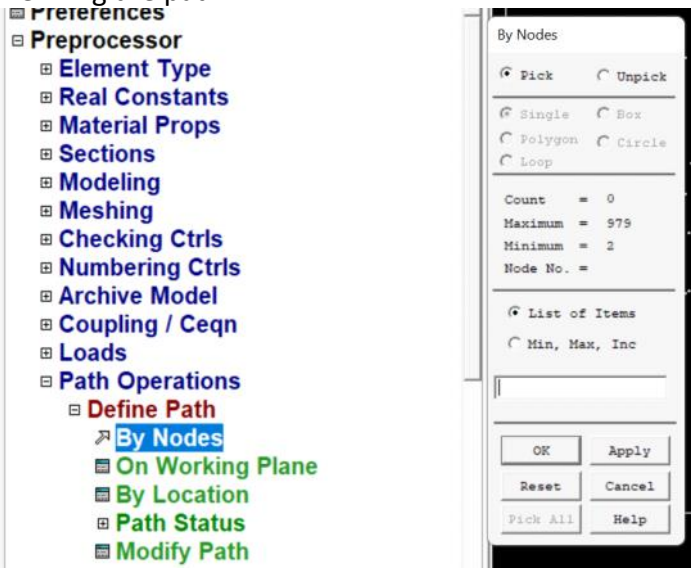
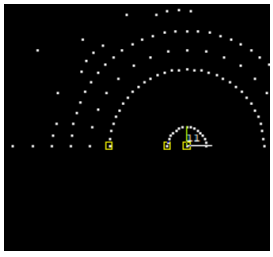
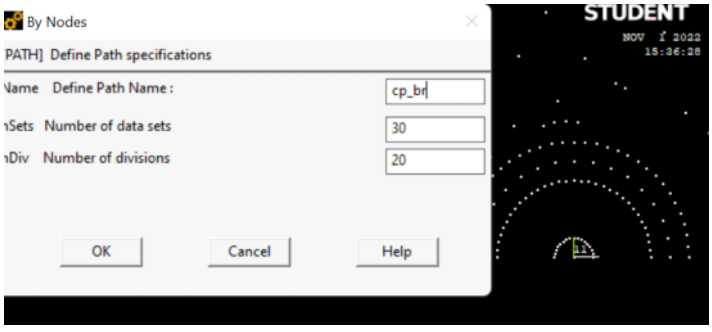
2. Defining the path:



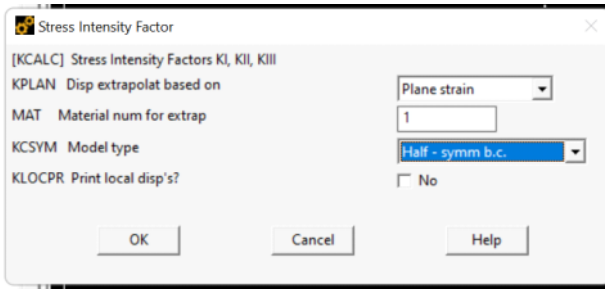




********** PATH STATUS **********

Path	nPts	nSets	nDiv
CP_BR	3	30	20

- General Postproc
  - Data & File Opts
  - Results Summary
  - Read Results
  - Failure Criteria
  - Plot Results
  - List Results
  - Query Results
  - Options for Outp
  - Results Viewer
  - Nodal Calcs
    - Total Force Sum
    - Sum @ Each Node
    - Summation Pt
    - Stress Int Factr



Mechanical APDL 2022 R2 Output Window

```
***** ROUTINE COMPLETED *****  CP =       25.359



        ***** MAPDL ANALYSIS DEFINITION (PREP7) *****

ENTER  /SHOW,DEVICE-NAME  TO ENABLE GRAPHIC DISPLAY
ENTER  FINISH             TO LEAVE PREP7
PRINTOUT KEY SET TO /GOPR (USE /NOPR TO SUPPRESS)

PARAMETER _Z1 =     3.000000000

Define Path  Name= CP_BR     nPts=    3  nSets=  30  nDiv=  20
PPATH  Point=  1  Node=     242  X,Y,Z=  5.000      0.000      0.000     cs= 11
PPATH  Point=  2  Node=     249  X,Y,Z=  4.950      0.000      0.000     cs= 11
PPATH  Point=  3  Node=     248  X,Y,Z=  4.800      0.000      0.000     cs= 11


***** ROUTINE COMPLETED *****  CP =       26.188



        ***** MAPDL RESULTS INTERPRETATION (POST1) *****

ENTER  /SHOW,DEVICE-NAME  TO ENABLE GRAPHIC DISPLAY
ENTER  FINISH             TO LEAVE POST1
```