2022/11/03

Thursday, November 3, 2022 2:32 PM

Second approach to compute K

3. J-integral

The J-Integral evaluation in ANSYS is based on the domain integral method. The domain integration formulation applies area integration for 2-D problems and volume integration for 3-D problems. In the following, the procedure to compute the J-integral is summarized.

It should be noted that the command syntax is all in UPPER CASE letters and the arguments which are entered by the user are in *lower case and italic*.

After creating the model (using keypoints, lines and areas), specifying the concentration keypoints to generate singular elements, defining the local coordinate for each crack tip and generating the mesh including singular elements around the crack tips, the following commands except the last one need to be issued in the command prompt in the utility menu at the preprocessor stage of the simulation. The last command (i.e., Step 8) is issued after solution and in the postprocessor stage. Unfortunately there is no way to apply these steps in GUI. It means these commands cannot be accessed from a menu.

1. Go to preprocessor, and start a new computation of a contour-based integral:
   CINT,NEW,id

Id is a number that you'd assign
Example

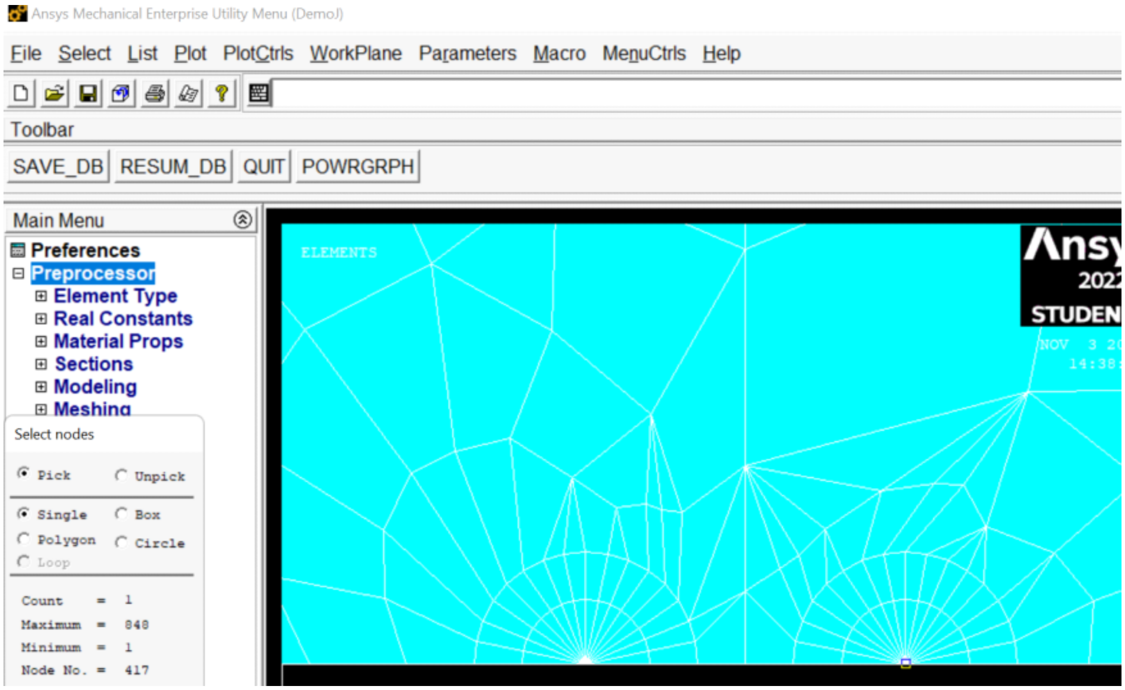
CINT,NEW,1

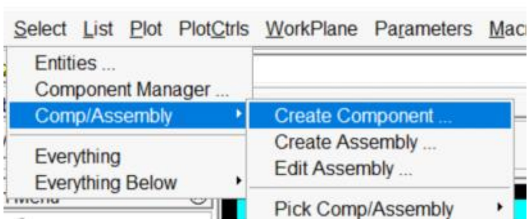
2. Specify that we are doing the J integral

CINT,TYPE,JINT

3. Define a component from the crack tip that we want to calculate the J-integral:

   Select -> entities -> nodes
   And select the crack tip

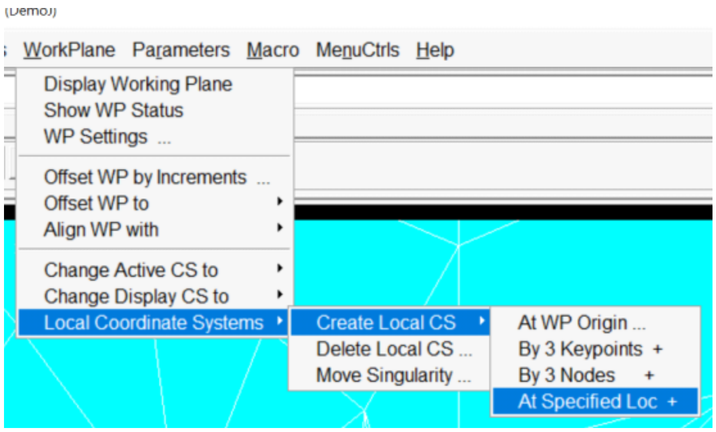


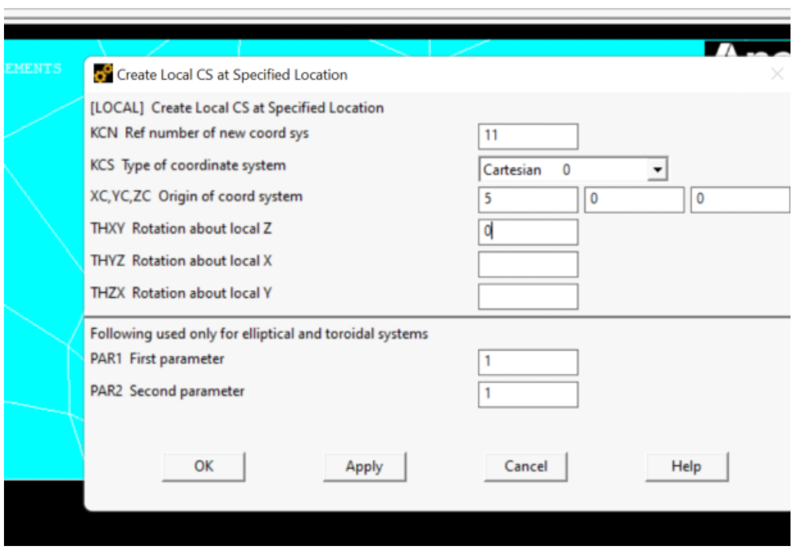


now the name of this crack tip is CTBR

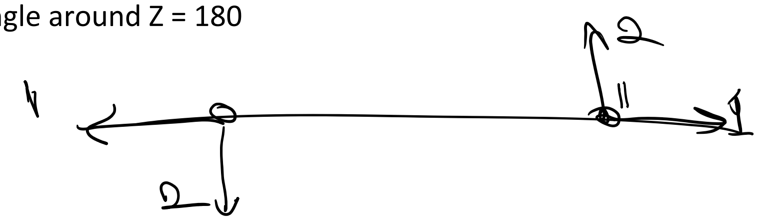
4. Associate the J integral with the crack tip
5. CINT,CTNC,CT_BR (or the name we specified above)

6. Define a local coordinate system whose direction 2 is along the crack and 2 is normal to it





Note for the crack tip on the left we do the same process with the angle around Z = 180

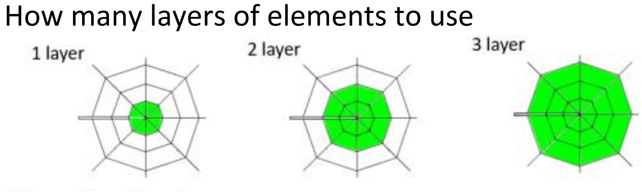


Now we need to specify the local coordinate system for J integral

CINT,NORM,coordinateID,direction normal 2 crack

CINT,NORM,11,2

7. How many layers of elements to use



CINT,NCON,n

For example
CINT,NCON,3

8. If the crack is not inside the domain (like here) we need to specify it
   CINT,SYMM,ON

9. Include J integral value in the results
   OUTRES,CINT

10. Solve the problem
11. Finally, to get the J integral value use the command
    PRCINT,id

    I used id 2
    PRCINT,2

$2a = 1$ $\quad a = .5 \qquad K_I = 10\sqrt{.5\pi} = 12.53$

$\sigma_\infty = 10$

$$J = \frac{K^2}{E'} \qquad E' = \begin{cases} E & \text{plane stress} \\ \frac{E}{1-\nu^2} & \text{" strain} \end{cases}$$

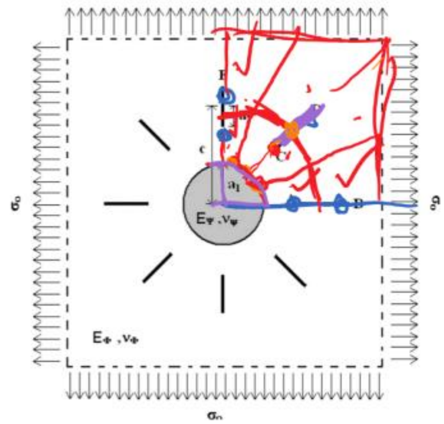
$$J = \frac{K^2}{\underset{100}{E}} = 1.5708$$

I got $\quad$ 1.45, 1.52

1 contour $\quad$ 2 contour

Project

Friday, November 13, 2020 4:33 PM



Assumptions

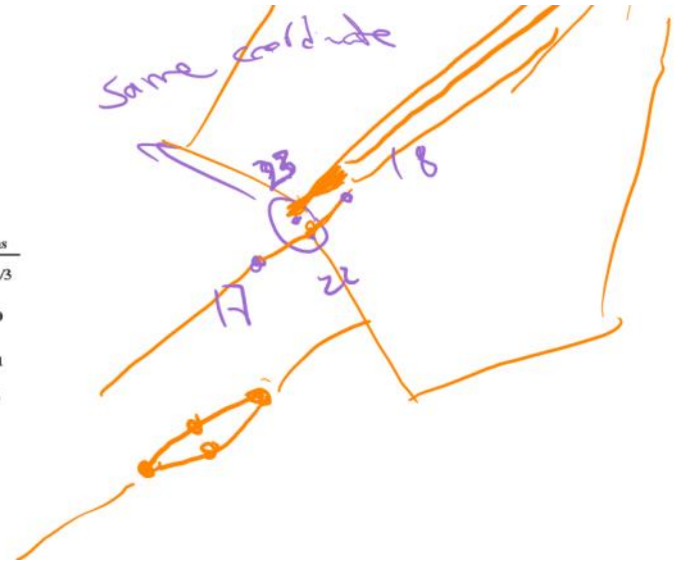
$\nu_\Phi = \nu_\Psi = 1/3$

$\frac{E_\Psi}{E_\Phi} = 10.0$

$a_2 = a_1 = 1$

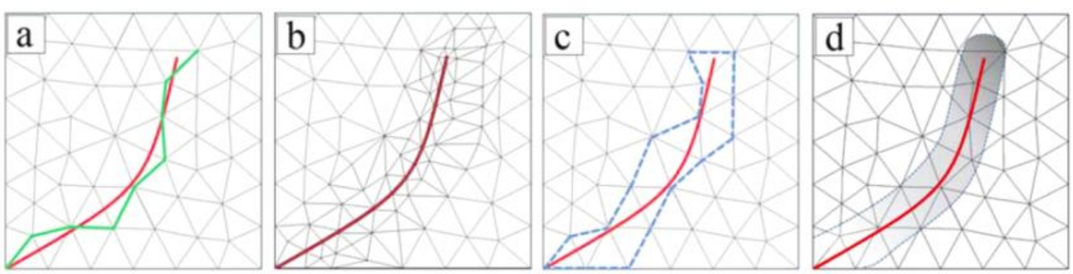
$\sigma_o = 1.0$

$c = 3$



XFEM and Cohesive models

Capturing/tracking cracks



Fixed mesh

Crack tracking

XFEM enriched elements

Brief overview in the next section

Crack/void capturing by bulk damage models

Brief overview in continuum damage models

Enriched methods allow the cracks to go inside the elements

Finite Elements for singular crack tip solutions

$$u = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(\kappa - 1 + 2\sin^2\frac{\theta}{2}\right)$$

$$v = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(\kappa + 1 - 2\cos^2\frac{\theta}{2}\right)$$

- Direct incorporation of singular terms: e.g. enriched elements by Benzley (1974), shape functions are enriched by K_I, K_{II} singular terms

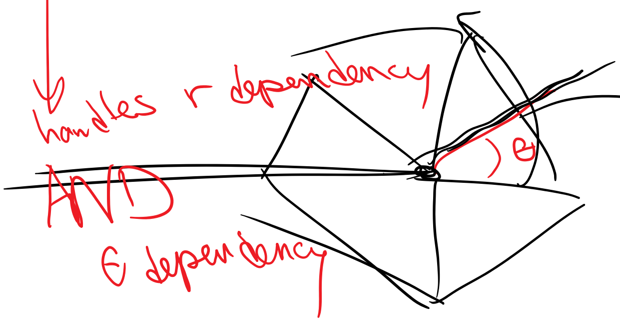
$$u_i = \sum_{k=1}^{4} f_k \bar{u}_{ik} + K_{\mathrm{I}}\left(Q_{1i} \sum_{k=1}^{4} f_k \bar{Q}_{1ik}\right) + K_{\mathrm{II}}\left(Q_{2i}\sum_{k=1}^{4} f_k \bar{Q}_{2ik}\right)$$

$$Q_{ij} = \frac{u_{ij}}{k_i}$$

e.g. enriched elements by Benzley (1974), shape functions are enriched by K_I, K_{II} singular terms

$$Q_{ij} = \frac{u_{ij}}{k_i}$$

$$u' = \left(1 + \frac{2x}{L} - 3\sqrt{\frac{x}{L}}\right) u_1' + \left(4\sqrt{\frac{x}{L}} - 4\frac{x}{L}\right) u_2' + \left(\frac{2x}{L} - \sqrt{\frac{x}{L}}\right) u_3'$$

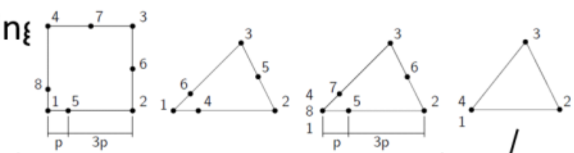


enrich these elements with LEFM type displacement field

Why is this better than 1/4 elements?

- Quarter point (LEFM) and Collapsed half point (Elastic-perfectly plastic) elements: By appropriate positioning of isoparametric element nodes create strain singularities

Can be easily used in FEM software



they hand $\frac{1}{\sqrt{r}}$ singular for σ ($\sqrt{r}$ for u for displacement)

LEFM: $\epsilon, \sigma : \frac{1}{\sqrt{r}}$ singular

Elastic-perfectly plastic: $\epsilon : \frac{1}{r}$ singular

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With XFEM, we allow the crack tip to be inside the element and do enrichment around the crack tip inside the element

Extended Finite Element Method (XFEM)

Belytschko et al 1999

set of enriched nodes

$$\mathbf{u}^h(\mathbf{x}) = \sum_{I \in \mathcal{S}} N_I(\mathbf{x})\mathbf{u}_I + \sum_{J \in \mathcal{S}^c} N_J(\mathbf{x})\Phi(\mathbf{x})\mathbf{a}_J$$

enrichment

standard part enrichment part

Partition of Unity (PUM)

enrichment function

$$\sum_J N_J(\mathbf{x}) = 1 \longrightarrow \sum_J N_J(\mathbf{x})\Phi(\mathbf{x}) = \Phi(\mathbf{x})$$

$\Phi(\mathbf{x})$ known characteristics of the problem (crack tip singularity, displaceme jump etc.) into the approximate space.

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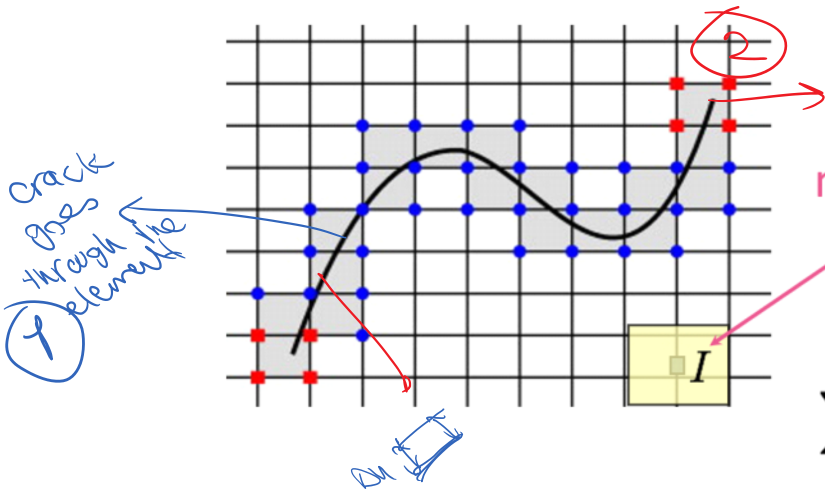
XFEM: enriched nodes



nodal support

$$\boxed{N_I(\mathbf{x}) \neq 0}$$

$$\sum_J N_J(\mathbf{x})\Phi(\mathbf{x}) = \Phi(\mathbf{x})$$

enriched nodes = nodes whose support is cut by the item to be enriched

enriched node I: standard degrees of freedoms (dofs) and additional dofs

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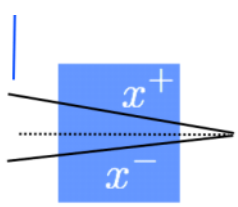
1.

crack edge



displacement: discontinuous across crack edge

$$\Phi_2 : \Phi_2(x^+) \neq \Phi_2(x^-)$$

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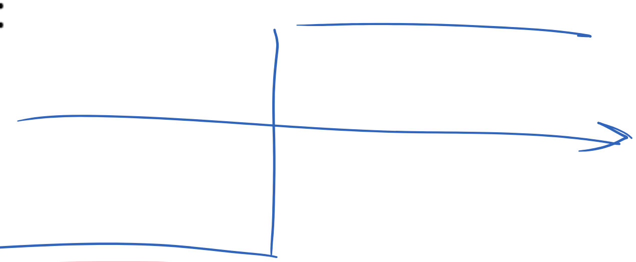
Crack edge enrichment functions:

$$\boxed{H(\mathbf{x}) = \begin{cases} +1 & \text{if } (\mathbf{x} - \mathbf{x}^*) \cdot \mathbf{n} \geq 0 \\ -1 & \text{otherwise} \end{cases}}$$



Crack tip enrichments

$$u = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(\kappa - 1 + 2\sin^2\frac{\theta}{2}\right)$$
$$v = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(\kappa + 1 - 2\cos^2\frac{\theta}{2}\right)$$

$\sqrt{r}\cos\frac{\theta}{2}$ $\quad$ $\sqrt{r}\cos\frac{\theta}{2}\sin^2\frac{\theta}{2}$

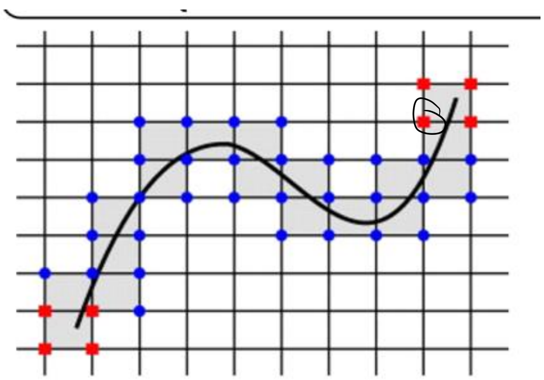
$\sqrt{r}\sin\frac{\theta}{2}$ $\quad$ $\sqrt{r}\sin\frac{\theta}{2}\cos^2\frac{\theta}{2}$

$$[B_\alpha] = \left[\sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\frac{\theta}{2}\sin\theta, \sqrt{r}\cos\frac{\theta}{2}\sin\theta\right]$$

$$\mathbf{u}^h(\mathbf{x}) = \sum_{I\in\mathcal{S}} N_I(\mathbf{x})\mathbf{u}_I + \sum_{J\in\mathcal{S}^c} N_J(\mathbf{x})H(\mathbf{x})\mathbf{a}_J + \sum_{K\in\mathcal{S}^t} N_K(\mathbf{x})\left(\sum_{\alpha=1}^{4} B_\alpha \mathbf{b}_K^\alpha\right)$$

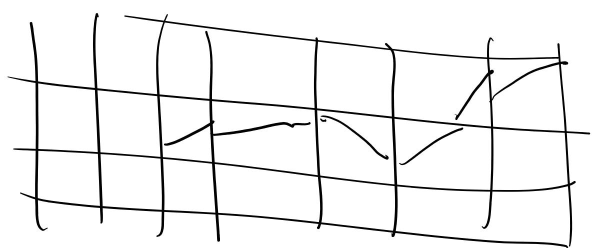
Jumps across element

crack tip enrichment



enrichment for red nodes

There are embedded discontinuity methods that enrich elements rather nodal dofs and they result in globally discontinuous solutions

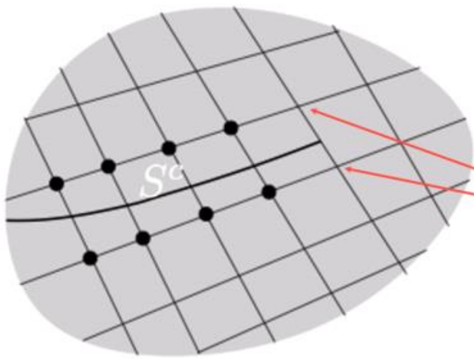


Some of difficulties with GFEMs/GFEMs:

1. For the crack tip to be inside the element (like red enrichments above), we need to have the particular form of enrichment functions that may not exist. We know these enrichments for LEFM, but what if we have a more complex material model, even cohesive models

Wells, Sluys, 2001

$$\mathbf{u}^h(\mathbf{x}) = \sum_{I \in \mathcal{S}} N_I(\mathbf{x})\mathbf{u}_I + \sum_{J \in \mathcal{S}^c} N_J(\mathbf{x}) H(\mathbf{x}) \mathbf{a}_J$$



No crack tip solution is known, no tip enrichment!!!

not enriched to ensure zero crack tip opening!!!

$$H(\mathbf{x}) = \begin{cases} +1 & \text{if } (\mathbf{x} - \mathbf{x}^*) \cdot \mathbf{n} \geq 0 \\ -1 & \text{otherwise} \end{cases}$$

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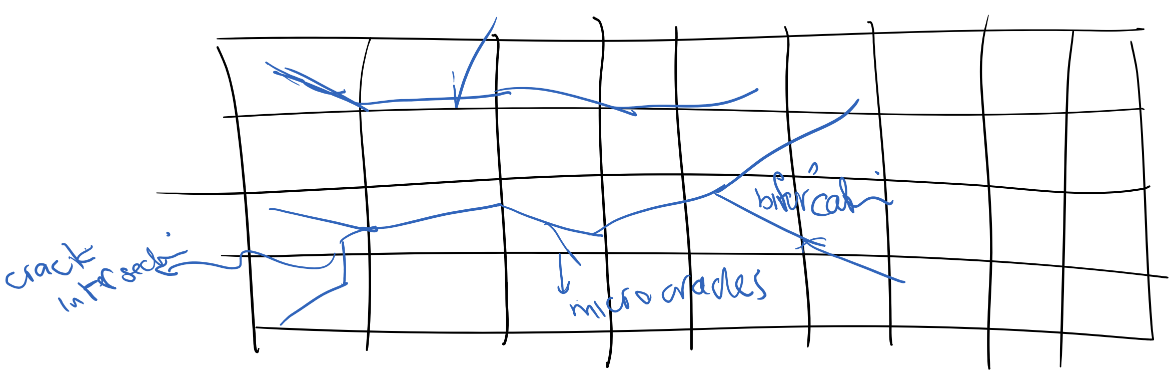
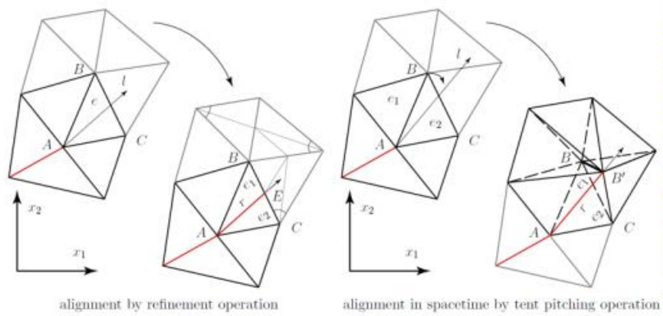
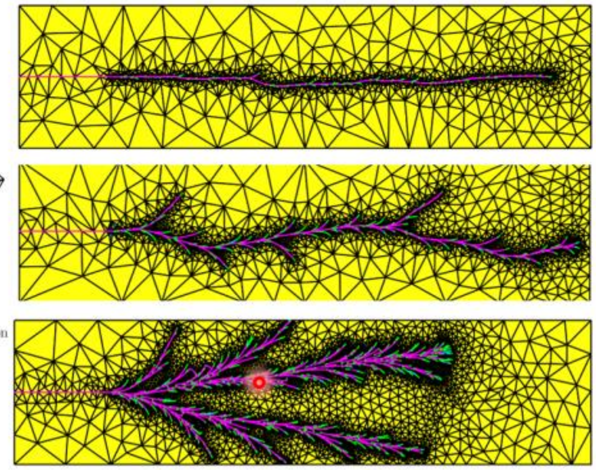
2. Complex fracture patterns



a 2d x time mesh is generated in spacetime

"patch" of elements is solved



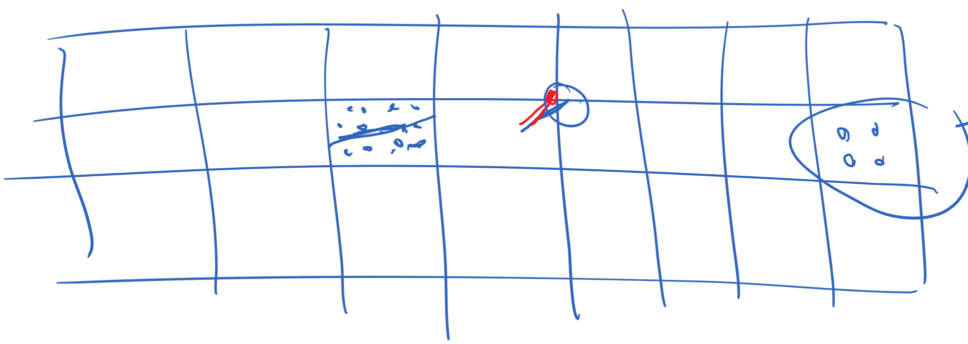


Spacetime mesh adaptive operations align element boundaries with crack path.

Challenging for XFEM

XFEM: more difficult to use it for dynamic fracture

3. Element quality (to some extent) and quadrature



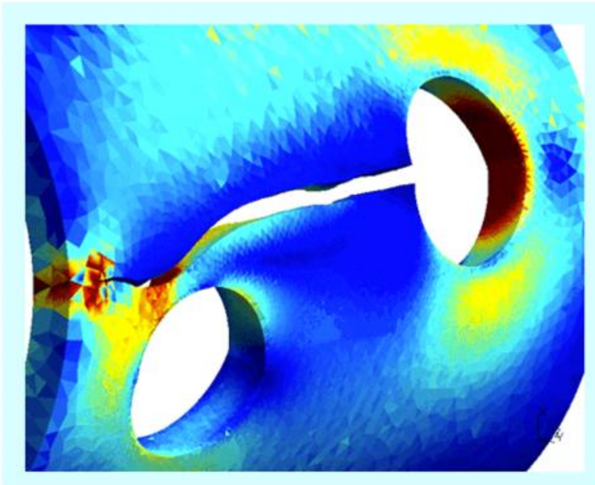
full integration scheme

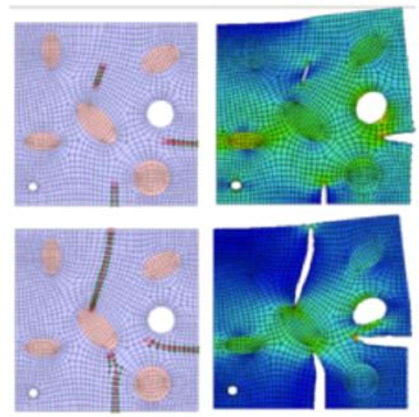
Mesh adaptive schemes and XFEM/GFEM are still the most powerful methods that can handle sharp interface cracks

Main disadvantage of adaptive schemes is implementation and handling of all crack intersection topologies, etc. especially in 3D

Examples from XFEM

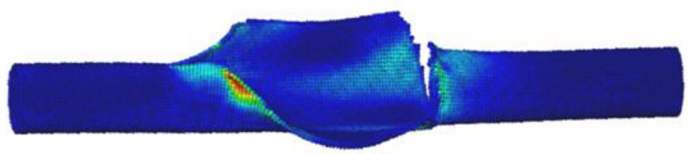
XFEM: examples





CENAERO, M. Duflot

Northwestern Univ.



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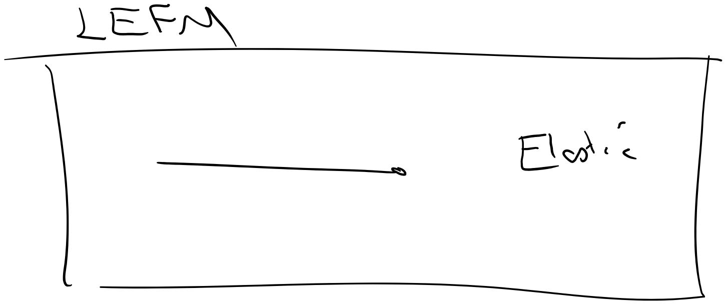
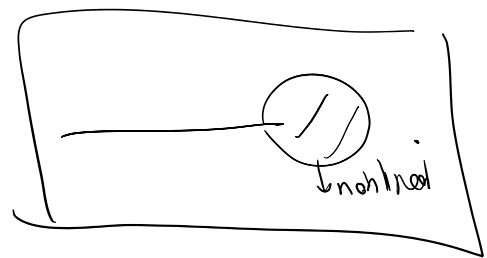
6.2. Traction Separation Relations (TSRs)

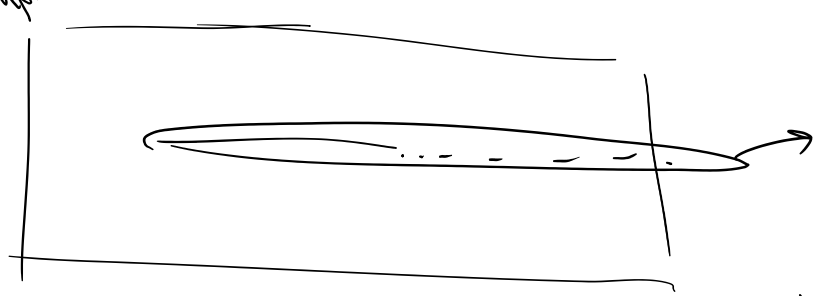
LEFM



reality



Compromise



nonlinearity limited to crack line

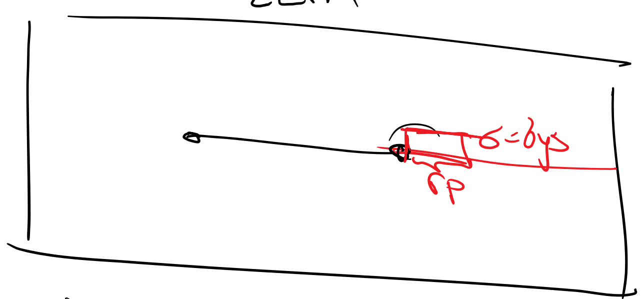
Strip yield model (Barenblatt & Dugdale models)

did this

LEFM

did this

LEFM



Cohesive models are very similar in that they are nonlinear constitutive equation ONLY ALONG THE CRACK and they too limit the stress value