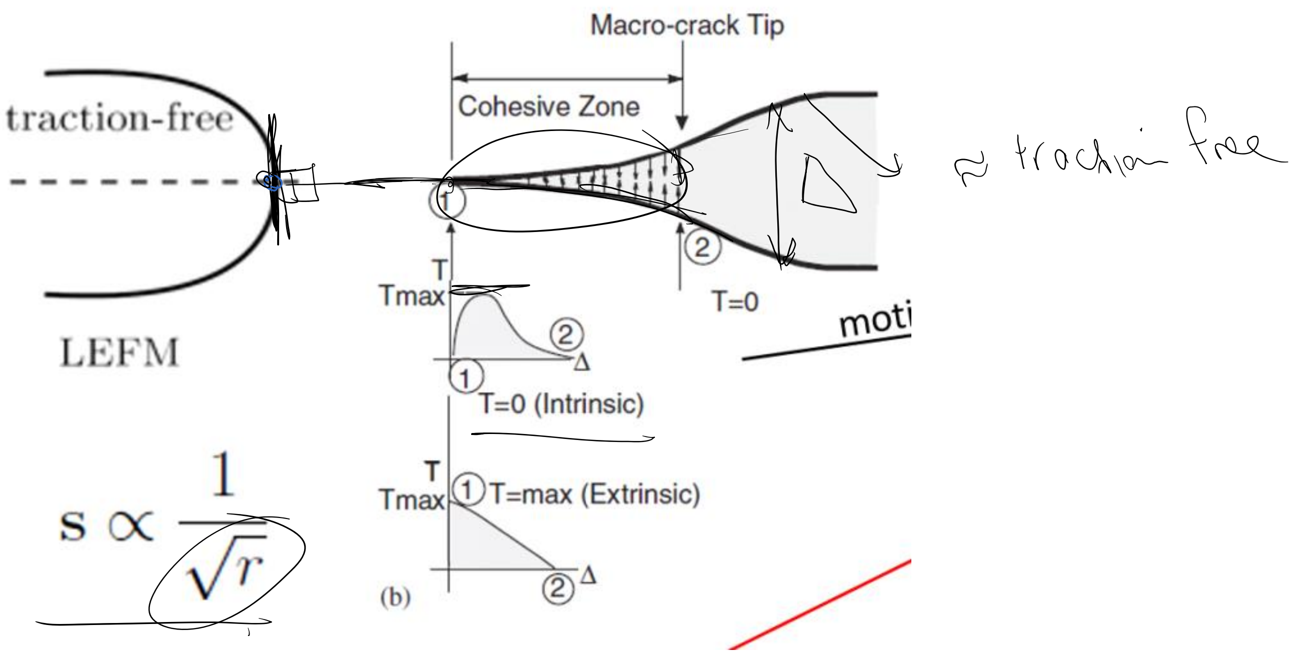
2022/11/08

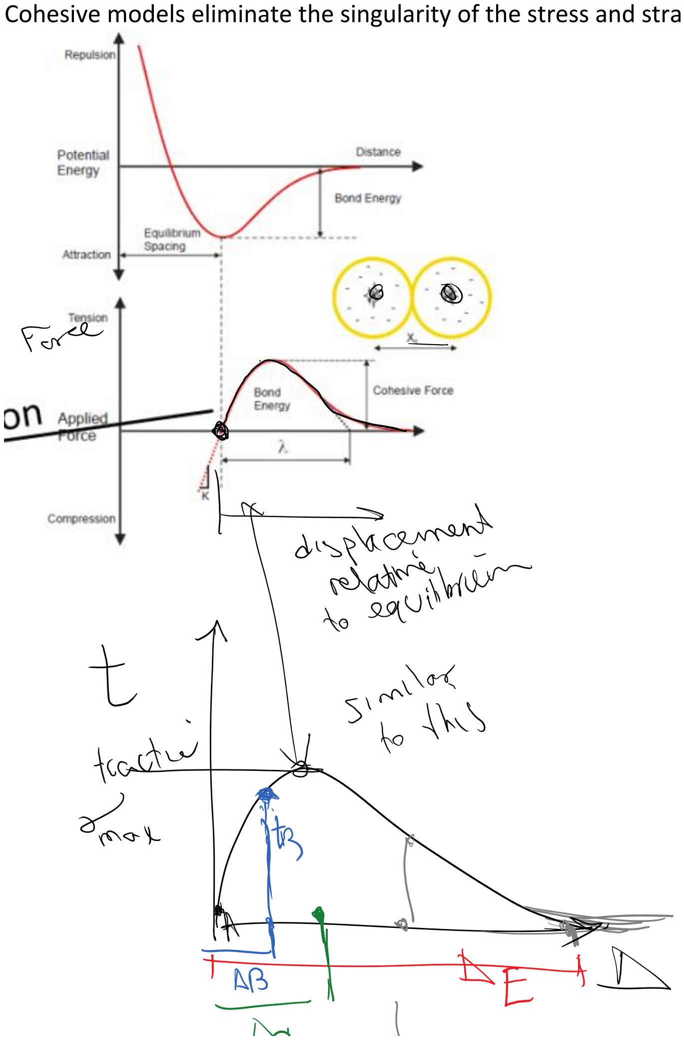
Tuesday, November 8, 2022 2:29 PM

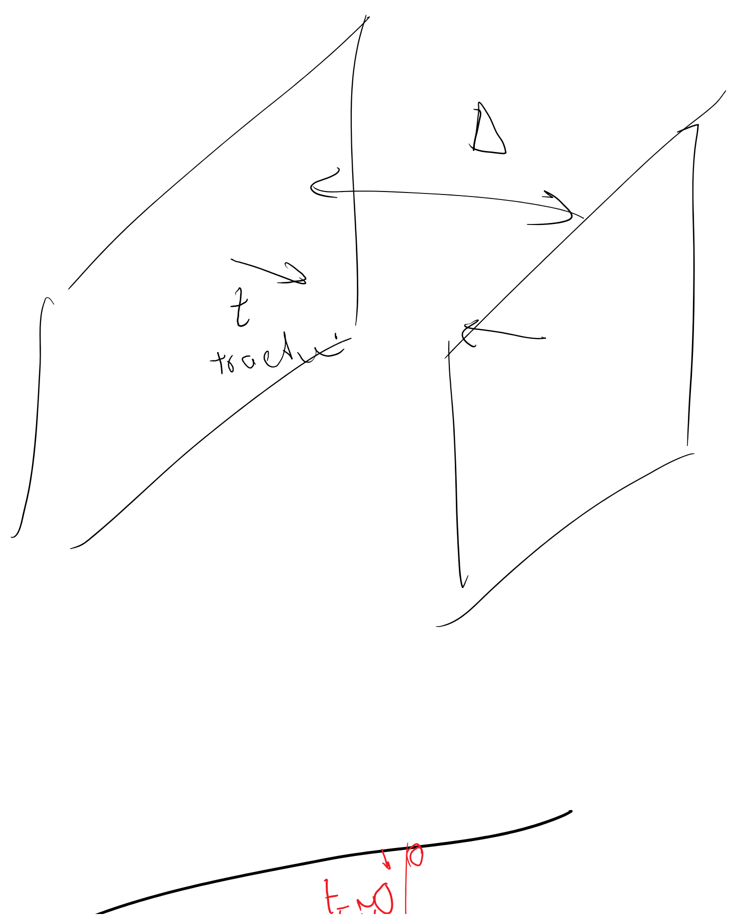
Cohesive models continued:

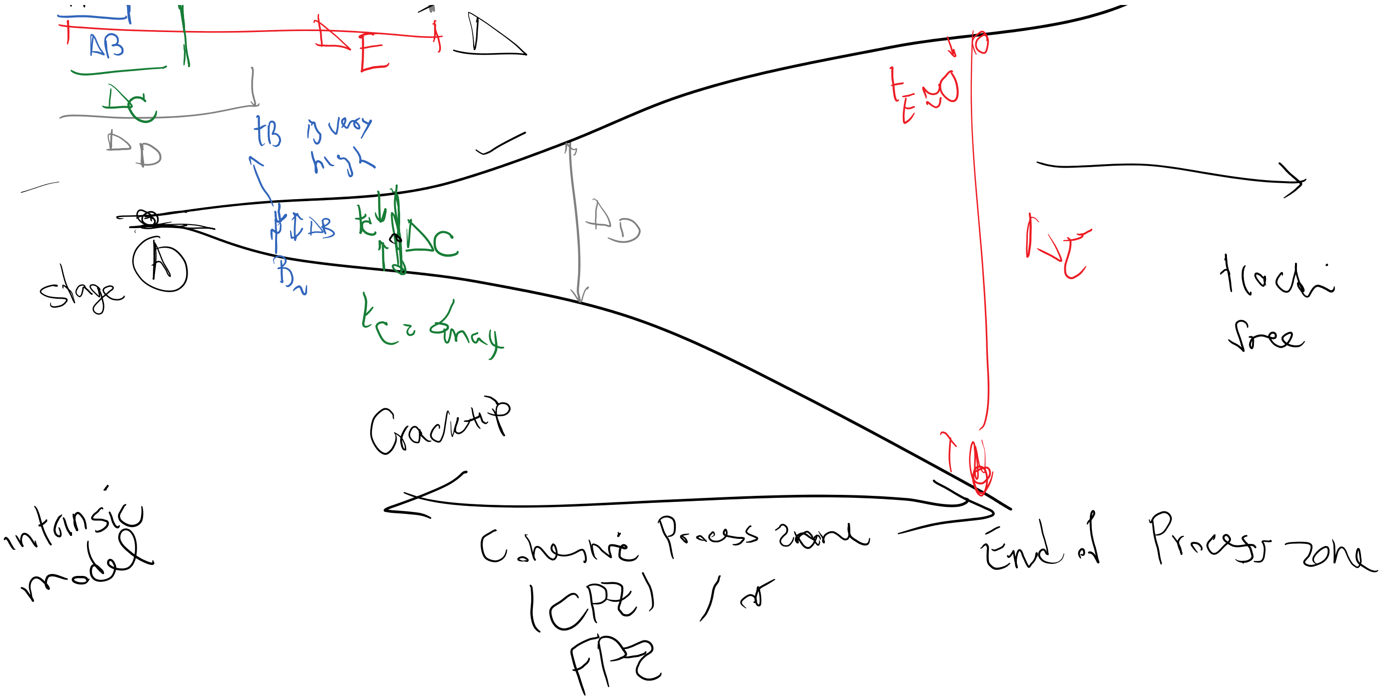


$$s \propto \frac{1}{\sqrt{r}}$$

Cohesive models eliminate the singularity of the stress and strain fields around the crack tip

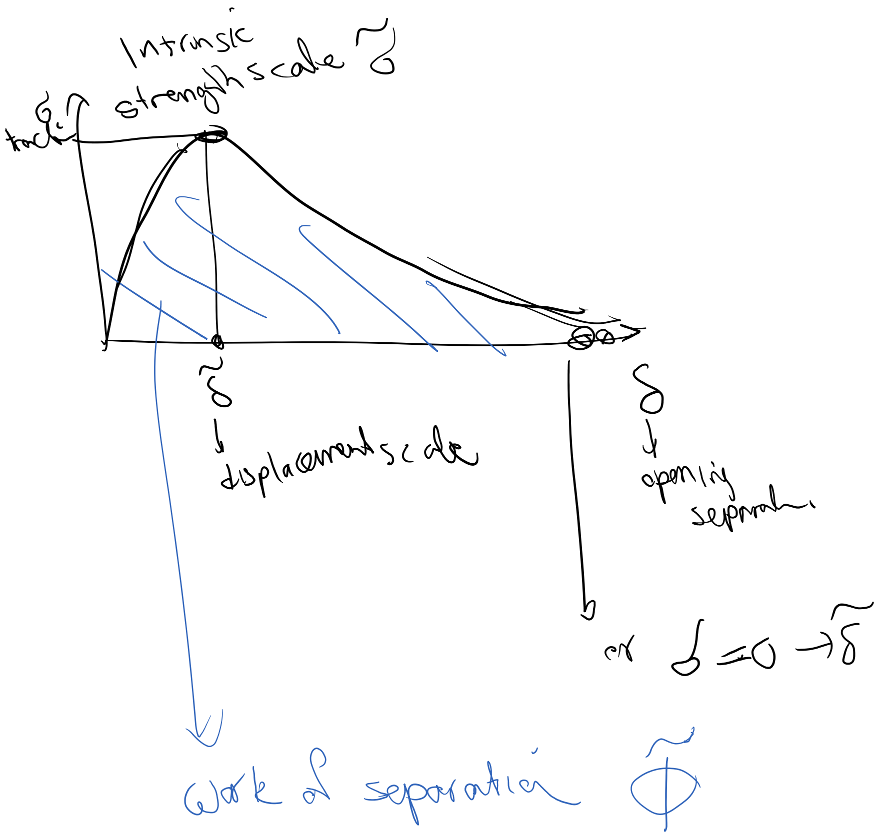




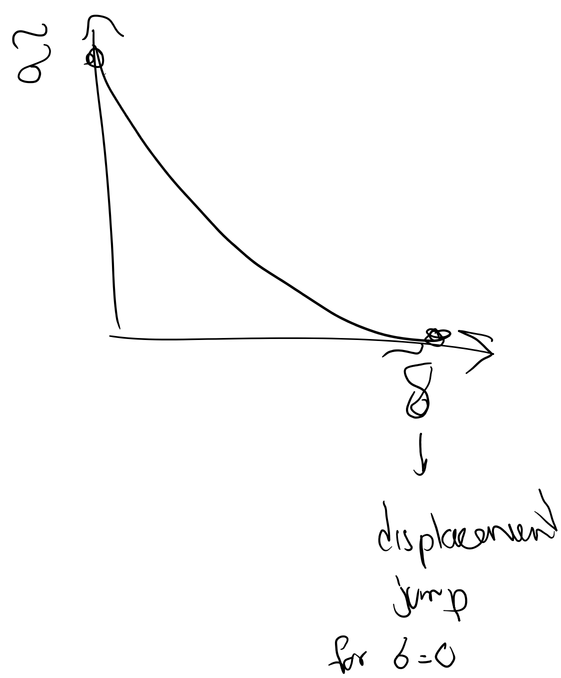


for intrinsic model

Parameters of Cohesive model



extrinsic



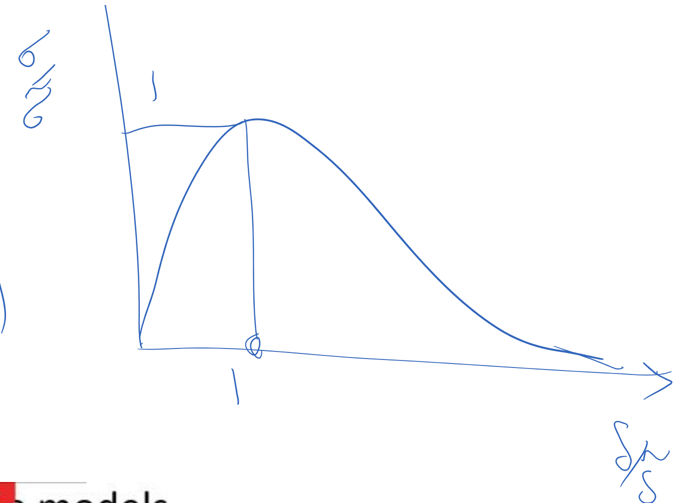
Cohesive models are defined by two out of these three parameters

$\int \sim \tilde{\sigma}, \tilde{\delta}$

σ

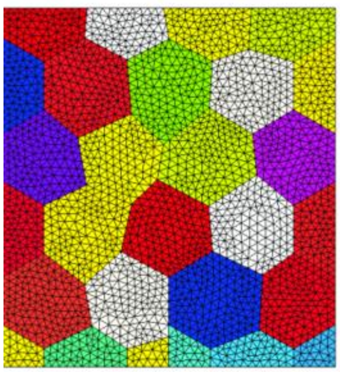
$$\left\{\begin{array}{l} -\tilde{\sigma},\ \tilde{\delta} \\ -\tilde{\sigma},\ \tilde{\phi} \\ \hline -\tilde{\delta},\ \tilde{\phi} \end{array}\right.$$

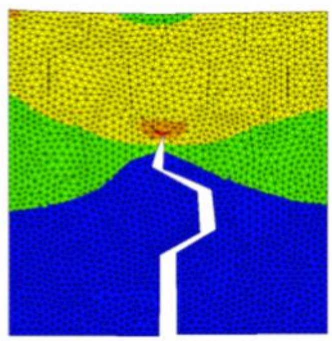
nondimensional expression $\sigma = \tilde{\sigma}\, f\left(\delta/\tilde{\delta}\right)$



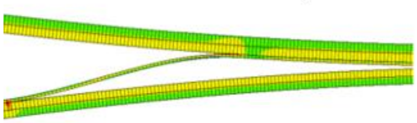
Sample applications of cohesive models

fracture of polycrystalline material

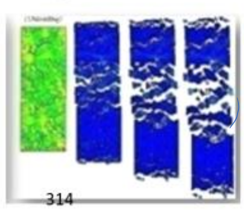




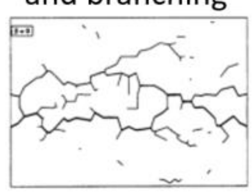
delamination of composites



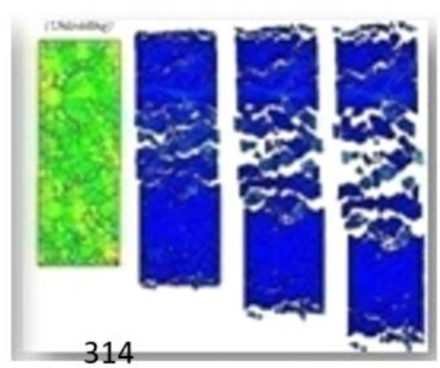
fragmentation



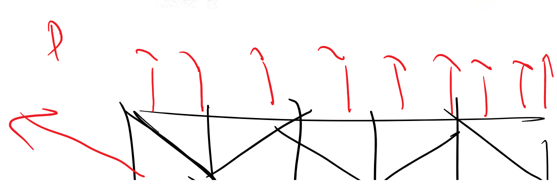
Microcracking and branching



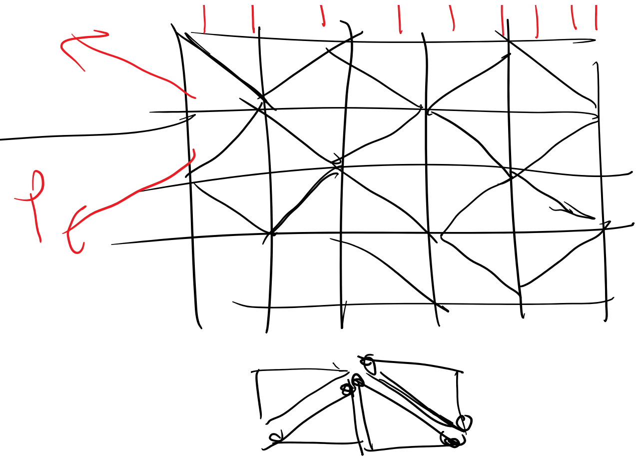
Extrinsic vs intrinsic models



all interelement ...mes



all inter elem boundaries have cohesive elements



Artificial compliance -> as the mesh gets finer the structure gets more and more compliant

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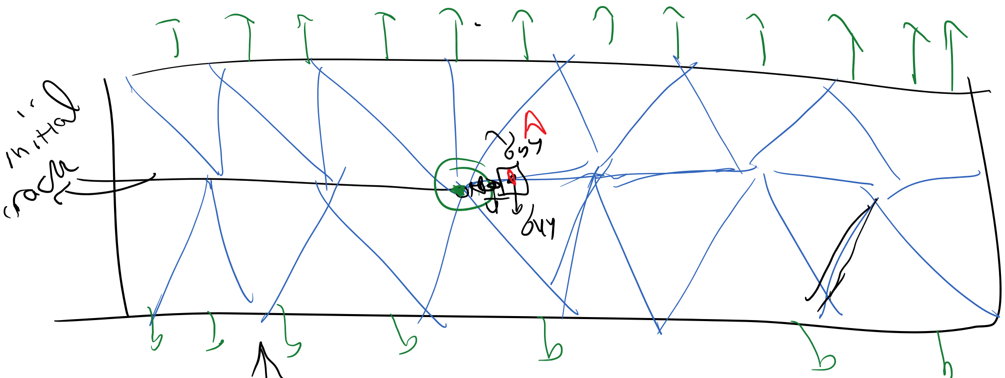
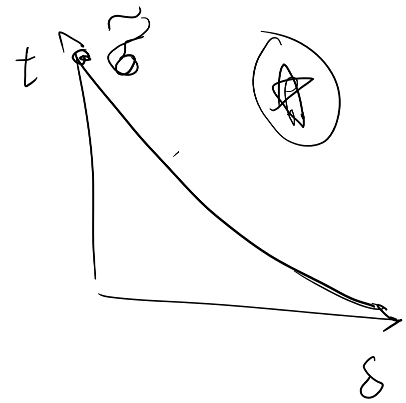
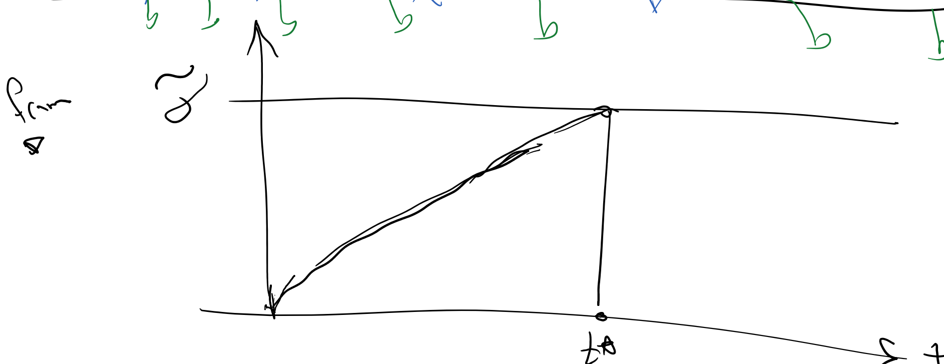
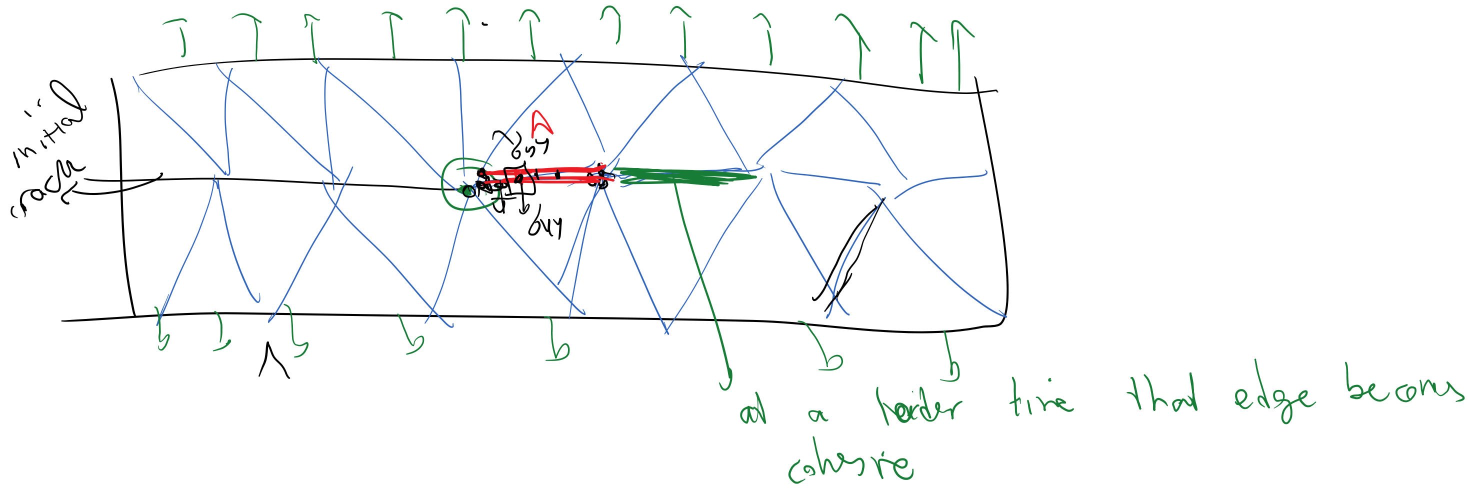
Extrinsic model



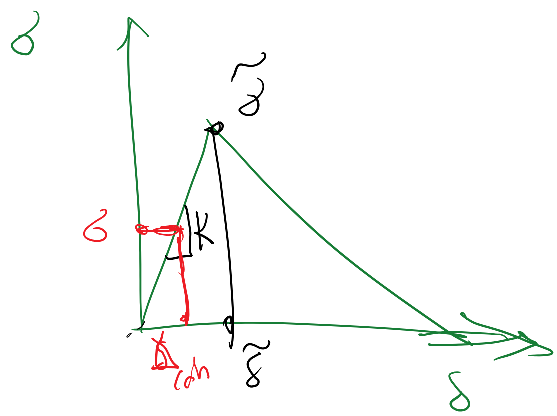


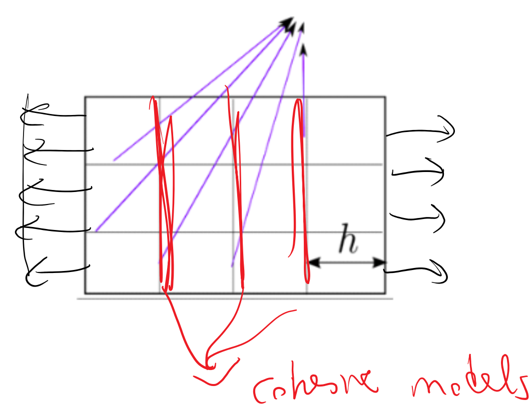


We insert a cohesive model at point A or the whole element edge



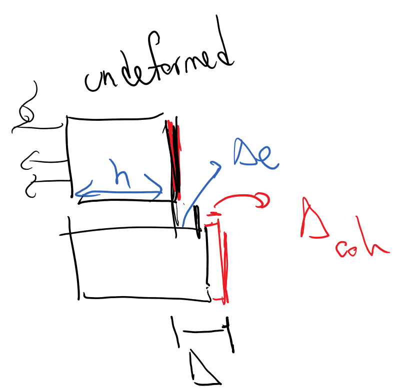
We can do similar fragmentation problems with extrinsic models, but handling the mesh connectivity and interactive insertion of cohesive surfaces is more difficult.





Bulk part

$$\Delta_e = \varepsilon h = \frac{\sigma}{E} h$$



$$\Delta_{coh} = \frac{\sigma}{K}$$

$$\Delta = \frac{\sigma}{E} h + \frac{\sigma}{K} = \sigma\left(\frac{h}{E} + \frac{1}{K}\right)$$

for a material with equivalent stiffness E_{eqn}

$$\Delta = \frac{\sigma}{E_{eq}} h$$

$$\sigma \quad h \quad \rightarrow \quad \frac{1}{\ } = \frac{1}{\ } + \frac{1}{\ }$$

$\sigma\left(\frac{1}{E}+\frac{1}{K}\right) = E_{eq}$

$E_{eq} \quad E \quad Kh$

$$\boxed{C_{eq} = C + C_{coh}}$$

compliance

compliance of original material

cohesive compliance

$$E_{eq} = \frac{1}{\frac{1}{E}+\frac{1}{Kh}}$$

$$\boxed{E_{eq} = E \cdot \frac{1}{1+\frac{E}{Kh}}}$$

Kh vs E

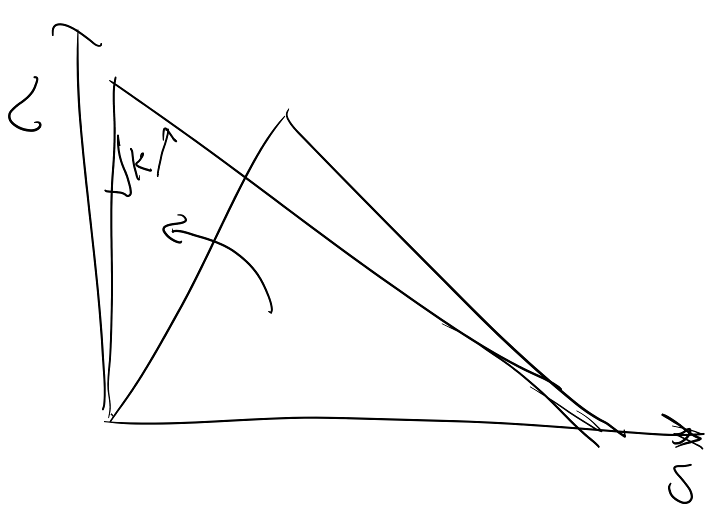
$\frac{Kh}{E} \to 0$

$E_{eq} \to 0 \quad (C_{eq} \to \infty)$

As $h \to 0$ (better FEM solution), $\frac{Kh}{E} \to 0 \longrightarrow$ Compliance $\nearrow$

What if for a given h

choose $\frac{Kh}{E}$ large enough, eg 10, 100



Unfortunately, we cannot use arbitrarily high K (penalties) in general, as the system matrices become difficult to solve (bad conditioning)

- Artificial compliance becomes important if cohesive surfaces are added between all elements for intrinsic models to find crack propagation path.
- The artifical compliance is computed as,

$$
\begin{aligned}
\Delta &= \Delta_e + \Delta_c, \quad \Delta_e = \text{elastic displacement}, \Delta_c = \text{cohesive separation} \quad \Rightarrow \\
\frac{\sigma}{E_{\text{eff}}}h &= \frac{\sigma}{E}h + \frac{\sigma}{K} \quad \Rightarrow \\
\frac{1}{E_{\text{eff}}} &= \frac{1}{E} + \frac{1}{Kh} \quad \Rightarrow
\end{aligned}
$$

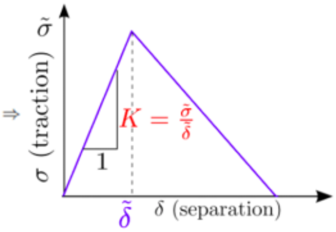
Artificial compliance is,

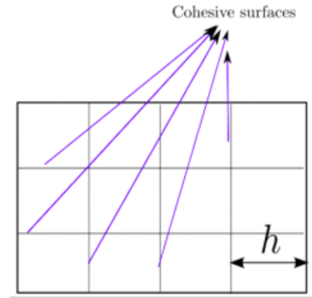
$$C_c = \frac{1}{Kh} = \frac{\tilde{\delta}}{\tilde{\sigma}h} = \frac{1}{E_c}, \text{ where}$$

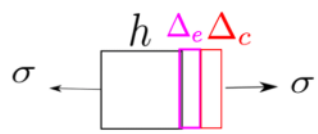
$$E_c = Kh = \frac{\tilde{\sigma}h}{\tilde{\delta}}, \text{ and effective elastic modulus is}$$

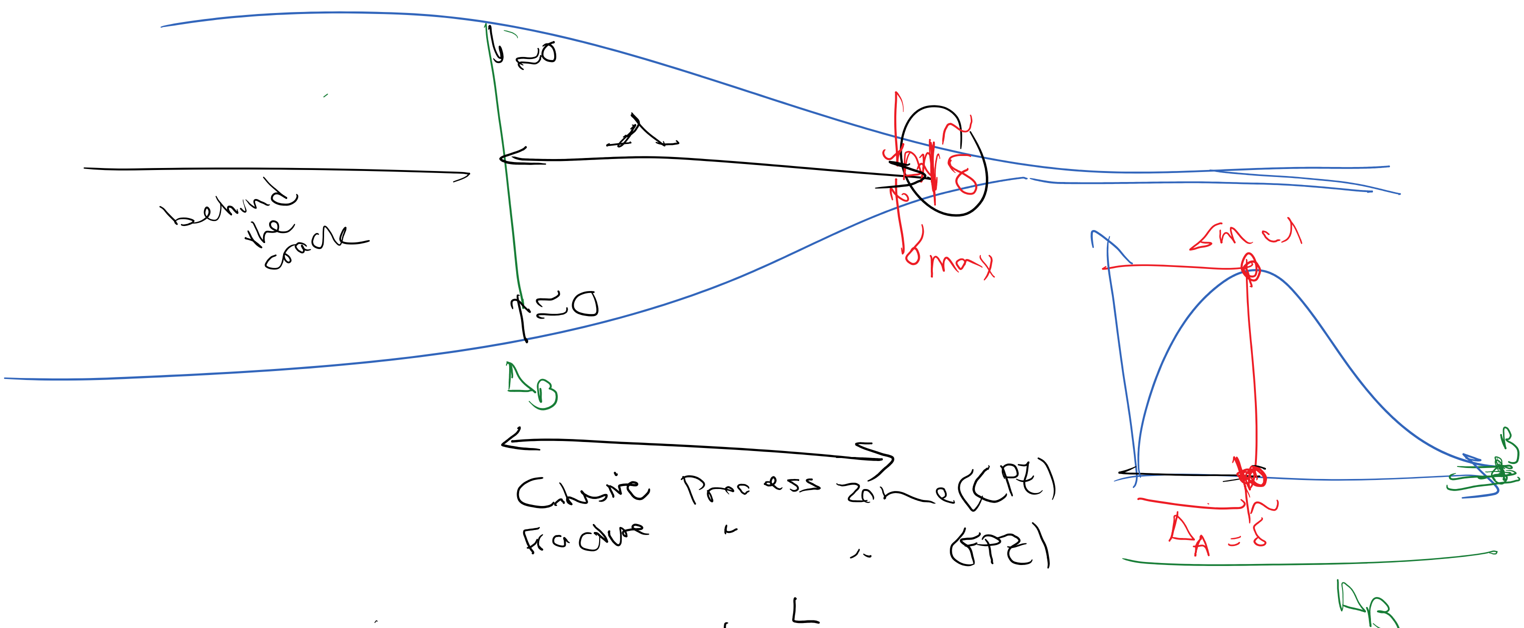
$$\frac{1}{E_{\text{eff}}} = \frac{1}{E} + \frac{1}{E_c}, \quad \Rightarrow E_{\text{eff}} = \frac{EE_c}{E + E_c}$$

- That is the smaller element spacing h or softer the initial slope K of TSR the higher artificial compliance (higher errors)
- While extrinsic cohesive models do not have the same problem, adaptive insertion of cohesive surfaces is more challenging for them.



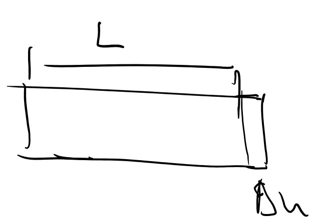






Cohesive Process zone (CPZ)
Fracture " " (FPZ)

$$\varepsilon = \frac{\Delta u}{L}$$



$$\underbrace{\Delta u}_{\text{change of length}} = \varepsilon L$$

$$u = \varepsilon L$$

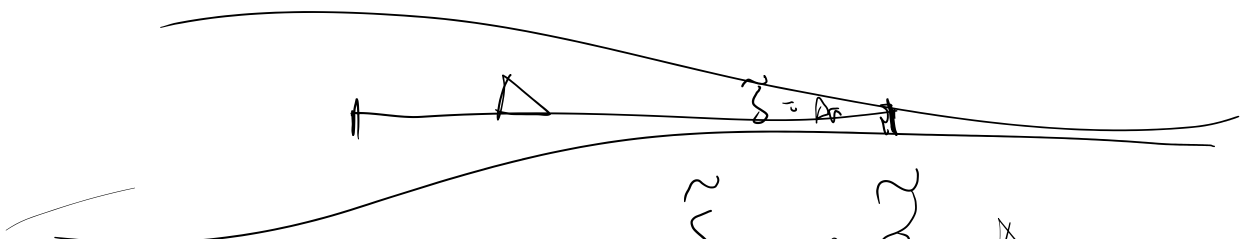
Magnitude of displacement

$$\frac{\text{Magnitude of displacement}}{\text{Magnitude of length scales}} = \text{Scale of strain}$$

$$\varepsilon = \frac{\sigma}{E}$$ for fracture $\sigma \rightsquigarrow \tilde{\sigma}$ strength

$$\boxed{\tilde{\varepsilon} = \frac{\tilde{\sigma}}{E}}$$ strain scale

$$\tilde{u} = \tilde{\varepsilon}\,\tilde{L} \qquad \boxed{\tilde{u} = \frac{\tilde{\sigma}}{E}\,\tilde{L}}$$

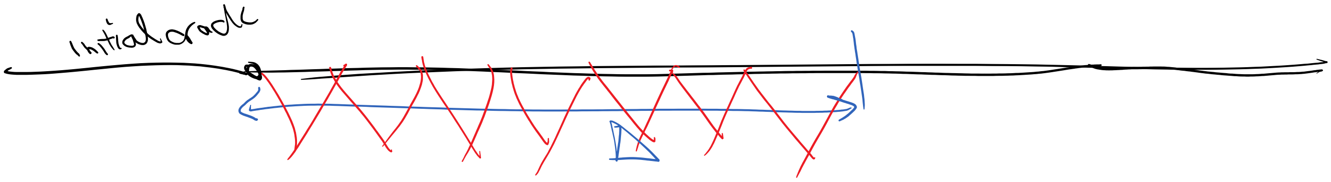


$$\tilde{\delta} \propto \frac{\tilde{\sigma}}{E}\,\Delta$$

$$\boxed{\Delta \propto \tilde{\delta}\,\frac{E}{\tilde{\sigma}}}$$

Finite Element

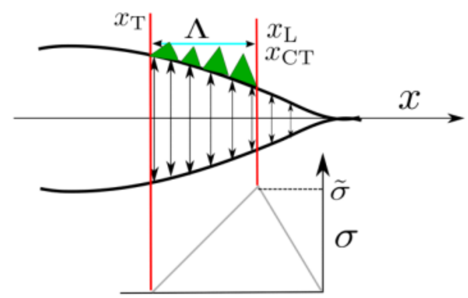
initial crack



recommendation 6–10 elements across Δ

Why process zone size is important?

- Importance of process zone size Λ
  - Static estimate:

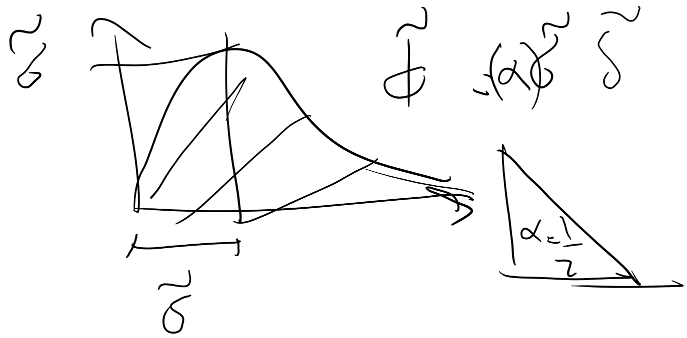


$$\Lambda = \varsigma \pi \frac{\mu}{1-\nu} \frac{\tilde{\phi}}{\tilde{\sigma}^2} \propto \tilde{L}$$

$$\varsigma = \begin{cases} \frac{1}{4} & \text{Dugdale model} \\ \frac{9}{16} & \text{Potential-based TSRs} \end{cases}$$

$$\propto E \frac{\tilde{\phi}}{\tilde{\sigma}^2} \propto E \frac{\tilde{\sigma}\tilde{\delta}}{\tilde{\sigma}^2}$$

$$\propto \left(\frac{E}{\tilde{\sigma}}\right) \tilde{\delta}$$

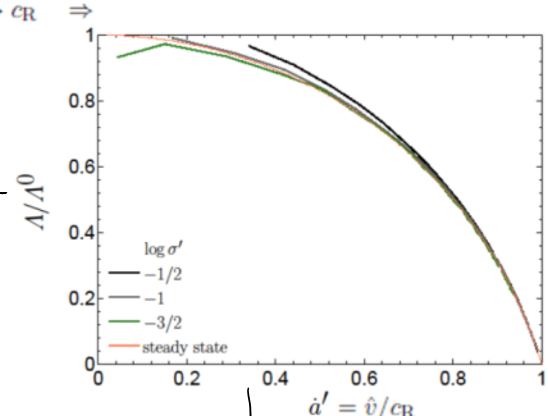


- Minimum number of elements in process zone size: There should be at least 4-10 elements along the PZ
- Dynamic estimate: PZS decreases as crack speed $\hat{v}$ approaches Rayleigh wave speed c_R

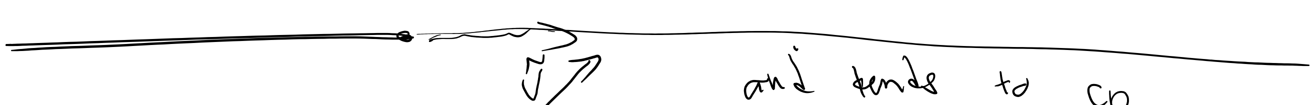
$$\Lambda(\hat{v}) = \frac{\Lambda}{A(\hat{v})}, \quad A(\hat{v}) \to 0 \text{ as } \hat{v} \to c_R \quad \Rightarrow$$

Smaller elements are needed in PZT as crack accelerates!

$$\frac{\Lambda}{\underbrace{\Lambda_0}_{\text{quasi-static process zone size}}}$$



normalized crack speed



and tends to c_R Rayleigh wave speed

$$\tilde{\phi} = \tilde{\sigma}\tilde{\delta}$$ Energy

2 out of the three are independent

$$\tilde{p} = \rho\tilde{v} = \frac{\tilde{\sigma}}{c_{\mathrm{d}}}$$ Linear momentum

$$\tilde{E} = \frac{\tilde{v}}{c_{\mathrm{d}}} = \frac{\tilde{\sigma}}{\rho c_{\mathrm{d}}^2} \propto \frac{\tilde{\sigma}}{\|\mathbf{C}\|}$$ scale Strain

$$\tilde{v} = \frac{\tilde{\delta}}{\tilde{\tau}} = \frac{\tilde{\sigma}}{\rho c_{\mathrm{d}}}$$ Velocity

$$\tilde{L} = c_{\mathrm{d}}\tilde{\tau} = \frac{\rho c_{\mathrm{d}}^2\tilde{\delta}}{\tilde{\sigma}} \propto \Lambda^0, \tilde{\delta}\ \frac{E}{\tilde{\sigma}}$$ Length

Process zone size Λ

$$\tilde{\tau} = \frac{\rho c_{\mathrm{d}}\tilde{\delta}}{\tilde{\sigma}}$$ Time

Influences time step for time marching methods