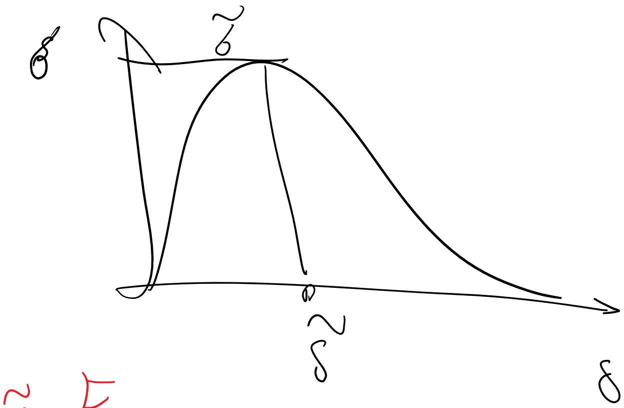
2022/11/10

Thursday, November 10, 2022 2:23 PM

From last time, the scales of fracture



Stress scale $\tilde\sigma$

displacement scale $\tilde\delta$

strain scale $\tilde\varepsilon = \frac{\tilde\sigma}{E}$

- Length scale $\tilde L = \frac{\tilde\delta}{\tilde\varepsilon} = \tilde\delta \frac{E}{\tilde\sigma}$

energy scale = $\tilde\sigma \tilde\delta$

fracture energy scale $G \cdot \tilde\sigma \tilde\delta$

velocity scale $= \frac{\tilde\sigma}{Z} = \frac{\tilde\sigma}{c_d \rho}$

impedence

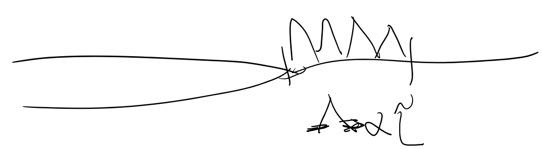
longitudinal wave speed

density

- time scale $\tilde t = \frac{\tilde\delta}{\tilde v} = \frac{\tilde\delta c_d \rho}{\tilde\sigma} \propto \frac{\tilde\phi c_d \rho}{\tilde\sigma^2}$

$\tilde L \propto$ fracture process zone size

\# element



$\tilde t$: time scale @ which fracture happens

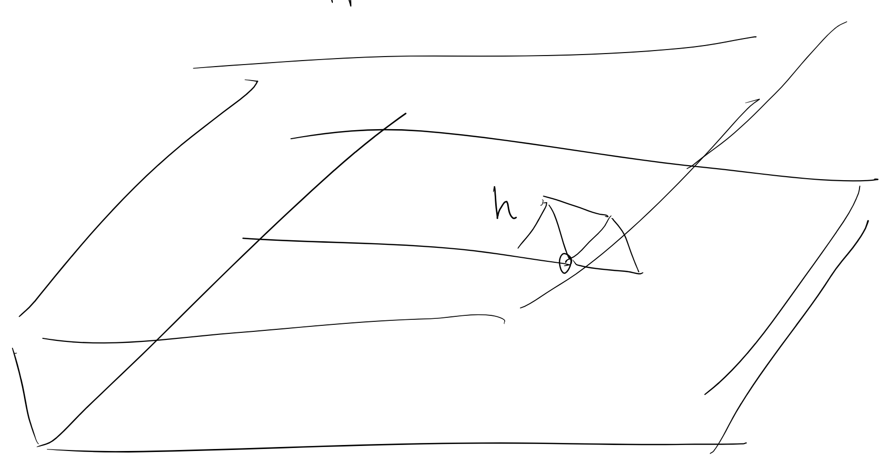
practical use

$\Delta t \propto \frac{h}{c}$

wave speed

needed for stability of an explicit method

Δt



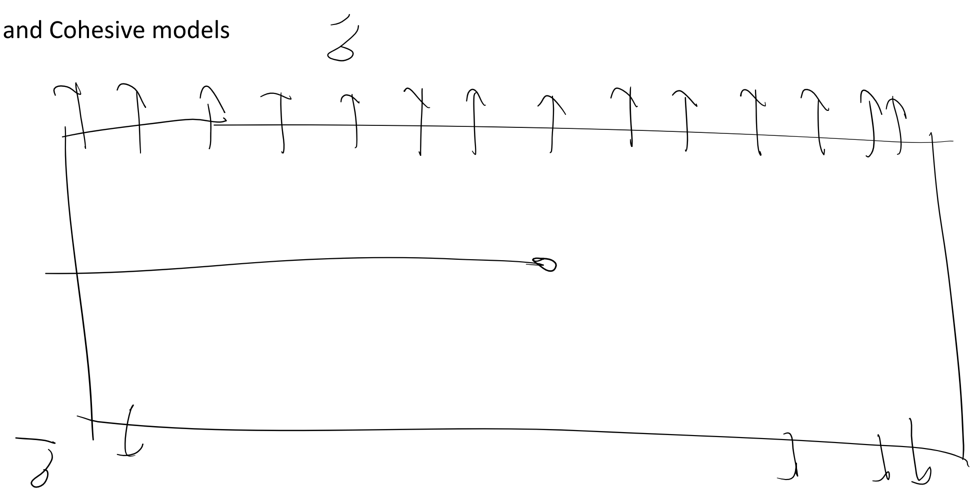
for stability of an explicit method

$$\Delta t \ll \tilde{\tau}$$

e.g. $\frac{\tilde{\tau}}{10}, \frac{\tilde{\tau}}{5}$

to ensure fracture is modeled accurately

Comparison of LEFM and Cohesive models

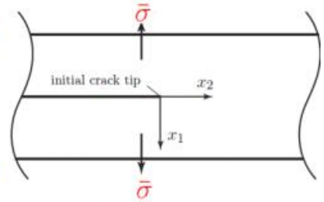


$\tilde{\sigma}$ = strength

$$\Delta \sim \frac{r_p}{r_s} \propto \left(\frac{\bar{\sigma}}{\tilde{\sigma}}\right)^2 \lesssim 0.1 \text{ e.g.}$$

$$\rightarrow \quad \frac{\bar{\sigma}}{\tilde{\sigma}} \lesssim 0.3$$ LEFM is OK.

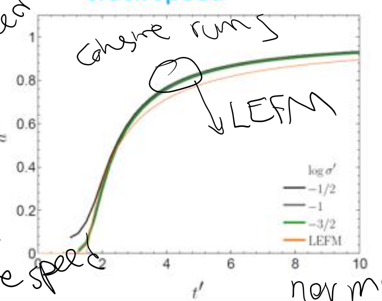
- When SSY condition is satisfied LEFM and Cohesive Fracture Mechanics (CFM) solutions are expected to be close ⇒



- When $\sigma' = \frac{\bar{\sigma}}{\sigma} \to 0$ LEFM & CFM are expected give similar results

crack speed

Normalized crack speed by Rayleigh wave speed

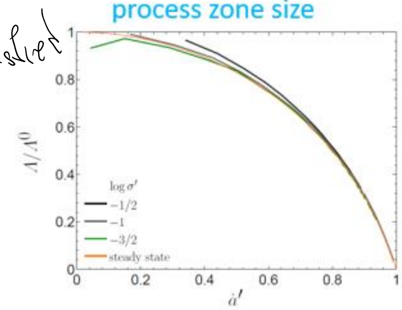


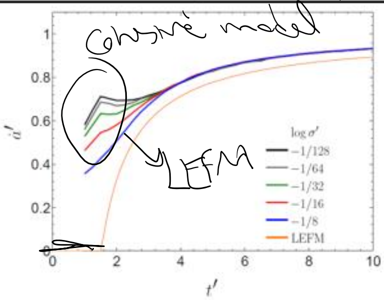
normalized time

SSY satisfied

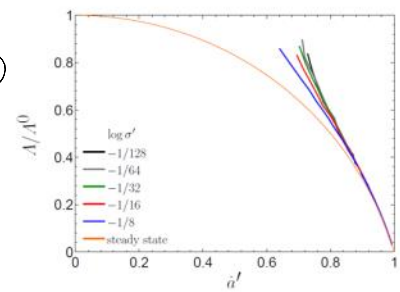
Low $\sigma' = \frac{\bar{\sigma}}{\sigma}$
SSY
LEFM ≈ CFM

process zone size

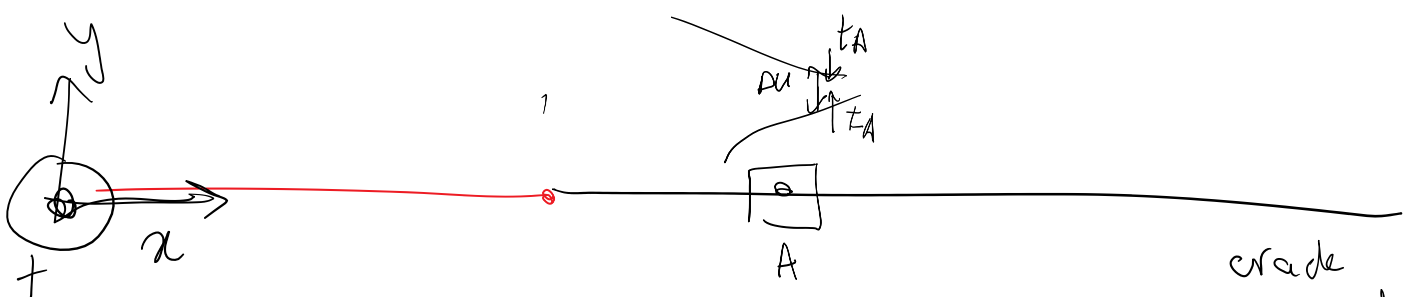


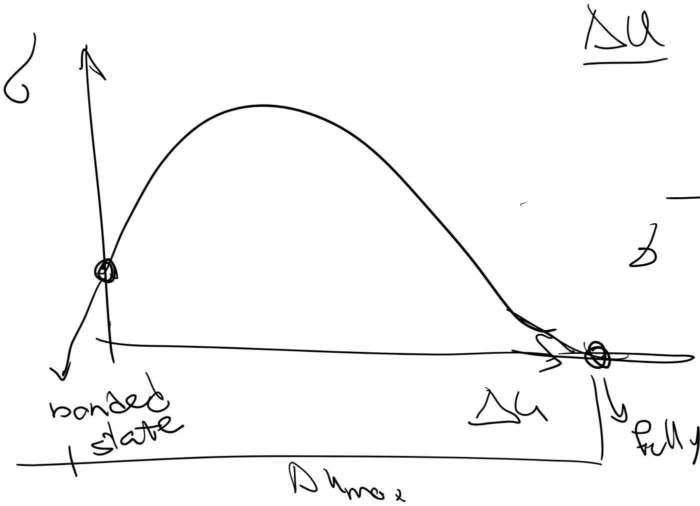


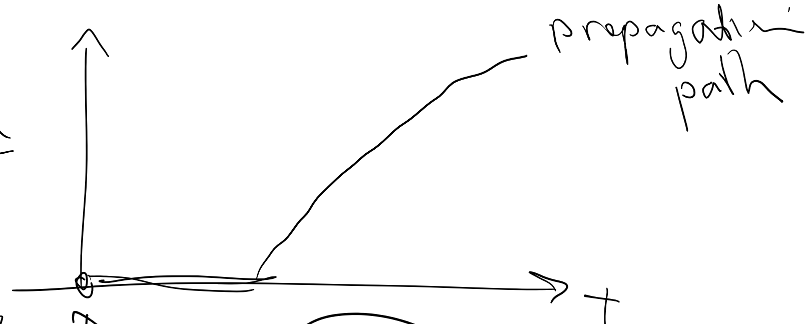
High $\sigma' = \frac{\bar{\sigma}}{\sigma}$
LSY
LEFM ≠ CFM

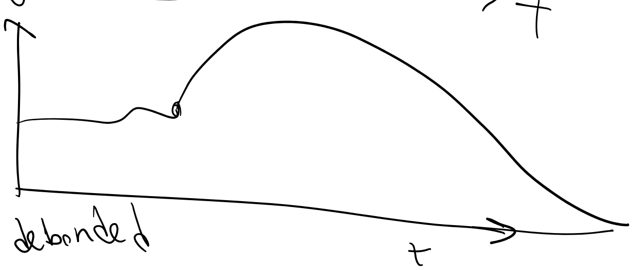


322



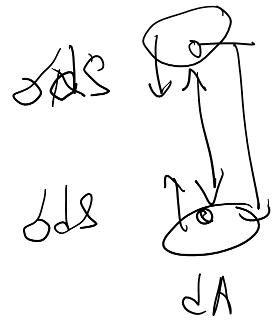




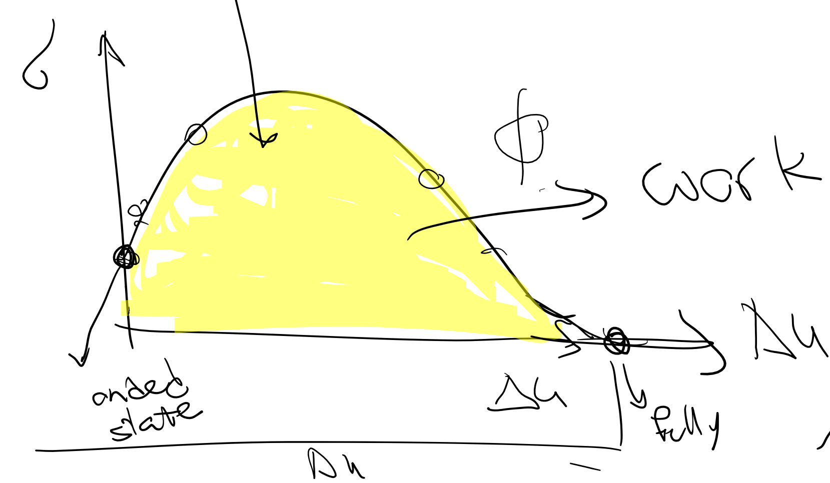


energy loss as point A goes from bonded to fully debonded energy loss as point A goes from bonded ($\Delta u = 0$) to fully debonded ($\sigma = 0$) for an infinitesimal area dS around A

$$\int_{\Delta u = 0}^{\Delta u = \Delta u_{max}} (\sigma dS)\, d\Delta u = dS \int_0^{\Delta u_{max}} \sigma \, d\Delta u$$



Δu is displacement jump



$\Phi \rightarrow$ work of separation

$$\sigma \Delta u = \frac{(\sigma \text{Area}) \cdot \Delta u}{\text{Area}} = \frac{\text{energy}}{\text{area}}$$

G = energy release rate = energy release as the crack propagate a unit area

Question 1: We know phi is energy dissipated for a fixed point in space as it goes from bonded to debonded state per area around it, G = energy released per unit area of crack. When would they be equal?

- Fracture toughness (Γ): LEFM: Energy needed to create one unit surface of crack
- Work of separation (ϕ): CFM: Energy needed to entirely debond a point in time per area (following a traction-separation-relation)
  - Relation between ϕ and G:

ERR — Work of separation

$$G = \frac{1}{\hat{v}} \int_{-\Lambda_{(k)}}^{0} \tilde{s}(\delta_k) \frac{\partial \delta_k}{\partial t}\, \mathrm{d}x + \int_0^{\delta_T} \tilde{s}(\delta_k)\, \mathrm{d}\delta_k = I_t + \tilde{\phi}_{(k)}$$

- Dynamic part () goes to zero when:
  - Steady state crack propagation (crack speed does not change). — $G = \Phi$
  - When the crack speed tends to Rayleigh wave speed (c_R) — $G \neq \Phi$

$$\left|\frac{I_t}{\tilde{\phi}_{(k)}}\right| \leq \frac{\tilde{\sigma}_{(k)}\Lambda_{(k)}}{\hat{v}\tilde{\phi}_{(k)}} \left|\frac{\partial \delta_k}{\partial t}\right|_\infty \approx \frac{\zeta_{(k)}\pi\mu}{(1-\nu)\hat{v}A_{(k)}(\hat{v})\tilde{\sigma}_{(k)}} \left|\frac{\partial \delta_k}{\partial t}\right|_\infty$$

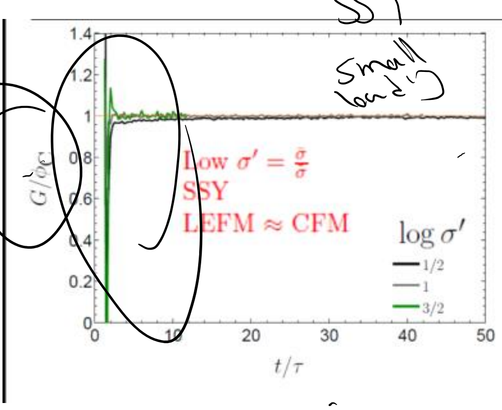
LEFM crack propagate $G = R$ — Γ

Cohesive mode uses Φ,

LEFM G = R fixes the ERR (G) to the value R
Whereas cohesive model fixes Work of Separation to the fixed value PHI.

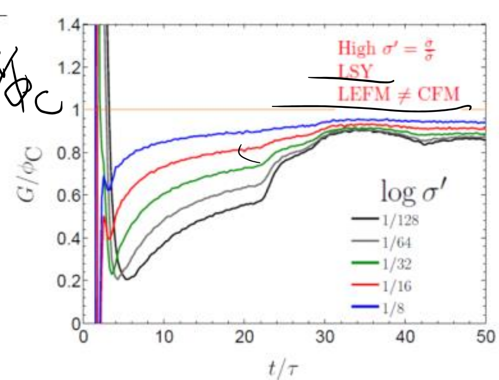
The two models do not predict the same crack dynamics, but if we want to compare LEFM and Cohesive models we need to set R = PHI,

SSY small loading



time

LEFM CFM comparison: set $\Gamma = \phi$ accurate except unsteady / low crack speed OR if SSY is not satisfied



large loading

time

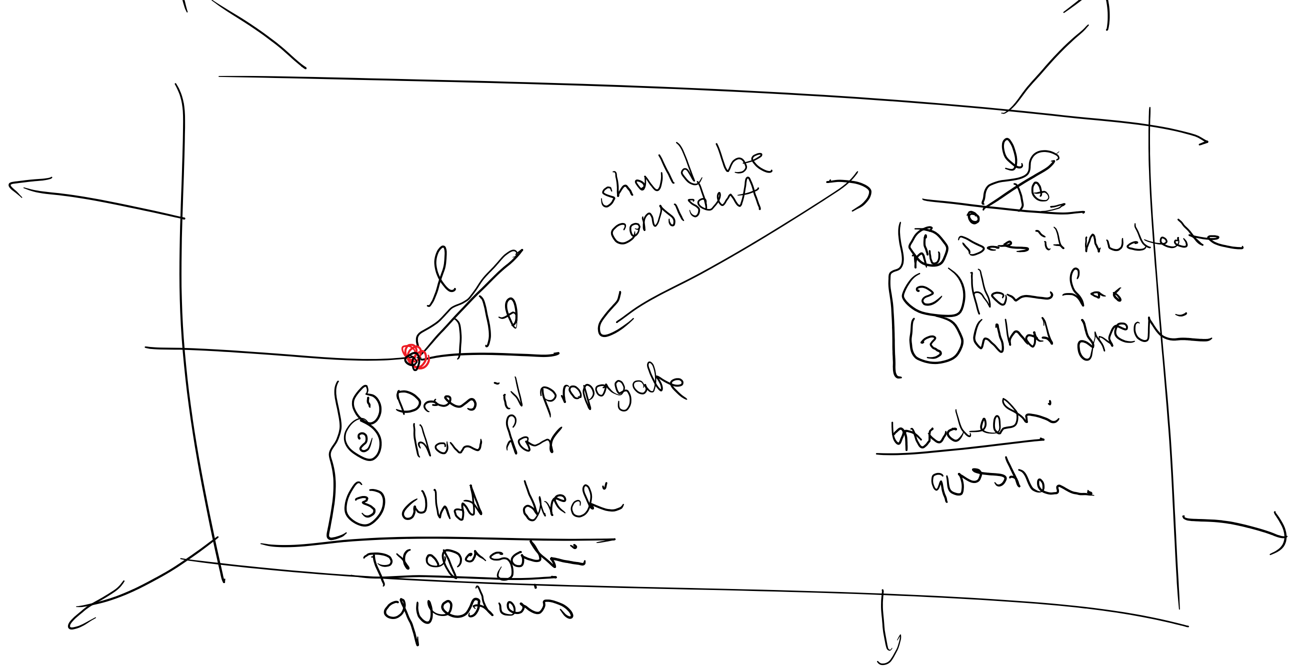
323

$$\left[\frac{J}{m^2}\right] = \left[\frac{N \cdot m}{m^2}\right] = \frac{[N]}{[m]} = [\sigma][m] = (R)$$

fracture toughness

$$[K] = [\sigma][\sqrt{m}]$$

Mixed mode fracture



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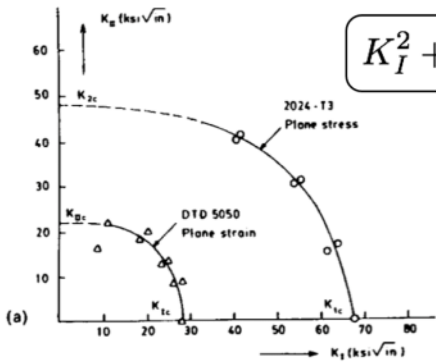
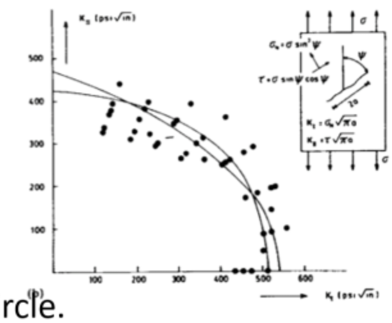
propagation criterion

Motivation: Experiment verification of the mixed-mode failure criterion



$$\boxed{K_I^2 + K_{II}^2 = K_{Ic}^2}$$

a circle in KI, KII plane



Data points do not fall exactly on the circle.

$$\left(\frac{K_I}{K_{Ic}}\right)^2 + \left(\frac{K_{II}}{K_{IIc}}\right)^2 = 1$$

self-similar growth

$$\boxed{G = \frac{(\kappa+1)K_I^2}{8\mu}}$$

327

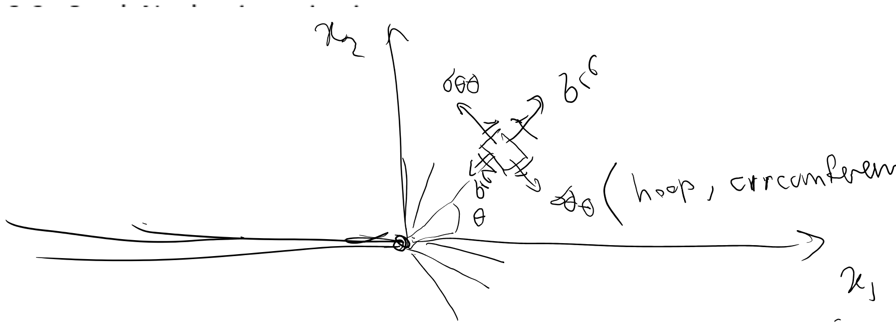
4.3 Mixed mode fracture

4.3.1 Crack propagation criteria

a) Maximum Circumferential Tensile Stress (Maximum hoop stress)

b) Maximum Energy Release Rate

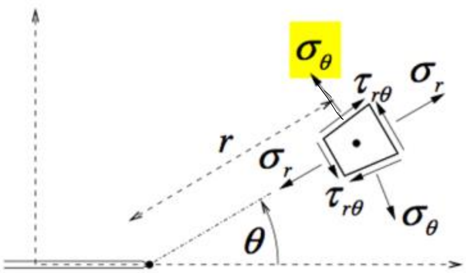
c) Minimum Strain Energy Density



$\sigma_{\theta\theta}$ (hoop, circumferential stress)

Crack propagates along the direction where $\sigma_{\theta\theta}$ is maximum

Erdogan and Sih



maximum circumferential stress criterion (maximum hoop stress criterion):
crack propagates in the direction perpendicular to the maximum circumferential stress
(evaluated on a circle of a small diameter centered at the tip)

the direction of propagation is given by the angle θ_c for which

$$\sigma_\theta(r,\theta_c) = \max_{-\pi<\theta<\pi} \sigma_\theta(r,\theta)$$

(from M. Jirasek)

principal stress ⟶ $\boxed{\tau_{r\theta} = 0}$

Maximum circumferential stress criterion

$$\sigma_r = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(-\frac{5}{4}\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2}\right) \quad (7.35a)$$

$$\sigma_\theta = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(-\frac{3}{4}\sin\frac{\theta}{2} - \frac{3}{4}\sin\frac{3\theta}{2}\right) \quad (7.35b)$$

$\frac{d\sigma_\theta}{d\theta} = 0$

$$\tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\left(\frac{1}{4}\cos\frac{\theta}{2} + \frac{3}{4}\cos\frac{3\theta}{2}\right). \quad (7.35c)$$

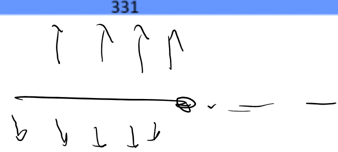
$\tau_{r\theta} = 0$

$$\tau_{r\theta} = 0 \longrightarrow \quad K_I\left(\sin\frac{\theta}{2} + \sin\frac{3\theta}{2}\right) + K_{II}\left(\cos\frac{\theta}{2} + 3\cos\frac{3\theta}{2}\right) = 0$$

$$\longrightarrow \quad \theta_c = 2\arctan\frac{1}{4}\left(K_I/K_{II} \pm \sqrt{(K_I/K_I I)^2 + 8}\right) \quad (1)$$

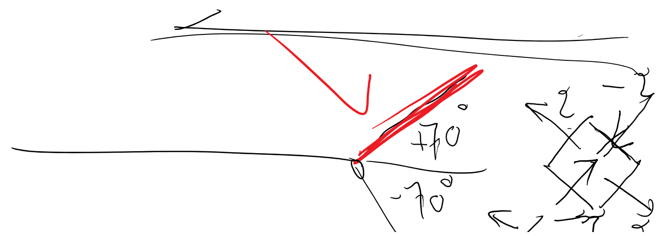
angle of propagation

$K_{II} = 0 \quad \theta = 0$



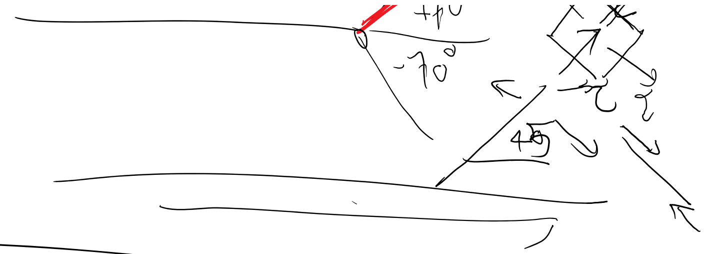
$K_I = 0$ pure mode 2

$\theta_c = 2\tan^{-1}(\quad 0 + \sqrt{8}$

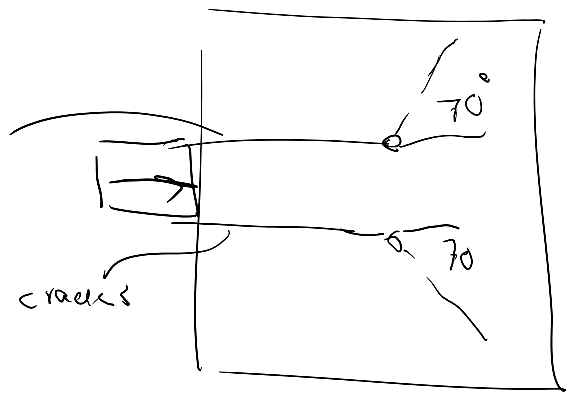


$$\theta_c = 2\tan^{-1}\frac{1}{4}\left(0 \pm \sqrt{8}\right)$$

$$= \pm 70^\circ$$

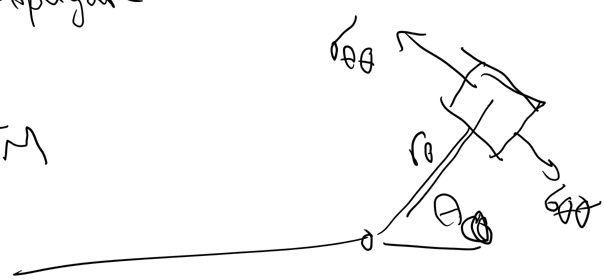


Kalthoff's example



ii) Does it propagate

LEFM



some strength parameter

$$\sigma_0 = (\sigma_{\theta\theta})_{max}(r_0, \theta_0) = \frac{1}{\sqrt{2\pi r_0}} \cos\frac{\theta_0}{2}\left[\cos^2\frac{\theta_0}{2} K_I - \frac{3}{2}\sin\theta_0 K_{II}\right]$$

K_{Ic}

(*) $\sigma_0\sqrt{2\pi r_0} = \cos\frac{\theta_0}{2}\left[\cos^2\frac{\theta_0}{2} K_I - \frac{3}{2}\sin\theta_0 K_{II}\right]$

what if we have pure mode I $\quad K_{II} = 0$

$$\sigma_0\sqrt{2\pi r_0} = K_I = K_{Ic} \quad \theta_0 = 0$$

because this is propagati

because this is propagate

$$\boxed{K_{Ic} = \cos\frac{\theta_0}{2}\left[\cos^2\frac{\theta_0}{2}K_I - \frac{3}{2}\sin\theta_0 K_{II}\right]}$$ (2)

θ_0 from (1)

Beyond LEFM