Fatigue fracture is prevalent!

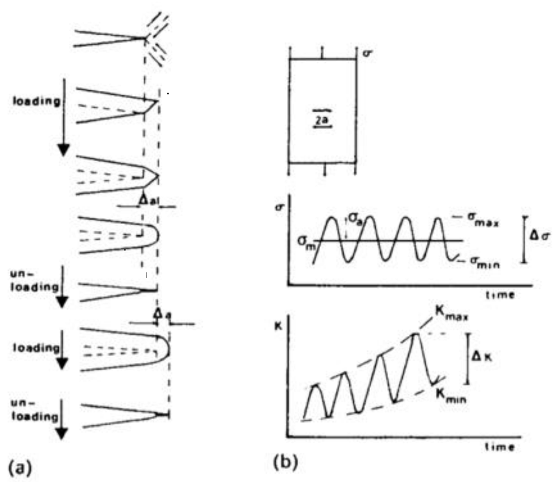
- Deliberately applied load reversals (e.g. rotating systems)
- Vibrations (machine parts)
- Repeated pressurization and depressurization (airplanes)
- Thermal cycling (switching off electronic devices)
- Random forces (ships, vehicles, planes)

(source: Schreurs fracture notes 2012)

Fatigue occurs always and everywhere and is a major source of mechanical failure

blunting

resharpening



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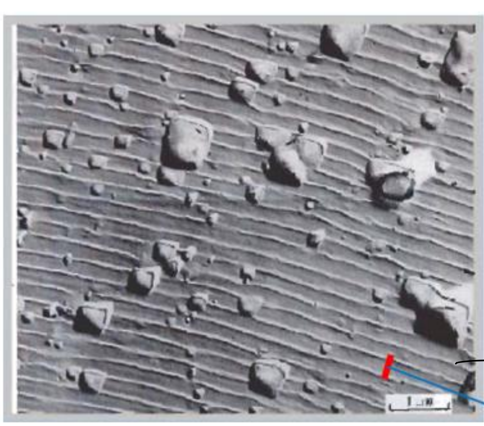
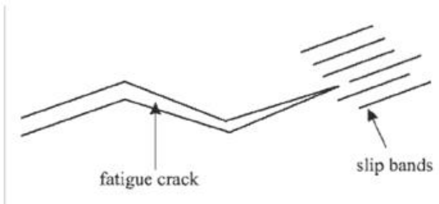


Fracture surface of a 2024-T3 aluminum alloy
(source S. Suresh MIT)

Incremental crack propagation
- Each time after sharpening the crack propagates a bit

Fatigue crack growth:

Microcrack formation in accumulated slip bands due to repeated loading



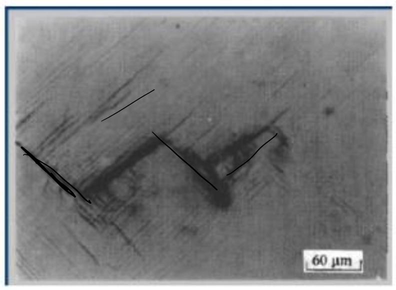
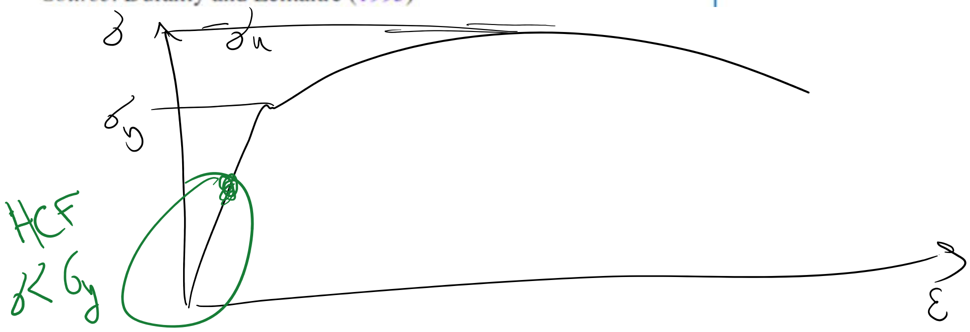
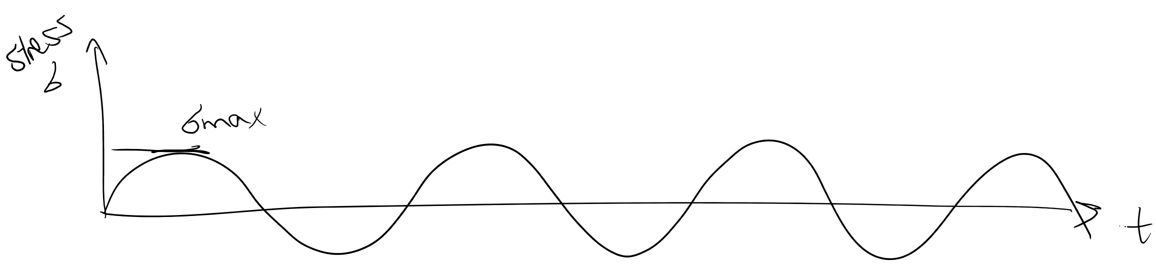


Table 7.1 Classification of fatigue damage

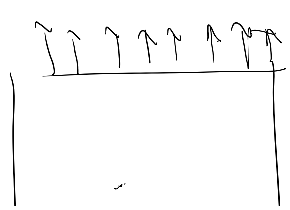
Fatigue	Failure cycles N_R	Pertinent stress	Strain ratio $\Delta\varepsilon^p/\Delta\varepsilon^e$	Energy ratio $\Delta W^p/\Delta W^e$
VHCF Very high cycle fatigue	$> 10^7$	$< \sigma_F$	≈ 0	≈ 0
High cycle fatigue	10^5 to 10^6	$< \sigma_Y$	≈ 0	≈ 0
Low cycle fatigue	10^2 to 10^4	σ_Y to σ_U	1 to 10	1 to 10
VLCF Very low cycle fatigue	1 to 20	$\approx \sigma_U$	10 to 100	10 to 100

Source: Dufailly and Lemaitre (1995)





HCF $\begin{cases} \sigma_{max} < \sigma_y \\ \quad\quad\;\; \ll \\ N \quad 10^6 - 10^7 \end{cases}$



N $10^6 - 10^7$

Almost all in elastic regime

LEFM model can be used

Stress-based : we study the evolution of stress

Plastic strain & energy ≈ 0

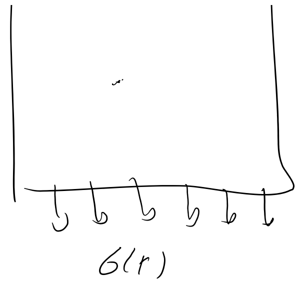


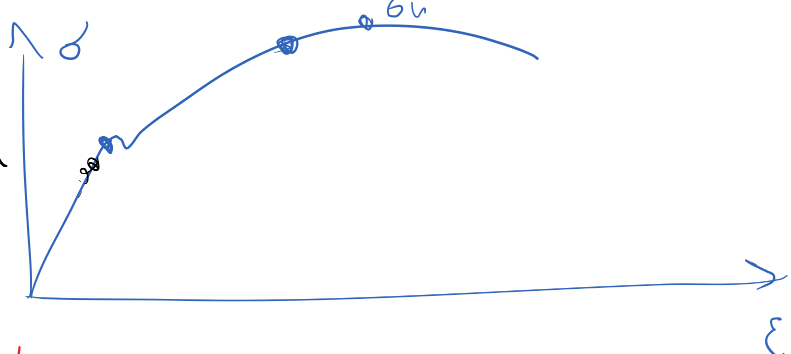
Table 7.1 Classification of fatigue damage

Fatigue	Failure cycles N_R	Pertinent stress	Strain ratio $\Delta\varepsilon^p/\Delta\varepsilon^e$	Energy ratio $\Delta W^p/\Delta W^e$
Very high cycle fatigue	$> 10^7$	$< \sigma_F$	≈ 0	≈ 0
High cycle fatigue	10^5 to 10^6	$< \sigma_Y$	≈ 0	≈ 0
Low cycle fatigue	10^2 to 10^4	σ_Y to σ_U	1 to 10	1 to 10
Very low cycle fatigue	1 to 20	$\approx \sigma_U$	10 to 100	10 to 100

Source: Dufailly and Lemaitre (1995)

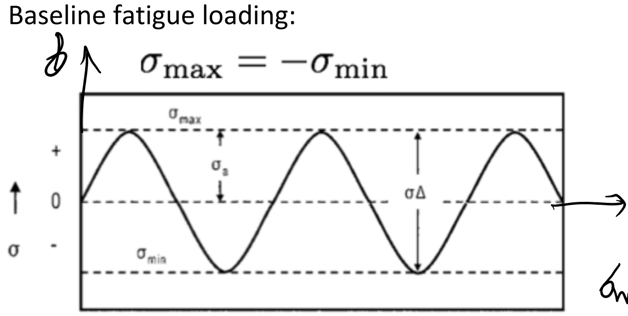
Low cycle fatigue

- ε_p, W^p very high
- LEFM (K) cannot be used
- Strain based

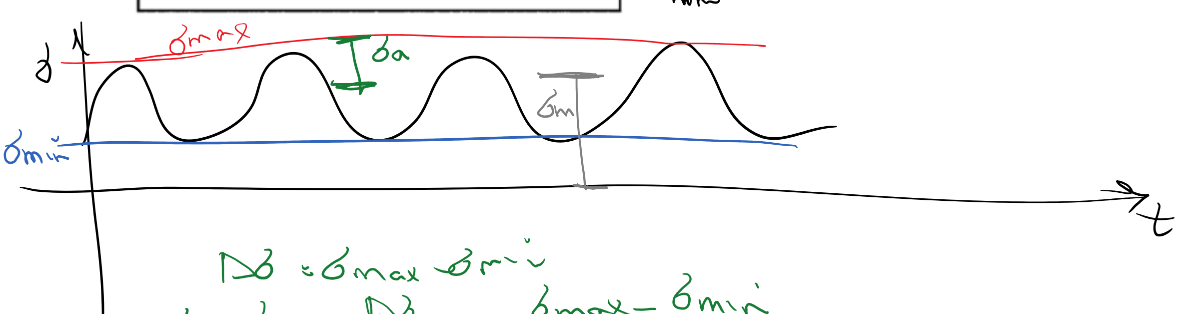


Baseline fatigue loading:

$\sigma_{max} = -\sigma_{min}$



$\sigma_{min} < \sigma_{max}$



$\Delta\sigma = \sigma_{max} - \sigma_{min}$

$\Delta\sigma$ ... $\sigma_{max} - \sigma_{min}$

τ

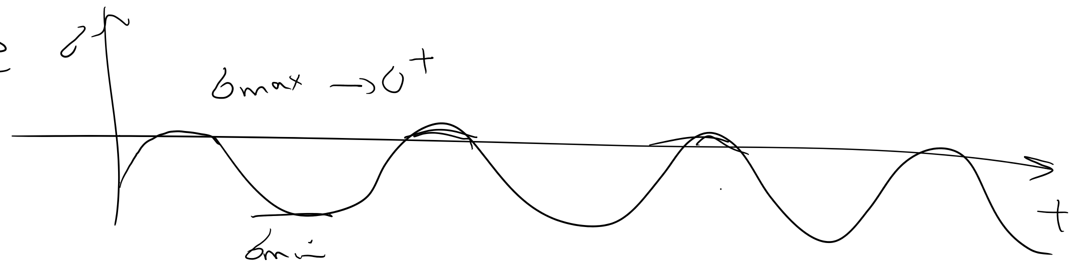
$\Delta\sigma = \sigma_{max} - \sigma_{min}$

$* \ \sigma_a = \frac{\Delta\sigma}{2} = \frac{\sigma_{max} - \sigma_{min}}{2}$

$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$

$R = \frac{\sigma_{min}}{\sigma_{max}}$ load ratio

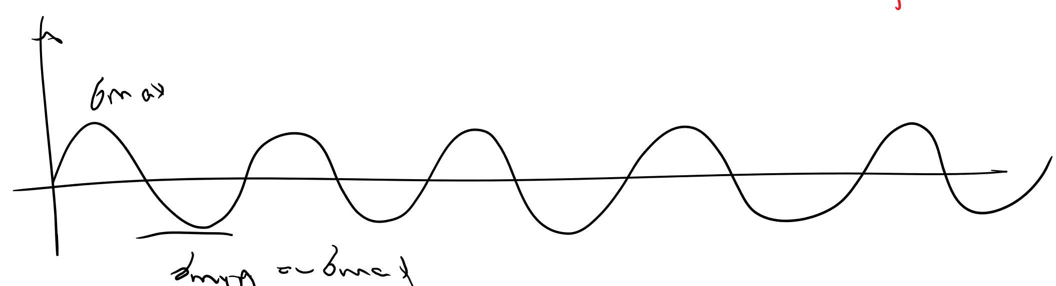
Cases



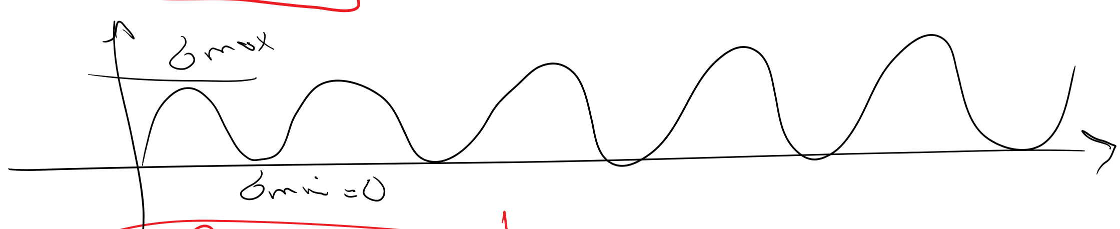
$R = \frac{\sigma_{min}}{\sigma_{max}} \to -\infty$ limit of all compressive loading

$R < -1$

Base line



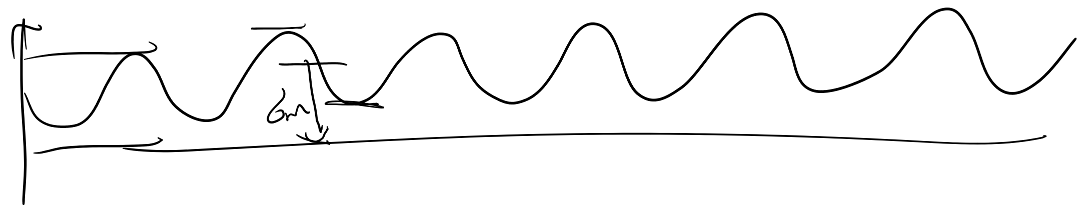
$\boxed{R = -1}$



$\boxed{R = \frac{\sigma_{min}}{\sigma_{max}} = 0}$

as $\sigma_m \nearrow$ $R \nearrow$

$R = \frac{1}{4}$



as $\sigma_m \to \infty$ $R \to 1^-$

$\boxed{R \in (-\infty, 1)}$

$R = -1$ sym base line
$R = 0$ touching $\sigma = 0$ all tensile

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min}$$

$$\sigma_a = 0.5(\sigma_{\max} - \sigma_{\min})$$

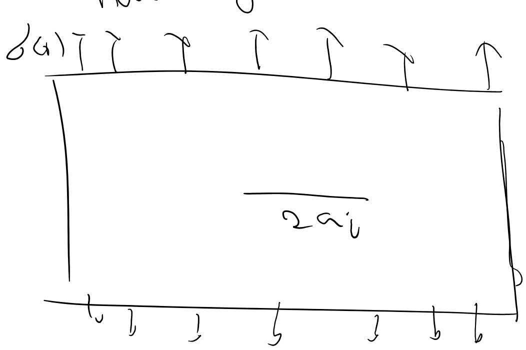
$$\sigma_m = 0.5(\sigma_{\max} + \sigma_{\min})$$

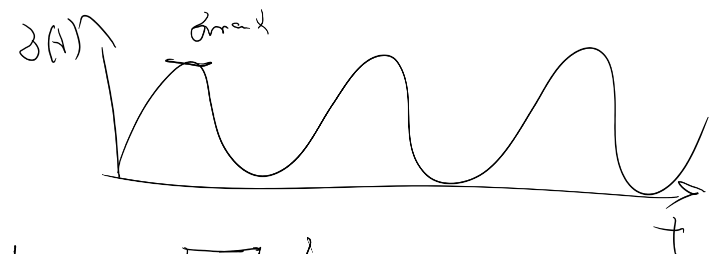
$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \quad \text{load ratio}$$

Cyclic vs. static loadings

- Static: Until K reaches K$_c$, crack will not grow
- Cyclic: K applied can be well below K$_c$, crack still grows!!!
- 1961, Paris Erdogan used the theory of LEFM to explain fatigue cracking successfully.
- Methodology: experiments first, then empirical equations are proposed.

How fatigue crack grows

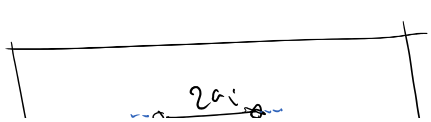




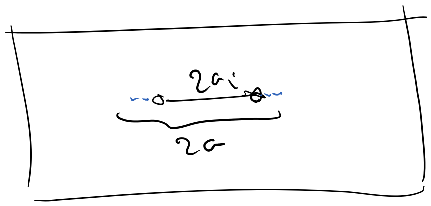
$$K_I(a,t) = \sqrt{\pi a}\,\langle \sigma(t) \rangle_t$$

$$\boxed{K_{max}(a) = \sqrt{\pi a}\,\sigma_{max}}$$

→ Compare $K_{max}(a)$ with K_{IC} to see if the crack grows from LEFM



$\Delta\sigma = \sigma_{max} - \sigma_{min}$ fixed



$$\Delta\sigma = \sigma_{max} - \sigma_{min} \quad \text{fixed}$$

$$\Delta K = \sqrt{\pi a}\,\Delta\sigma \quad \nearrow \text{ as } a \text{ grows}$$

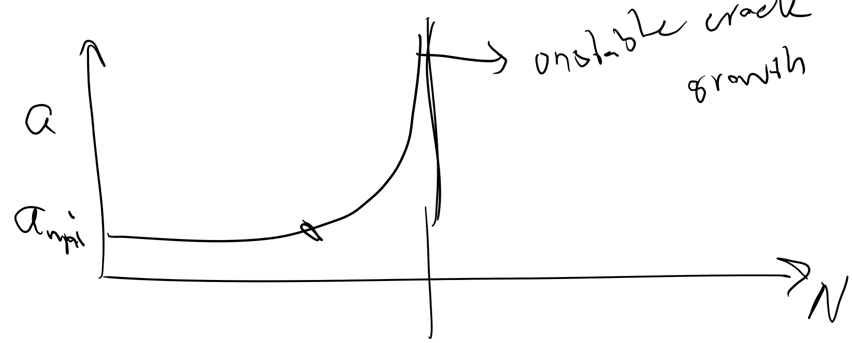
[illegible]

$$K_{max} = \sqrt{\pi a}\,\sigma_{max}$$

From a_i up to some crack length (a_c) crack grows slowly by fatigue process and a_c it unstably propagates.

$$K_{max}(a_c) = \sqrt{\pi a_c}\,\sigma_{max} = K_{IC}$$

$$\boxed{a_c = \frac{1}{\pi}\left(\frac{K_{IC}}{\sigma_{max}}\right)^2}$$

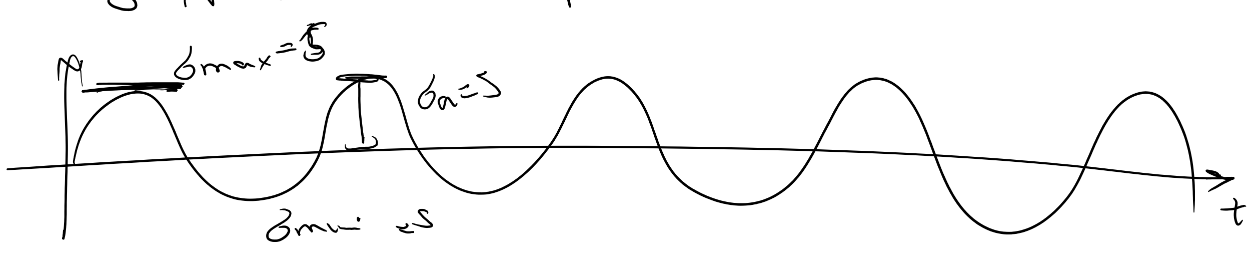


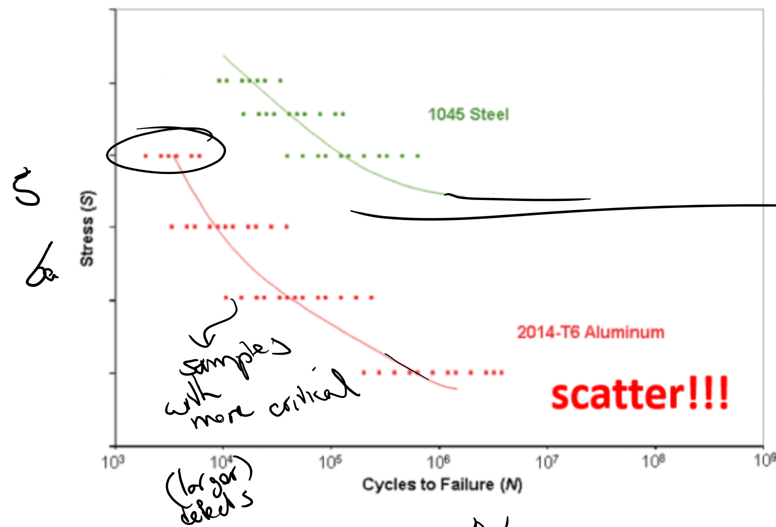
There are two approaches for modeling fatigue

Older approach:
- Stress-based
- Does not incorporate initial material flaws

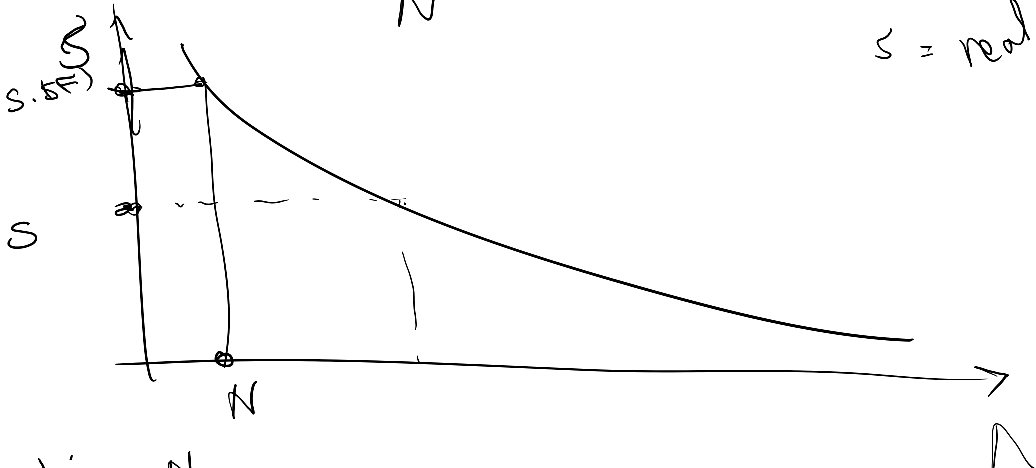
uses ↓ S–N or S–N–P plots

Newer approach:
- K-based (SIF-based)
- We start with some initial crack length





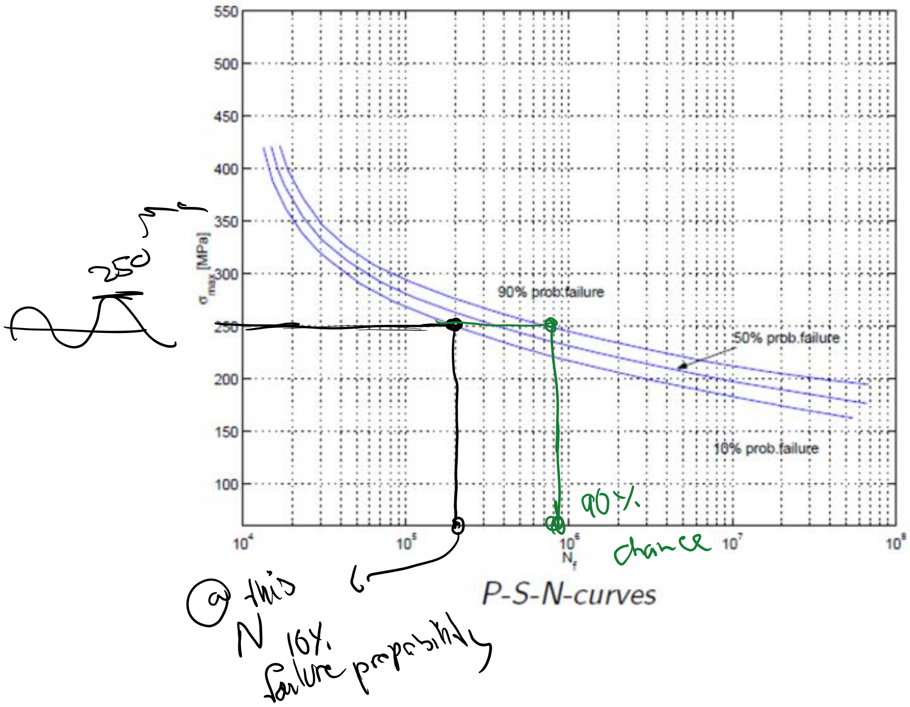
Some like steel have a threshold



s = real s :

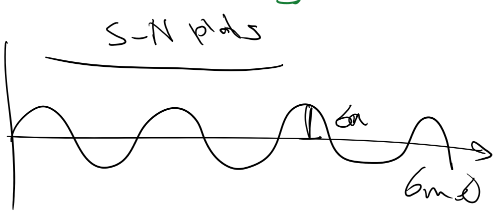
Inspection interval = $\frac{N}{SF_2}$

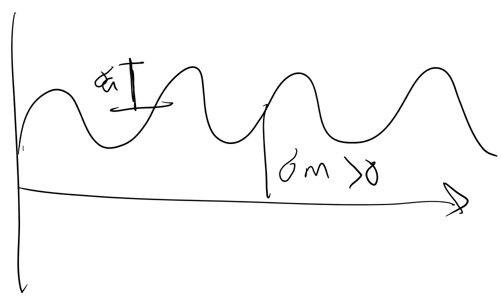
S-N-P curve: scatter effects



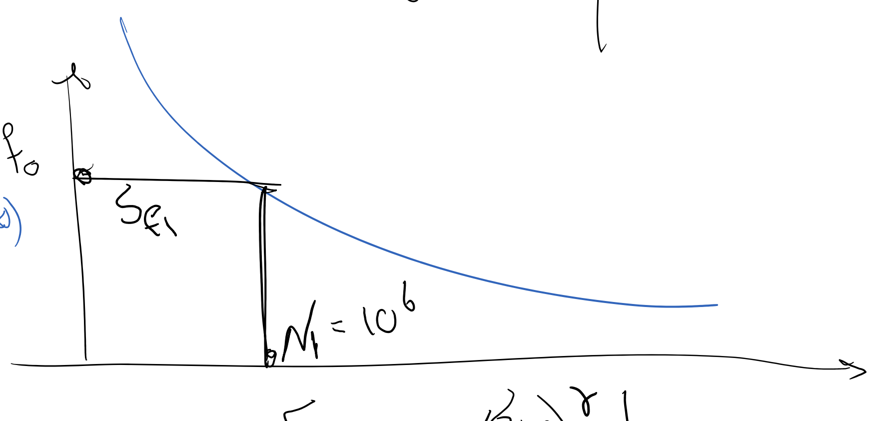
P-S-N-curves

S-N (and S-N-P) plots are calibrated for R = -1.
How to use them when R ≠ -1



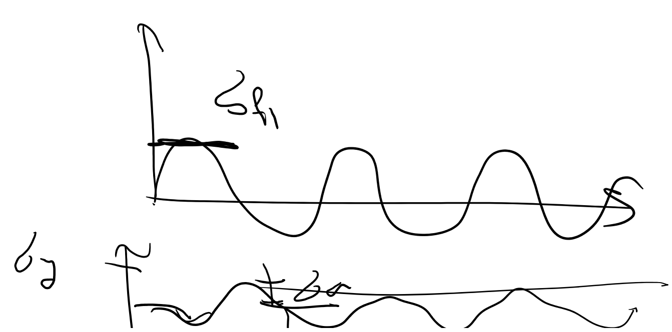


① get S_{f_0} = (S for $\sigma_m = 0$)

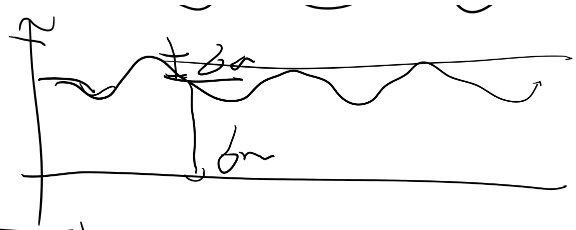


② $\sigma_a = S_{f_0}\left[1 - \left(\frac{\sigma_m}{\sigma_u}\right)^\gamma\right]$ or σ_y

$\sigma_m \to \sigma_y$, $\sigma_a \to 0$

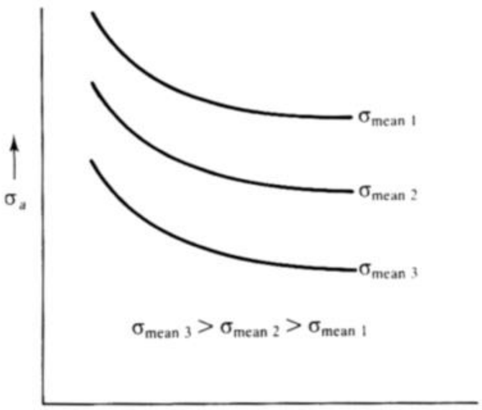


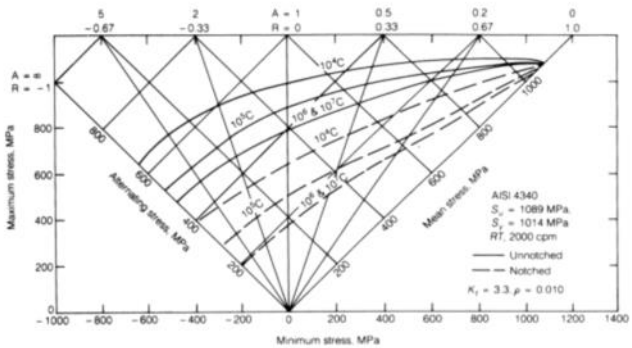
$\sigma_a = .2\, S_{f1}$



So as $\sigma_m \to \sigma_y$ or σ_u $\quad \sigma_a \to 0$ and we tend to steady state yielding failure

Effect of mean stress





Approach 1:
Master diagram

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

$$A = \frac{\sigma_a}{\sigma_m} = \frac{1-R}{1+R}$$

Approach 2:
Correction-factor formulas

$$\sigma_a = \sigma_{f0}\left[1 - \left(\frac{\sigma_m}{\sigma_u}\right)^r\right]$$

where σ_a is the amplitude of allowable stress (alternating stress).

σ_{f0} is the stress at fatigue fracture when the material under zero mean stress cycled loading

σ_m is the mean stress of the actual loading.

σ_u is the tensile strength of the material.

r = 1 is called Goodman line which is close to the results of notched specimens.

r = 2 is the Gerber parabola which better represents ductile metals.

Other correction factor

Gerber (1874)	$\frac{\sigma_a^*}{\sigma_a} = 1 - \left(\frac{\sigma_m}{\sigma_u}\right)^2$
Goodman (1899)	$\frac{\sigma_a^*}{\sigma_a} = 1 - \frac{\sigma_m}{\sigma_u}$
Soderberg (1939)	$\frac{\sigma_a^*}{\sigma_a} = 1 - \frac{\sigma_m}{\sigma_{y0}}$

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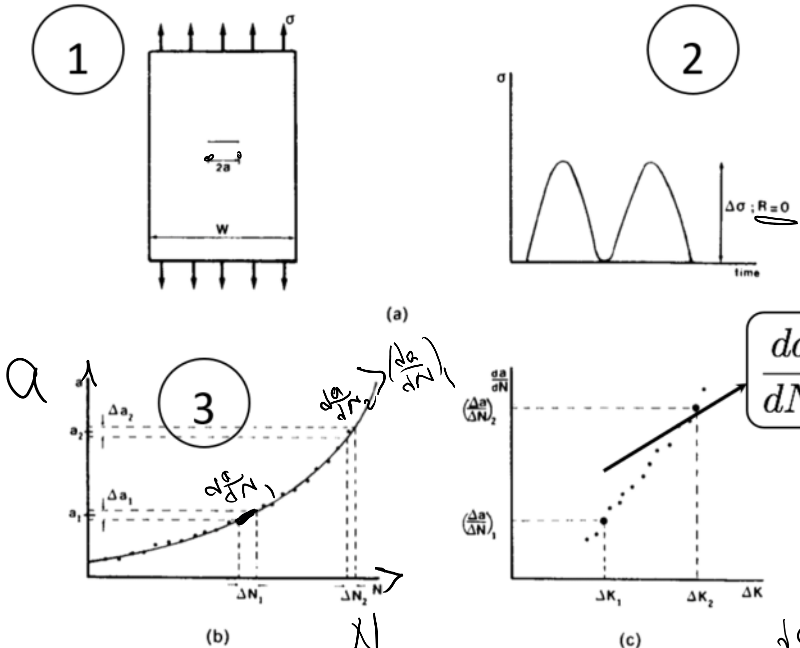
New Approach

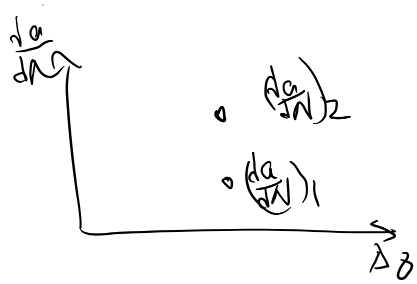
Crack growth data

$K = \sigma\sqrt{\pi a}$

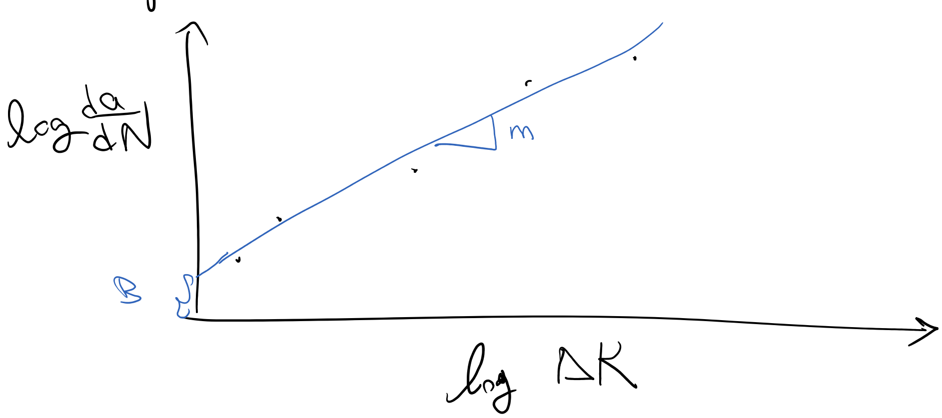


$$\boxed{\frac{da}{dN} = f_1(\Delta K, R)}$$

Hypothesis $\frac{da}{dN}$ depends on $\Delta\sigma$



$\frac{da}{dN}$ should be characterized with some thing other than $\Delta\sigma$



$$\log \frac{da}{dN} = B + m \log \Delta K$$

$$e^{(\quad)} \quad e^{(\quad)}$$

$$\frac{da}{dN} = e^B \cdot e^{m \log \Delta K} = \underbrace{e^B}_{C} (\Delta K)^m$$

$\frac{da}{dN} = C\,\Delta K^m$ Paris Erdogan

$$\boxed{\frac{da}{dN} = C\,\Delta K^m \quad \text{Paris-Erdogan relation}}$$

$K = \sigma\sqrt{\pi a}\,Y$

$\Delta K = \Delta\sigma\, Y\sqrt{\pi a}$

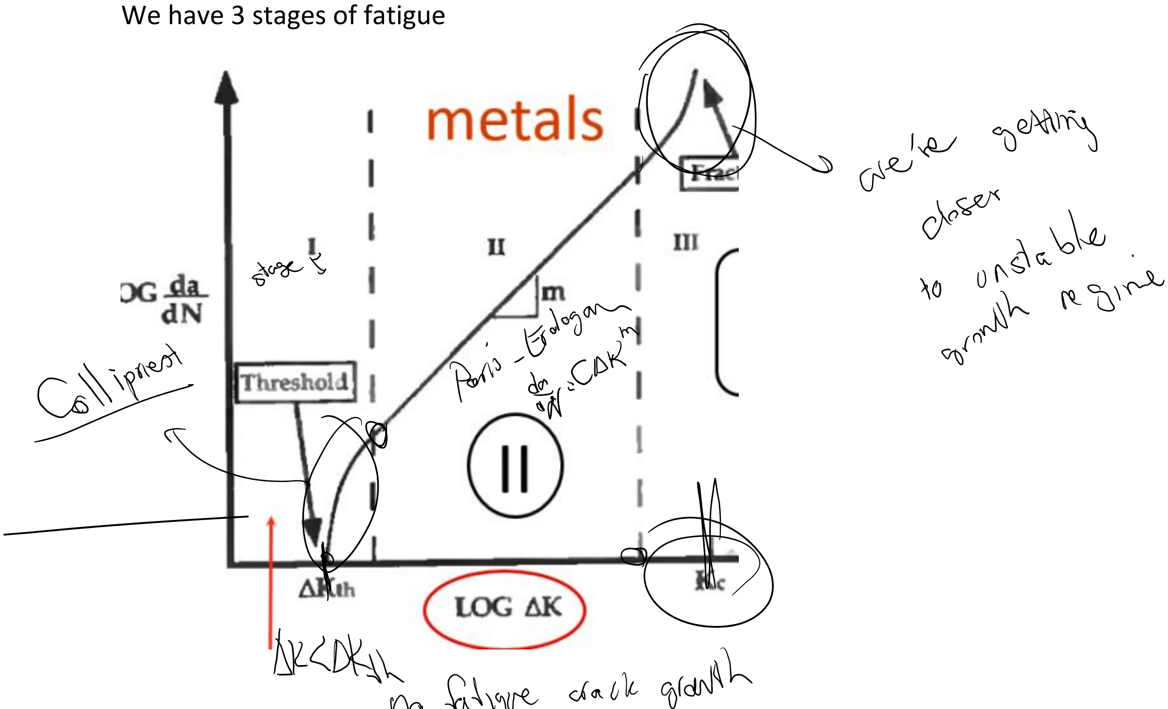
$\left[\frac{da}{dN}\right] = L$ length

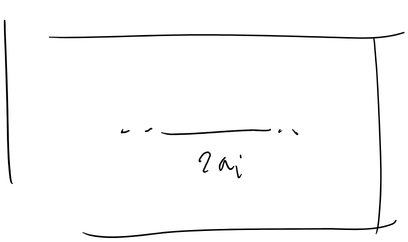
$[\Delta K]^m \rightarrow [\sigma \cdot \sqrt{L}]^m$ } → C is dimensional

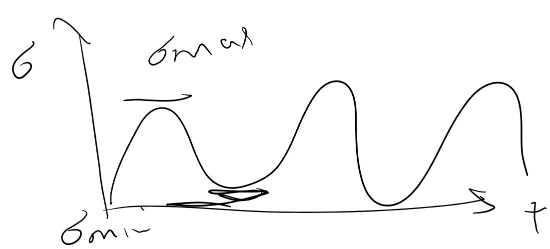
We need to clearly mention what units are used to calculate C.

* m is constant for a material

We have 3 stages of fatigue







$K_{max} = \sigma_{max}\sqrt{\pi a}$

$K_{max} = \sigma_{max}\sqrt{\pi a}$

stage 3 ; $K_{max} = \sigma_{max}\sqrt{\pi a_c} = K_c$

$a_c = \frac{1}{\pi}\left(\frac{K_c}{\sigma_{max}}\right)^2$