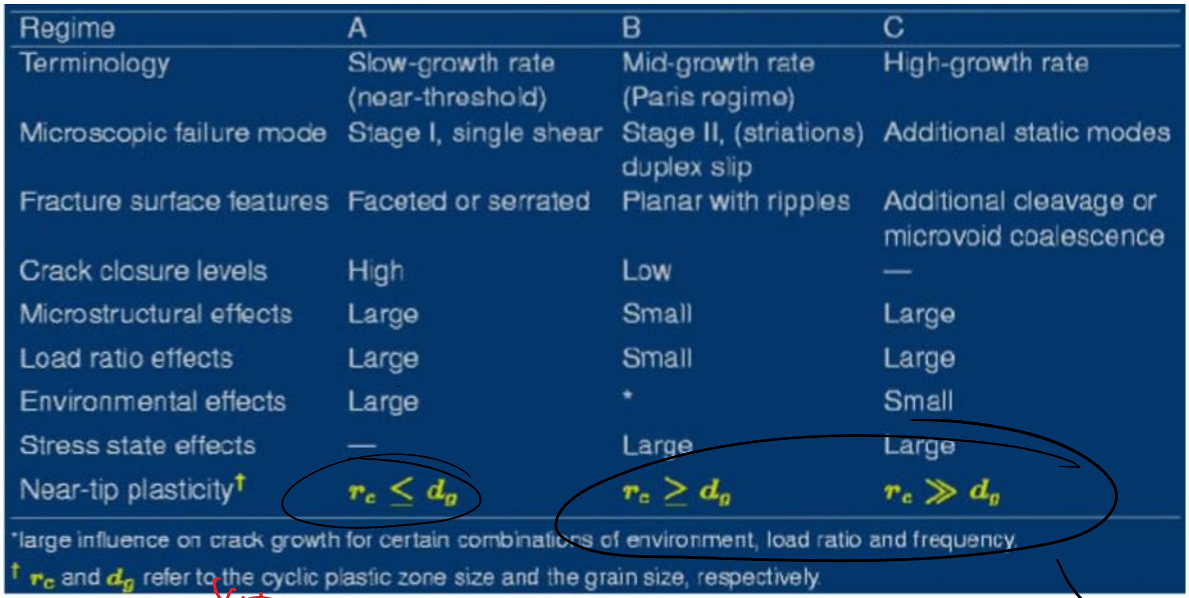
2022/11/22b

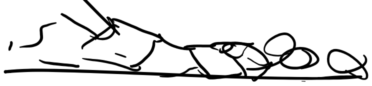
Tuesday, November 22, 2022 2:25 PM

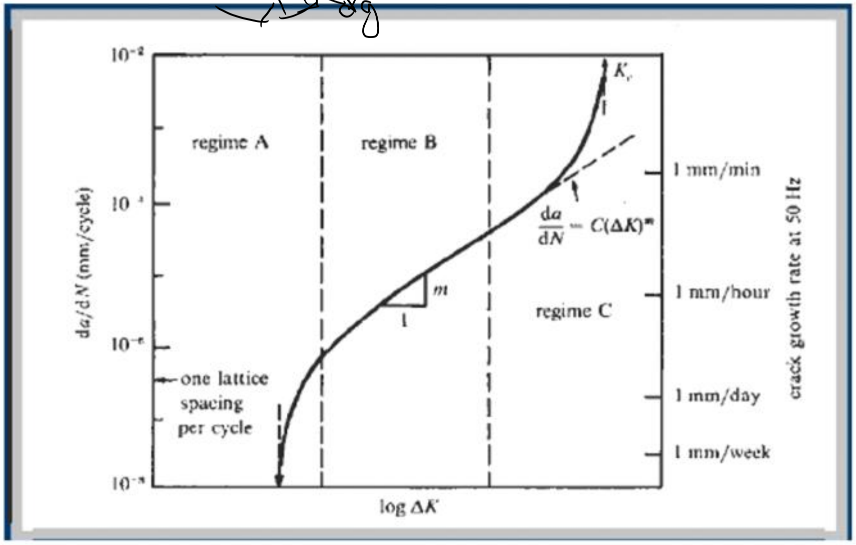


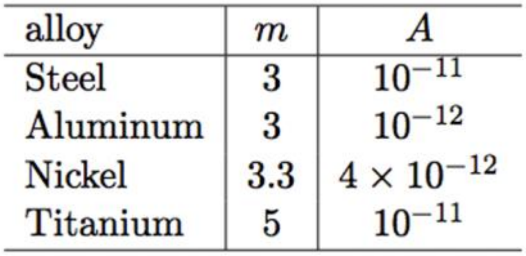
Regime	A	B	C
Terminology	Slow-growth rate (near-threshold)	Mid-growth rate (Paris regime)	High-growth rate
Microscopic failure mode	Stage I, single shear	Stage II, (striations) duplex slip	Additional static modes
Fracture surface features	Faceted or serrated	Planar with ripples	Additional cleavage or microvoid coalescence
Crack closure levels	High	Low	—
Microstructural effects	Large	Small	Large
Load ratio effects	Large	Small	Large
Environmental effects	Large	*	Small
Stress state effects	—	Large	Large
Near-tip plasticity†	$r_c \le d_g$	$r_c \ge d_g$	$r_c \gg d_g$

*large influence on crack growth for certain combinations of environment, load ratio and frequency

† r_c and d_g refer to the cyclic plastic zone size and the grain size, respectively







alloy	m	A
Steel	3	10^{-11}
Aluminum	3	10^{-12}
Nickel	3.3	4×10^{-12}
Titanium	5	10^{-11}

C, m

are material properties that must be determined experimentally from a log(delta K)-log(da/dN) plot.

m

2-4 metals

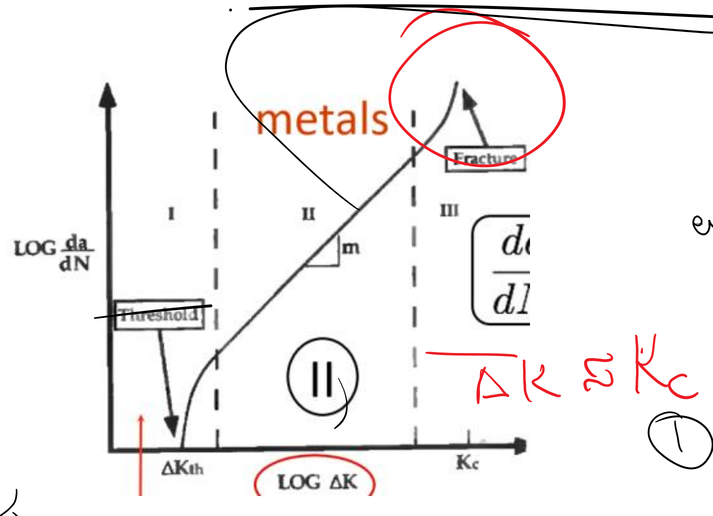
4-100 ceramics/ polymers

sensitive.

determined experimentally from a log(delta K)-log(da/dN) plot.

4-100 ceramics/ polymers

more sensitive



Paris-Erdogan relation

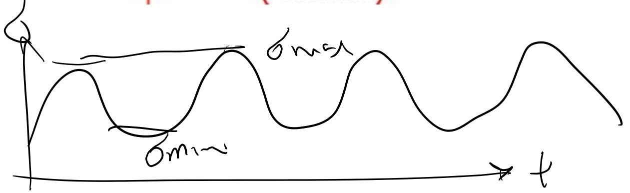
$$\frac{da}{dN} = C\,\Delta K^m \qquad \text{stage II}$$

we can extend this to stage II & III

Forman's model

$\Delta K \cong K_c$

$$① \quad \frac{da}{dN} = \frac{C(\Delta K)^m}{[\underbrace{(1-R)}_{\frac{K_{min}}{K_{max}}}K_c - \underbrace{\Delta K}_{K_{max}-K_{min}}]} \rightarrow \text{correction}$$



$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

$K_{min} = \sigma_{min}\sqrt{\pi a}\,Y$

$K_{max} = \sigma_{max}\sqrt{\pi a}\,Y$

Y → geometry correction factor

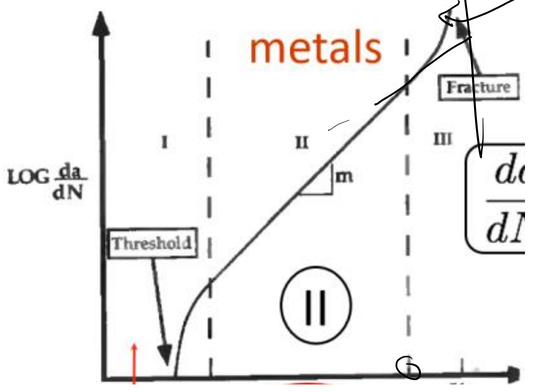
$$\frac{K_{min}}{K_{max}} = \frac{\sigma_{min}}{\sigma_{max}} = R$$

$$① \quad \frac{da}{dN} = \frac{C(\Delta k)^m}{\left(1-\frac{K_{min}}{K_{max}}\right)K_c - (K_{max}-K_{min})} = \frac{C\Delta K^m}{\underbrace{(K_{max}-K_{min})}_{\Delta K}\left(\frac{K_c}{K_{max}}-1\right)}$$

Forman's model

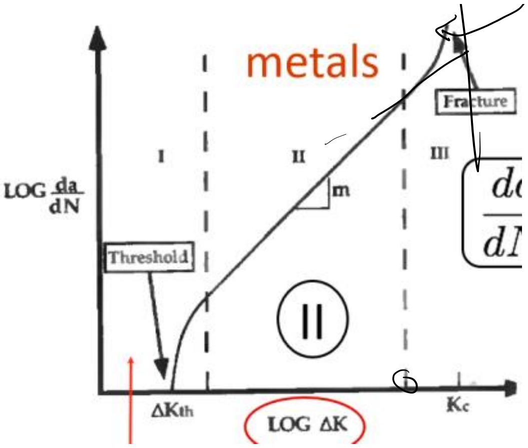
$$\boxed{\frac{da}{dN} = \frac{C\Delta K^{m-1}}{\frac{K_c}{K_{max}}-1}}$$

represents smooth region



as $K_{max} \longrightarrow K_c$

$$\frac{da}{dN} \rightarrow \infty$$



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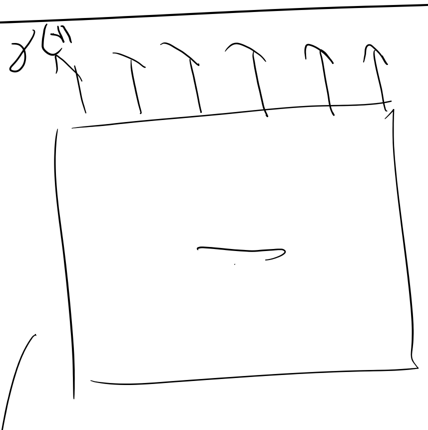
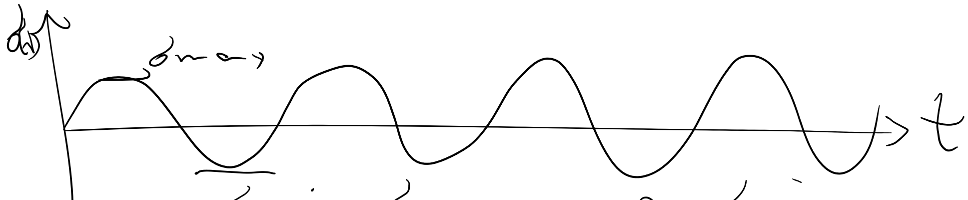




$\sigma_{min} = -\sigma_{max}$

$R = \frac{\sigma_{min}}{\sigma_{max}} = -1$

$K_{max} = \sqrt{\pi a}\ \sigma_{max}$

$K_{min} = \cancel{\sqrt{\pi a}\ \sigma_{min} < 0}$

$K_{min} = 0$

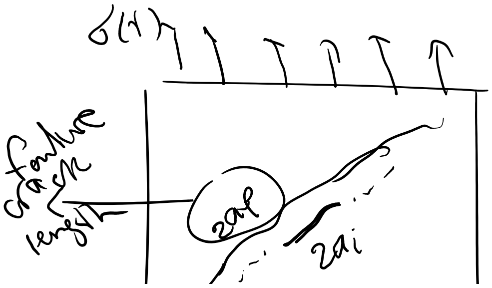
Kmin cannot be negative.

$K_{min} = \max(K_{min}(\text{corresponding to } \sigma_{min}), 0)$

$$\boxed{\Delta K = (\sigma_{max} - \max(\sigma_{min}, 0))\sqrt{\pi a}}$$

Solving $\frac{da}{dN} = \Delta K^m \longrightarrow a(N) = \ldots$

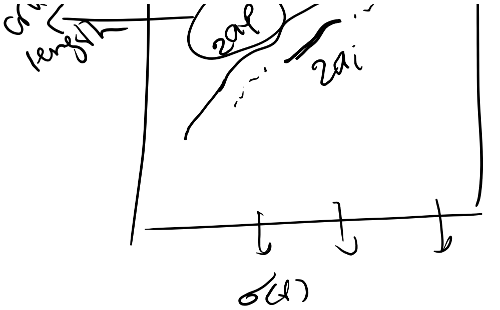
Ordinary differential Equ



a_i start crack length

a_f sudden failure crack length

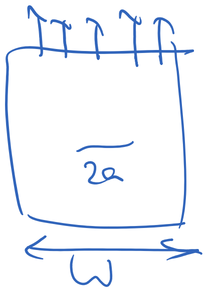
$K_{max}(a) = \sigma_{max}\sqrt{\pi a}\ Y(a)$ geometry correction factor



$K_{max}(a) = \sigma_{max}\sqrt{\pi a}\;Y(a)$ correction factor

$K_{min}(a) = \sigma_{min}\sqrt{\pi a}\;Y(a)$

$$\left.\begin{array}{l}\rightarrow \boxed{\Delta K = \Delta\sigma\sqrt{\pi a}\;Y(a)} \\ \boxed{\frac{da}{dN} = C\Delta K^m}\end{array}\right\} \rightarrow$$



$Y(a) = \frac{1}{\cos\left(\frac{\pi a}{W}\right)}$

✪

$$\frac{da}{dN} = C\left(\Delta\sigma\sqrt{\pi a}\;Y(a)\right)^m$$

ODE

a_i given: $a(N=0) = a_i$

a_f corresponds to failure

$$\sigma_{max}\sqrt{\pi a_f}\;Y(a_f) = K_c$$

For the general case that Y(a) is not constant (e.g. finite domains, ...) we need to solve this numerically.

Assume Y(a) is almost constant (large domain, ...)

$$\frac{da}{dN} = C\left(\Delta\sigma\sqrt{\pi a}\right)^m Y^m \rightarrow$$

$$= \underbrace{\left(C\,\Delta\sigma^m Y^m \pi^{\frac{m}{2}}\right)}_{A} a^{\frac{m}{2}}$$

$$\frac{da}{dN} = A\,a^{\frac{m}{2}}$$

$$\rightarrow \frac{da}{a^{\frac{m}{2}}} = A\,dN$$ integrate this

$$\int_{a_i}^{a} \tilde{a}^{-\frac{m}{2}}\,d\tilde{a} = \int_{0}^{N} A\,d\tilde{N}$$

$$\left.\frac{a^{1-\frac{m}{2}}}{1-\frac{m}{2}}\right|_{a_i}^{a} = AN$$

In general $m > 2$

$$\frac{a}{1-\frac{m}{2}}\Big|_{a_i} = AN$$

In general m > 2

$$\left(\frac{1}{a^{\frac{m}{2}-1}} - \frac{1}{a_i^{m/2-1}}\right)\frac{-1}{\frac{m}{2}-1} = AN$$

$$\longrightarrow \quad N(a) = \frac{1}{\left(\frac{m}{2}-1\right)A}\left(\frac{1}{a_i^{m/2-1}} - \frac{1}{a^{m/2-1}}\right)$$

$$A = C\,\Delta\sigma^m Y^m \pi^{\frac{m}{2}}$$

$$N(a) = \overset{F}{\frac{1}{\left(\frac{m}{2}-1\right)C\,\pi^{m/2}\,\Delta\sigma^m Y^m}}\left(\frac{1}{a_i^{m/2-1}} - \frac{1}{a^{m/2-1}}\right)$$

for a_f:

$$K_{max} = K_I$$

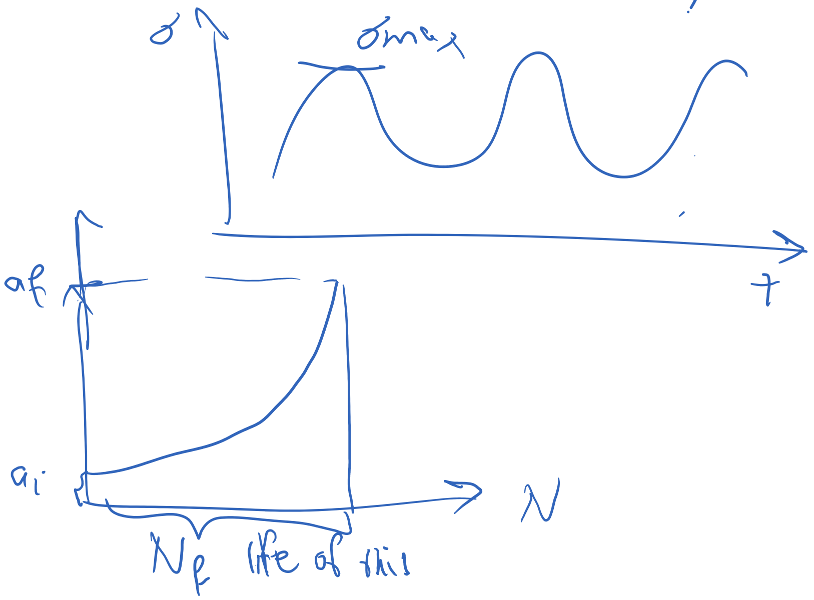
$$\downarrow$$

$$Y\sqrt{\pi a_f}\,\sigma_{max} = K_I$$

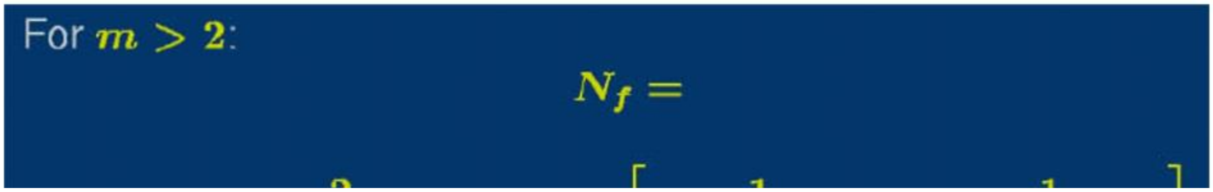
$$a_f = \frac{1}{\pi}\left(\frac{K_I}{Y\sigma_{max}}\right)^2$$

$$N(a_i) = F\left(\frac{1}{a_i^{m/2-1}} - \frac{1}{a_f^{m/2-1}}\right)$$

a $a_i \to 0$, $N(a_i) \to \infty$



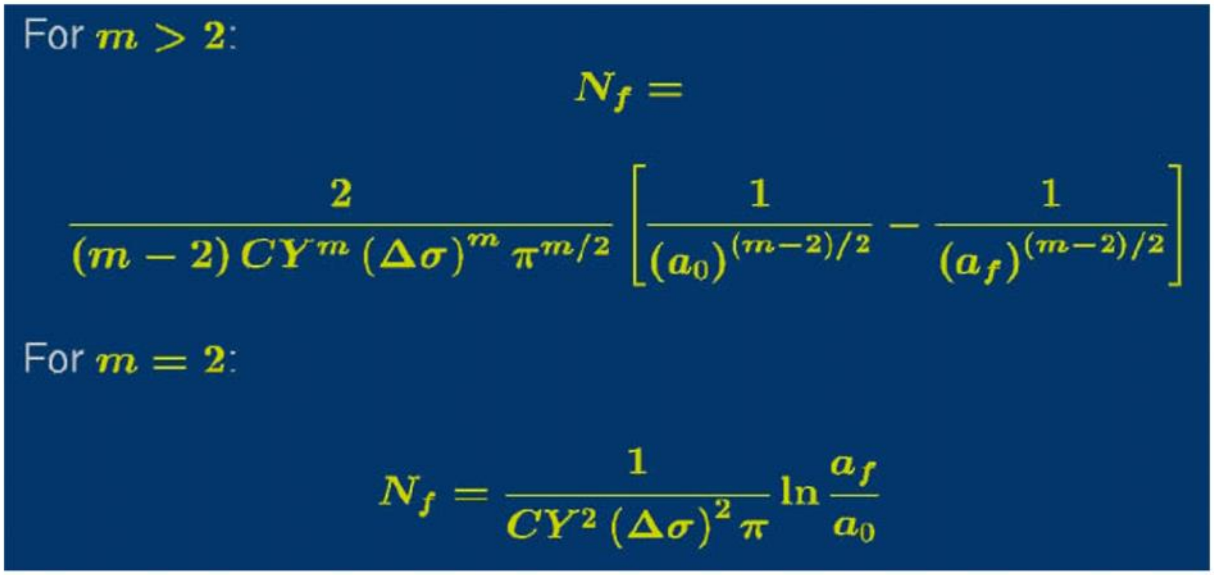
Fatigue life calculation



For $m > 2$:

$N_f =$

Fatigue life calculation



For $m > 2$:

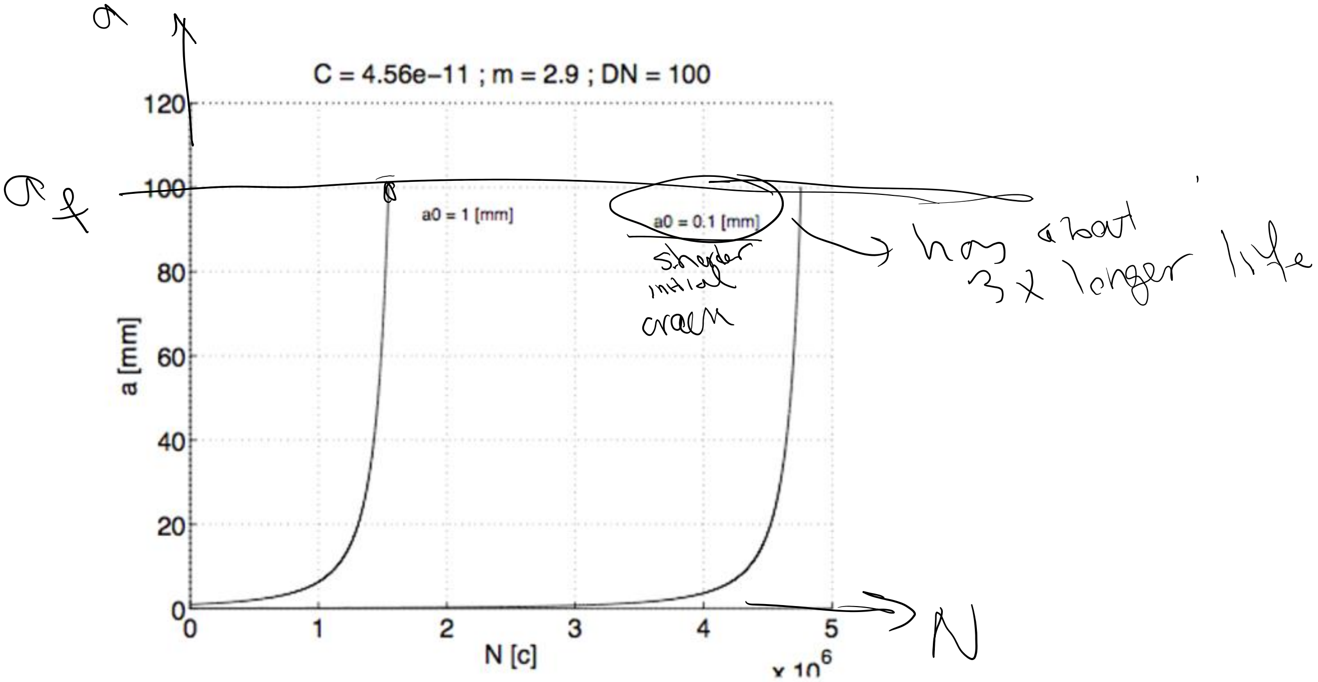
$$N_f = \frac{2}{(m-2)\,CY^m\,(\Delta\sigma)^m\,\pi^{m/2}}\left[\frac{1}{(a_0)^{(m-2)/2}} - \frac{1}{(a_f)^{(m-2)/2}}\right]$$

For $m = 2$:

$$N_f = \frac{1}{CY^2\,(\Delta\sigma)^2\,\pi}\ln\frac{a_f}{a_0}$$

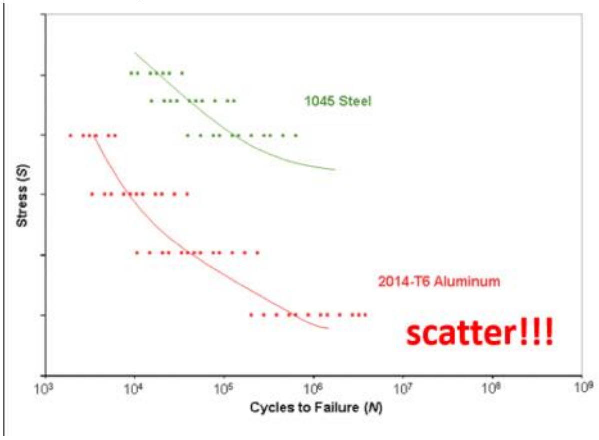
(source Course presentation S. Suresh MIT)

Importance of initial crack length



aluminum ; 380$\Delta\sigma = 50$ [MPa]

The example above is the source of the scatter below



How do we choose ai?
Do Nondestructive evaluation (NDE)

case 1
we see the crack

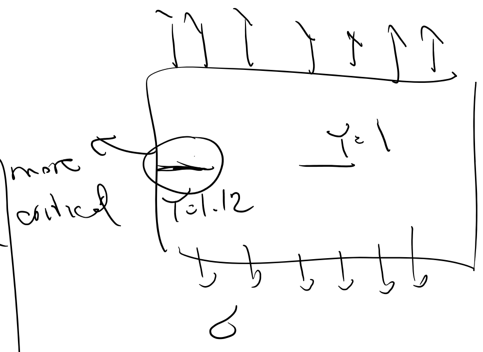
most critical

use this

Case 2

what if we don't detect a crack

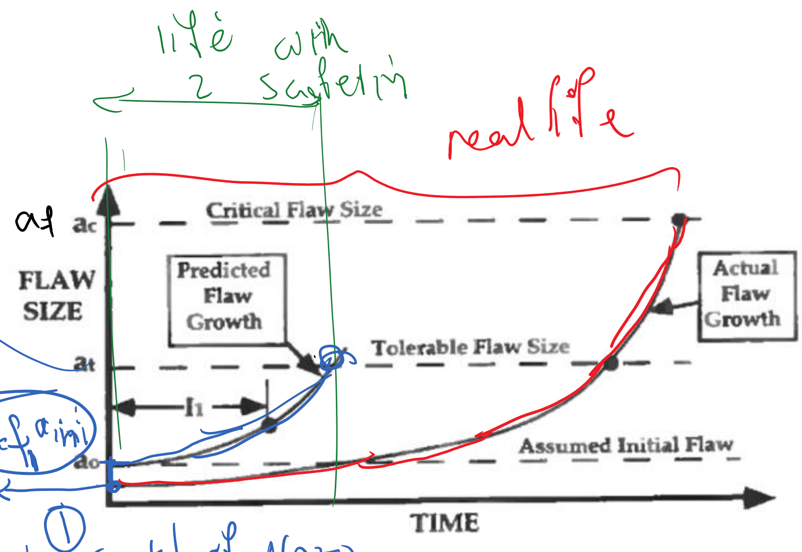
Use the tolerance of NDE



We already discussed how to build factor of safety using S-N or S-N-P plots

How about Paris-Erdogan model

② we go to $\frac{1}{2}$ (or other factor) of a_c



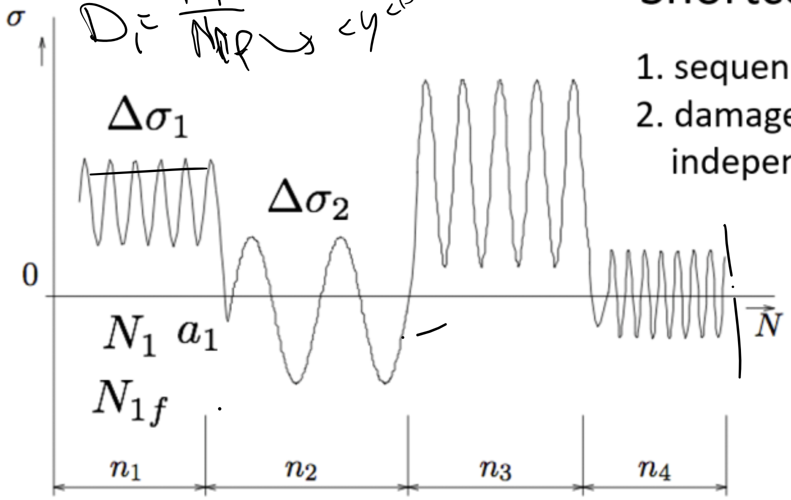
$f_1 > 1$. $a_0 = f_1 a_{ini}$

initial flaw size a_{ini} (either measured ① or tol of NDE)

Miner’s rule for variable load amplitudes

1945

$D_i = \frac{N_1}{N_{1f}}$ → real # cycle → cycles we can have



Shortcomings:

1. sequence effect not considered
2. damage accumulation is independent of stress level

N_i/N_{if} : damage

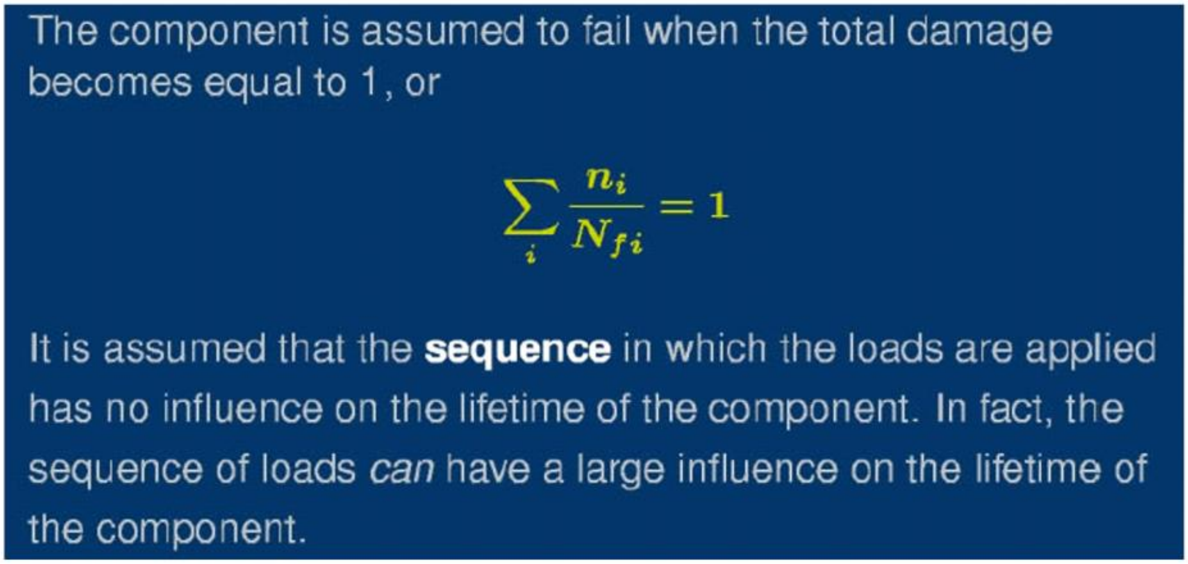
$$\boxed{\sum_{i=1}^{n} \frac{N_i}{N_{if}} = 1}$$

$\Delta\sigma_i$

N_i number of cycles a_0 to a_i

N_{if} number of cycles a_0 to a_c

Influence of sequence of loading



The component is assumed to fail when the total damage becomes equal to 1, or

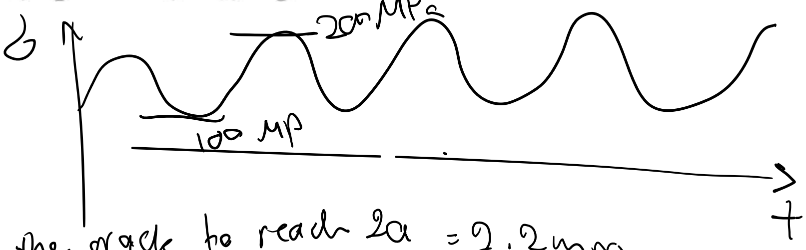
$$\sum_i \frac{n_i}{N_{fi}} = 1$$

It is assumed that the **sequence** in which the loads are applied has no influence on the lifetime of the component. In fact, the sequence of loads *can* have a large influence on the lifetime of the component.

Examples for Fatigue

A large plate contains a crack of length $2a_0$ and is subjected to a constant-amplitude tensile cyclic stress normal to the crack which varies between 100 MPa and 200 MPa. The following data were obtained: for $2a_0 = 2$ mm it was found that $N = 20,000$ cycles were required to grow the crack to $2a_{f_{\text{inal}}} = 2.2$ mm, while for $2a = 20$ mm it was found that $N = 1000$ cycles were required to grow the crack to $2a_{f_{\text{inal}}} = 22$ mm. The critical stress intensity factor is $K_c = 60$ MPa $\sqrt{m}$. Determine the constants in the Paris (Equation (9.3)) and Formam (Equation (9.4)) equations.

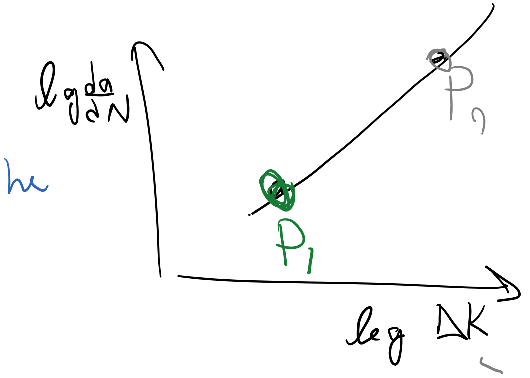
$K = (\sqrt{\pi a})\sigma$



P1 { $2a_0 = 2$ mm
it takes 20,000 cycles for the crack to reach $2a = 2.2$ mm

P2 [$2a_0 = 20$ mm
" = 1000 " " " 22 mm

Calibrate Paris_Erdogan law



- By having two points we can calibrate the model.

$\frac{da}{dN} \approx \frac{\Delta a}{\Delta N}$

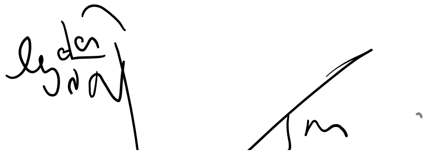
$\left(\frac{da}{dN}\right)_1 = \frac{\Delta a}{\Delta N} = \frac{(1.1\,m - 1.0\,m)\,1e{-3}}{20000} = 5e{-9}\ m$

$\Delta K = \Delta\sigma\sqrt{\pi a} = (200-100)Ma\sqrt{\pi(1e{-3}m)} = 5.6\ MPa\sqrt{m}$

$\left(\frac{da}{dN}\right)_2 = \frac{\Delta a}{\Delta N} = \frac{11e{-3} - 10e{-3}}{1000} = 1e{-6}$

$\Delta K_2 = \Delta\sigma\sqrt{\pi a} = 100\ MPa\sqrt{\pi(10e{-3}m)} = 17.72\ MPa\sqrt{m}$

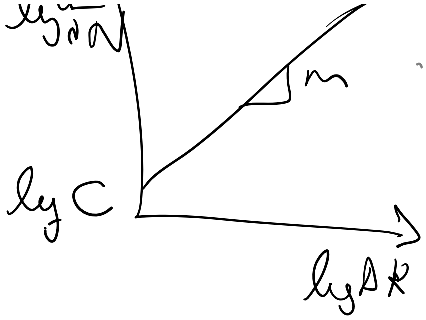
$\log\frac{da}{dN} = \log C + m\log\Delta K$



$$\log \frac{da}{dN} = \log C + m \log \Delta K$$

$\hookrightarrow$ $\log 5\text{e-}9 \;\; \log C + m \log 5.6$

$\hookrightarrow$ $\log 1\text{e-}6 = \log C + m \log 17.77$



$$\begin{cases} -8.3 = \log C + .748\, m \\ -6 = \log C + 1.248\, m \end{cases}$$

$$\boxed{\begin{array}{c} m = 4.6 \qquad C = 1.82\text{e-}12 \dfrac{m}{(MPa\sqrt{m})^{4.6}} \\ \dfrac{da}{dN} = C\, \Delta K^{m} \end{array}}$$