2022/12/06

Tuesday, December 6, 2022 2:31 PM

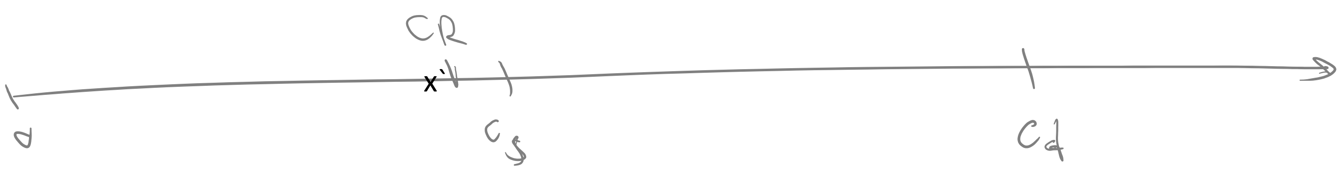
Approximate Rayleigh wave speed:

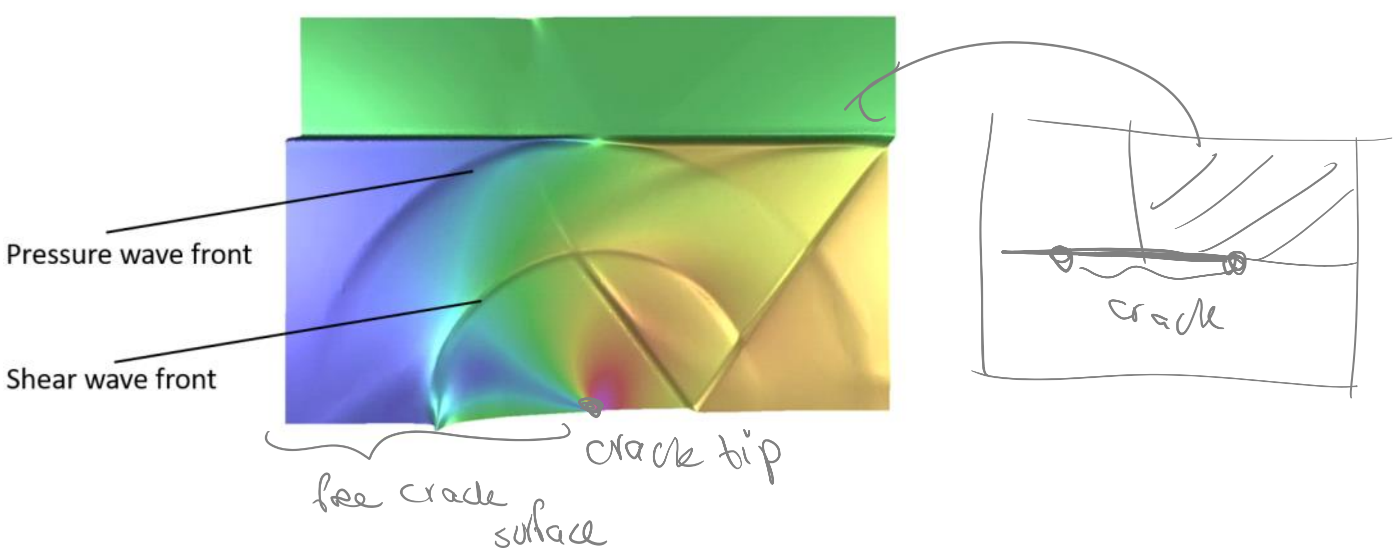
$$c_{\mathrm{d}} = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad c_{\mathrm{s}} = \sqrt{\frac{\mu}{\rho}}.$$

Poisson ratio

3D and 2D plane strain $c_d = \sqrt{\dfrac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}}, \quad c_s = \sqrt{\dfrac{E}{2\rho(1+\nu)}}, \quad c_R \approx c_s \dfrac{0.862 + 1.14\nu}{1+\nu}.$

Often cd is about 2x or higher than cs and cR is about
90%, 95% of cs

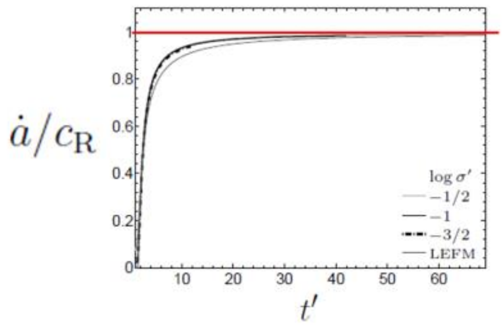




$$\Sigma_I^{11} = -\frac{1}{D}\left\{(1+\alpha_{II}^2)^2\frac{\cos\frac{1}{2}\theta_I}{\sqrt{\gamma_I}} - 4\alpha_I\alpha_{II}\frac{\cos\frac{1}{2}\theta_{II}}{\sqrt{\gamma_{II}}}\right\}$$

For one material under mode I maximum possible crack speed is Rayleigh wave speed where angular functions tend to infinity:

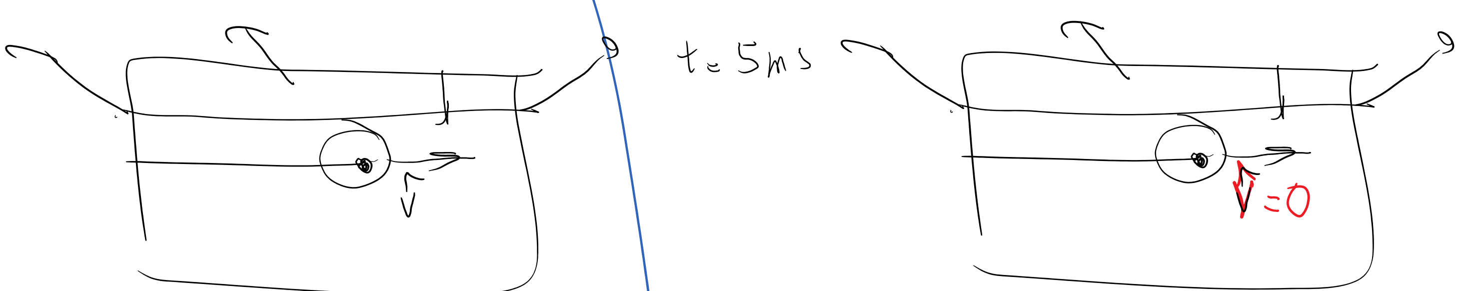
It can be shown that the *Rayleigh wave speed*, denoted by c_R, equals the non-zero value of $\hat{v}$ at which D vanishes (Rayleigh, 1885).



Dynamic stress intensity factor

k=1 mode I
k=2 " II

$$\boxed{\underbrace{K_{(k)}(t,a,\hat{v})}_{\text{Dynamic SIF for crack speed } \hat{v}} = k_{(k)}(\hat{v})\underbrace{K_{(k)}(t,a,0)}_{\text{quasi-static SIF for } \hat{v}=0}}$$



ratio of dynamic SIF to quasi-static SIF *stationary stress intensity factor*, $K_{(k)}(t, a, 0)$, is the stress intensity factor that would result from the same applied loading if the crack tip were stationary at the instantaneous position corresponding to the crack length a

$k_{(k)}(\hat{v})$ is a universal function of crack-tip speed for mode-(k) crack growth that is independent of the loading and the geometry of the body and that can be approximated as

$$\boxed{k_{(k)}(\hat{v}) \approx (1 - \hat{v}/c_{\mathrm{R}})/\sqrt{1 - \hat{v}/c_{(k)}}}$$

$$k_I(\hat{v}) \approx \frac{\left(1 - \frac{\hat{v}}{c_R}\right)}{\sqrt{1 - \frac{\hat{v}}{c_d}}} \qquad c_{(1)} = c_d$$

$$k_{II}(\hat{v}) = \frac{1 - \frac{\hat{v}}{c_R}}{\sqrt{1 - \frac{\hat{v}}{c_s}}} \qquad c_{(2)} = c_s$$

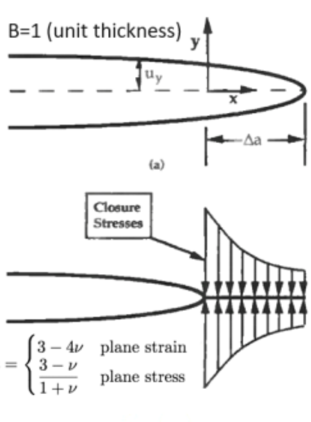
Note that k(v) approaches 0 as the crack speed tends to Rayleigh wave speed.

ERR for dynamics

Recall

K-G relationship

Irwin



$$\kappa = \begin{cases} 3 - 4\nu & \text{plane strain} \\ \frac{3-\nu}{1+\nu} & \text{plane stress} \end{cases}$$

$$G = \lim_{\Delta a \to 0} \left(\frac{\Delta U}{\Delta a} \right)_{\text{fixed load}}$$

work of crack closure $\quad \Delta U = \int_0^{\Delta a} dU(x)$

$$dU(x) = 2\frac{1}{2}\sigma_{yy}(x) u_y(x) dx$$

$$u_y = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(\kappa + 1 - 2\cos^2\frac{\theta}{2}\right), \ \theta = \pi \ \Rightarrow$$

$$u_y = \frac{(\kappa + 1)K_I(a + \Delta a)}{2\mu}\sqrt{\frac{\Delta a - x}{2\pi}}$$

$$\sigma_{yy} = \frac{K_I(a)}{\sqrt{2\pi x}}$$

$\kappa = \begin{cases} 3-4\nu & \text{plane strain} \\ \frac{3-\nu}{1+\nu} & \text{plane stress} \end{cases}$

$$\sigma_{yy} = \frac{K_I(a)}{\sqrt{2\pi x}}$$

$$G = \lim_{\Delta a \to 0} \frac{(\kappa+1)K_I^2}{4\pi\mu\Delta a} \int_0^{\Delta a} \sqrt{\frac{\Delta a - x}{x}}\, dx \Rightarrow \boxed{G = \frac{(\kappa+1)K_I^2}{8\mu}}$$

Dynamic energy release rate

- K, G relation:

$$\boxed{G = \frac{1-\nu}{2\mu}\left[A_I(\hat{v})K_I^2 + A_{II}(\hat{v})K_{II}^2\right]}$$

because now $\sigma_{ij}^{(k)} = \frac{K_{(k)}}{\sqrt{2\pi r}} \Sigma_{ij}^{(k)}(\theta, \hat{v})$

as $\hat{v} \to 0$ (quasi-static)

$A_I(\hat{v})$ & $A_{II}(\hat{v}) \to 1$

$\to \infty$ as $\hat{v} \to c_R$

We're back to the original equation we had before

$$G = \frac{1-\nu}{2\mu}\left[K_I^2 + K_{II}^2\right]$$

- Static limit: $A_{(k)} \to 1 \text{ as } \hat{v} \to 0^+$
- Rayleigh speed limit (G tends to infinity)

$$A_{(k)} = O[(c_R - \hat{v})^{-1}]$$

as $\hat{v} \to c_R$ $\quad A_{(k)} \propto \frac{1}{c_R - \hat{v}}$

---

consider a mode I problem

$$G = \frac{1-\nu}{2\mu}\left[A_I(\hat{v}) K_I^2(\hat{v})\right]$$

$$K_I(\hat{v}) = k_I(\hat{v}) \underbrace{K_I(\hat{v}=0)}_{\text{Static SIF}}$$

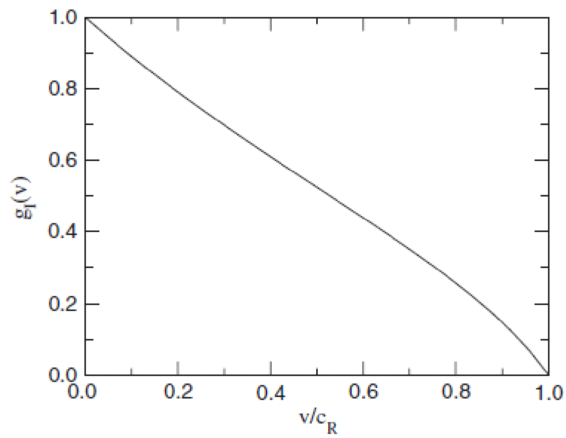
Static

$$G = \frac{1-\nu}{2\mu} \underbrace{\left[A_I(\hat{v}) k_I^2(\hat{v}) \right]}_{g_I(\hat{v}) \approx 1 - \frac{\hat{v}}{c_R}} K_I(\hat{v}=0)$$

$$G = \frac{1-\nu}{2\mu} g_I(\hat{v}) K_I(\hat{v}=0)$$

$$g_I(\hat{v}) \approx 1 - \frac{\hat{v}}{c_R}$$

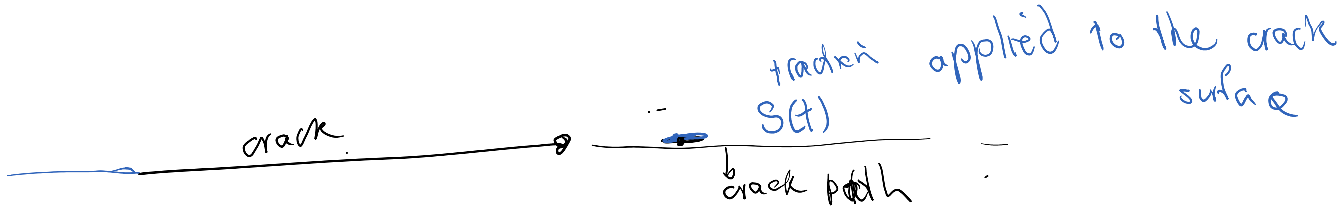
$$\boxed{g(\hat{v}) \approx 1 - \hat{v}/c_R \text{ for } 0 \le \hat{v} \le c_R}$$



Example:

Consider crack propagation under far field loading

Mode I



infinite crack

time

crack length

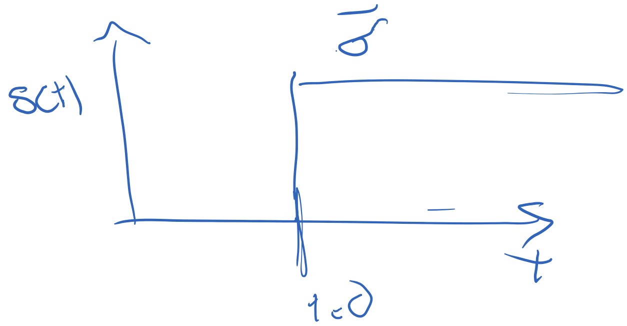
$$K_I(t, a, \hat{v}=0) = C\sqrt{2\pi c_d} \int_{-\infty}^{t} s(z)\sqrt{t-z}\, dz$$

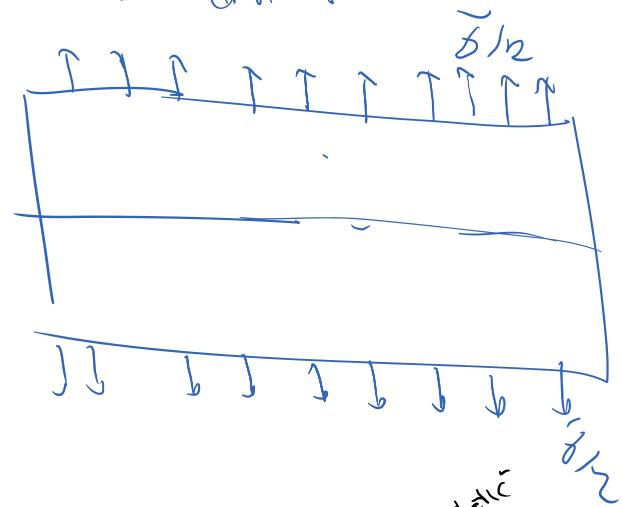
quasi-static SIF

constant

K_I quasi-static SIF $\qquad \to \infty$ constant

Consider far field loading





static

$\bar{\sigma}\sqrt{\pi a}$

$$K(t, \hat{v}=0) = C\sqrt{2\pi c_d t}\,\bar{\sigma}$$

dynamic length scale

$$C = \sqrt{\frac{2(1-2\nu)}{\pi(1-\nu)}}$$

$$G = g(\hat{v})\, K^2(\hat{v}=0)\,\frac{1-\nu}{2\mu} = \Gamma \rightarrow \text{Resistance}$$

if $=$ crack can propagate

$$1-\frac{\hat{v}}{c_R} \simeq g(\hat{v}) = \frac{2\mu\Gamma}{(1-\nu)K^2(\hat{v}=0)} = \frac{\mu\Gamma}{(1-\nu)\pi c_d C^2 \bar{\sigma}^2 t} = \frac{\tau_0}{t}$$

τ_0

$$\tau_0 = \frac{\mu\Gamma}{(1-\nu)\pi c_d C^2}$$

$$\boxed{\tau_0 = \frac{\mu\Gamma c_d}{4(c_s\bar{\sigma})^2} \qquad g(\hat{v}) = 1-\frac{\hat{v}}{c_R} = \frac{\tau_0}{t}}$$

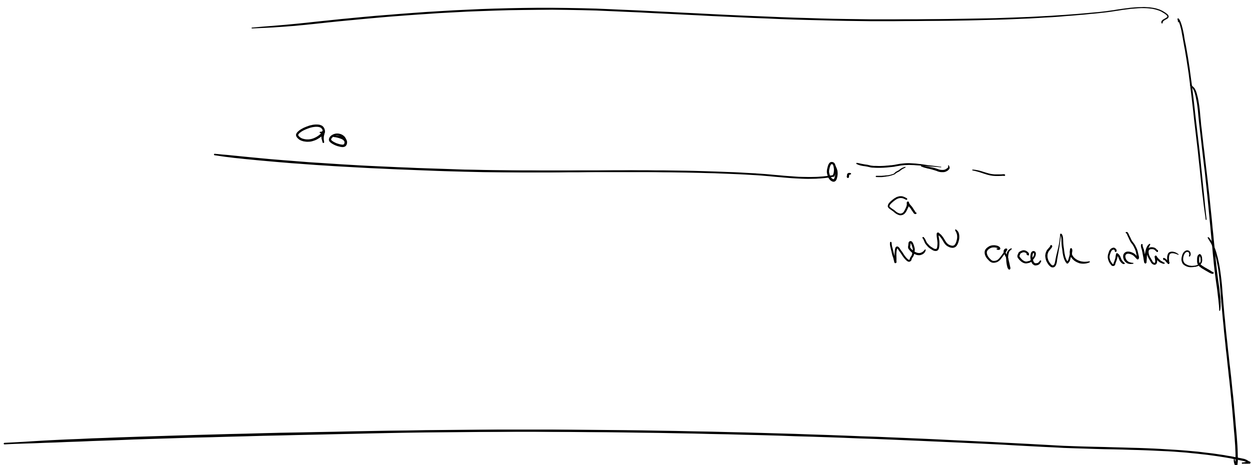
only if crack propagates

at $t=0$ LHS $\hat{v}=0$ $\quad 1$

RHS $\quad \infty$

as $t \nearrow$ RHS $\searrow$ until it reaches 1

From time $t = \tau_0$ the crack can propagate



time scale τ_0

velocity " c_R

length scale $= c_R \tau_0 = l_0$

$$1 - \frac{\hat{V}}{c_R} = \frac{\tau_0}{t} \quad \frac{1}{t'} \qquad \hat{V} = \frac{d(\text{crack length})}{d \text{ time}} = \frac{da}{dt}$$

normalized crack length $= a/l_0$

$$\frac{d\frac{a}{l_0}\, l_0}{d\frac{t}{\tau_0}\, \tau_0} = \boxed{\frac{d\,a'}{d\,t'}\, c_R = \hat{V}}$$

normalized time scale $= \dfrac{t}{\tau_0}$

$$1 - \frac{da'}{dt'} = \frac{1}{t'} \longrightarrow \frac{da'}{dt'} = 1 - \frac{1}{t'}$$

integrate

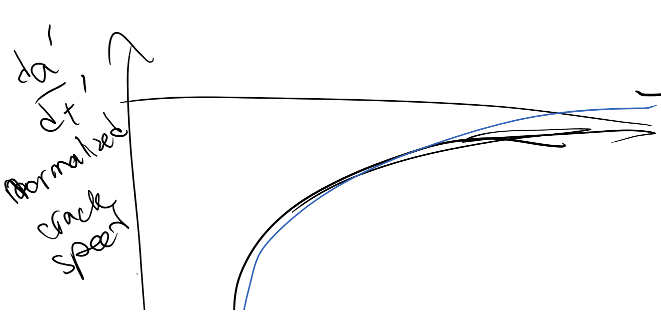
$t' > 1$

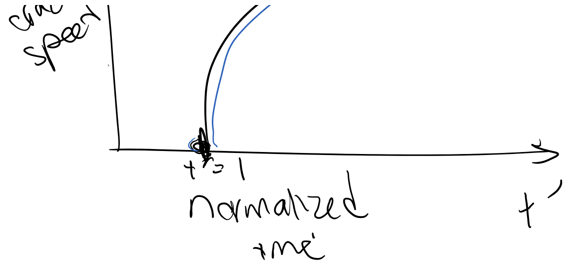
$$a' = t' - 1 - \ln t'$$

$t' < 1$

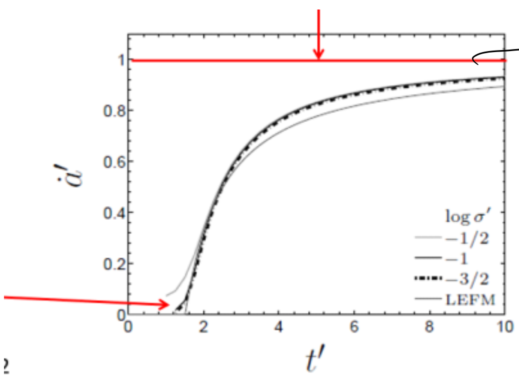
$$\frac{da'}{dt'} = 0$$

$a' = 0$





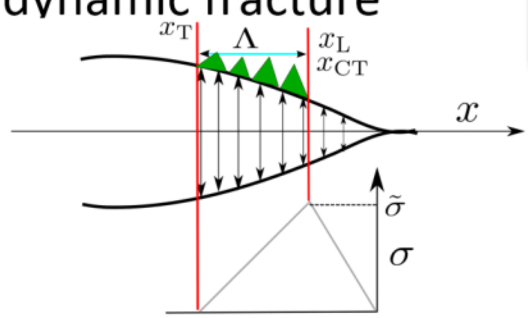
$$\boxed{\begin{aligned}\frac{da}{dt'} &= 0\\ a' &= 0\end{aligned}}$$



Rayleigh wave speed limit

Implication of FPZ size tending to zero in dynamic fracture

Reminder:
Fracture process zone in dynamic fracture



- Importance of process zone size Λ
  - Static estimate:

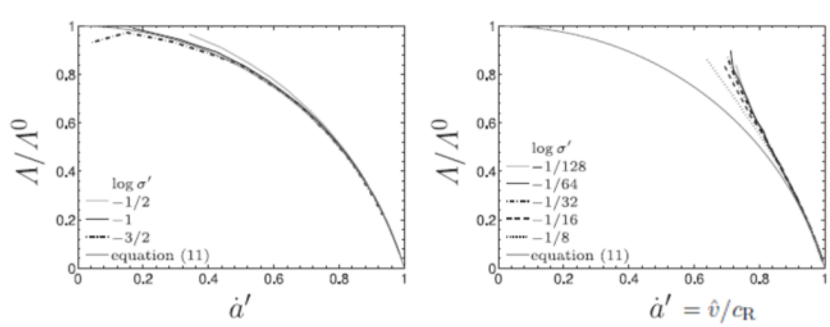
$$\Lambda = \varsigma\pi\frac{\mu}{1-\nu}\frac{\tilde\phi}{\tilde\sigma^2} \propto \tilde L$$

$$\varsigma = \begin{cases}\frac{1}{4} & \text{Dugdale model}\\ \frac{9}{16} & \text{Potential-based TSRs}\end{cases}$$

  - Minimum number of elements in process zone size: There should be at least 4-10 elements along the PZ
  - Dynamic estimate: PZS decreases as crack speed $\hat v$ approaches Rayleigh wave speed c_{R}

$$\Lambda(\hat v) = \frac{\Lambda}{A(\hat v)}, \quad \Lambda(\hat v) \to 0 \text{ as } \hat v \to c_{\mathrm{R}} \quad \Rightarrow$$

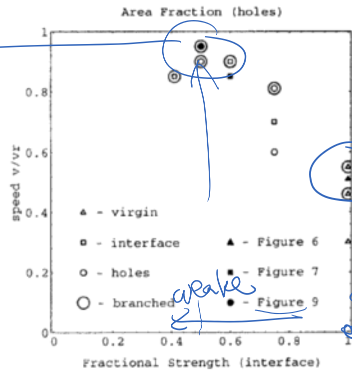
Smaller elements are needed in PZT as crack accelerates!



(a) Low-amplitude loading, $\bar t_\infty \ll \tilde\sigma$. (b) High-amplitude loading, $\bar t_\infty \to \tilde\sigma^-$.

- For a homogeneous solid, crack speed cannot exceed Rayleigh wave speed (c_R).
  - In practice, speed often does not exceed 50% of c_R.
  - Experiment with two weakly joined identical solids, where the weak interface confines the crack to the plane, an interfacial crack indeed approaches c_R.

gets close to theoretical limit



Washabaugh, P. D. & Knauss, W. G. 1994 A reconciliation of dynamic crack velocity and Rayleigh wave speed in isotropic brittle solids. Int. J. Fracture 65, 97{114

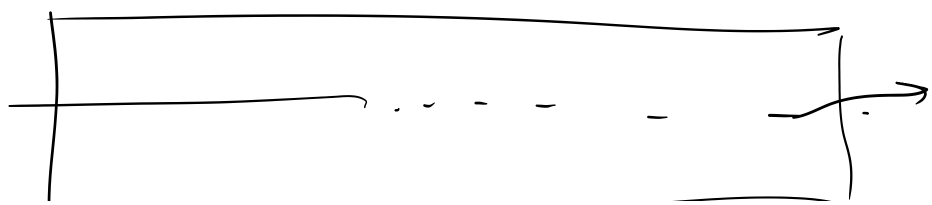
50% CR

fully bonded

weak

interface strength

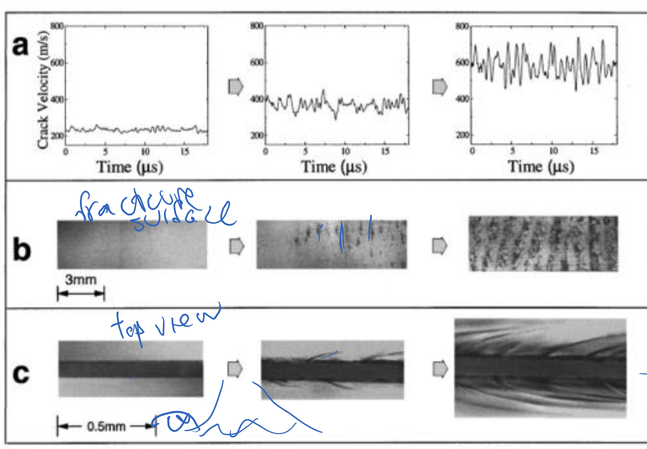
Fig. 8. Dependence of the crack velocity on the property of the material (i.e. fractional strength or area fraction) preceding the crack front.



weak plane

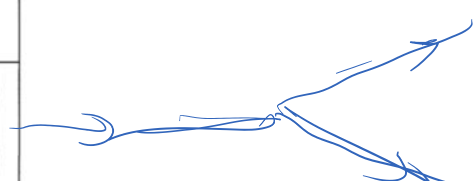
Do cracks do really reach Rayleigh speed?

Sharon Fineberg:
mirror, mist, hackle patterns as the crack accelerates



$v_C = 340$m/s

$\boxed{(\text{or } 0.36V_R)}$



V<Vc V⩾Vc V>Vc

- Crack starts oscillating well before reaching Rayleigh wave speed V_R (c_R)
- Crack speed does not reach V_R (c_R)!
- For this material critical speed $v_c = 0.36\ V_R$

Eran Sharon & Jay Fineberg

$$\Gamma = G(l)A(v) \approx G(l)\left(1 - \frac{v}{v_R}\right) \qquad (1)$$

$$v(l) = v_R\left(1 - \frac{\Gamma}{G(l)}\right)$$

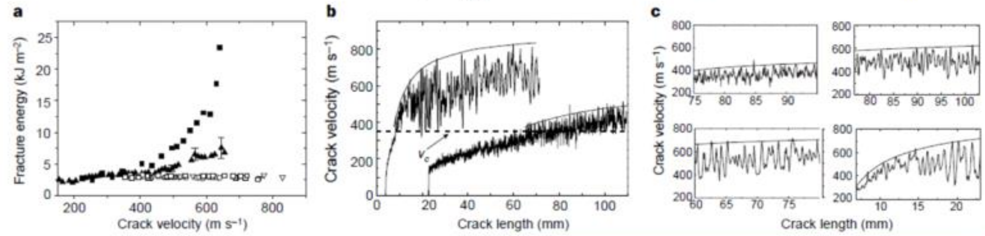


Figure 3 A comparison of theory with measurements in PMMA, for $v > v_c$. **a**, The total measured[14] fracture energy, $\Gamma(v)$, (filled squares) compared with derived values, obtained using equation (1) with the velocity measurements shown in Fig. 2a as input. The derived values, $\Gamma_d(v)$, (filled triangles) coincide with $\Gamma(v)$ for $v < 400\,\mathrm{m\,s^{-1}}$ ($1.17v_c$), and diverge for $v > 400\,\mathrm{m\,s^{-1}}$. This divergence occurs simultaneously with increased 'scatter' in $\Gamma_d(v)$, indicating that Γ_d is no longer a well-defined function of v. Thus, due to micro-branch formation, the single-crack assumption necessary for equation (1) is invalid when the average velocity is used. As a crack possesses no inertia, instantaneous single-crack states that correspond to the highest velocity peaks beyond v_c are still described by equation (1). The derived fracture energy (open symbols), using the peak velocities presented in **b** and **c**, indeed collapses to the well-defined value of $3{,}000\,\mathrm{J\,m^{-2}}$. This value equals $0.9\Gamma(v_c)$, the same as the value of Γ (per unit fracture surface) obtained in ref. 14 for $v > v_c$. **b**, **c**, Full (**b**) and close-up (**c**) measurements of the instantaneous velocities (thin lines) corresponding to Fig. 2a, compared to the single-crack predictions of equation (2) (heavy lines) using $\Gamma = 3{,}000\,\mathrm{J\,m^{-2}}$. With no adjustable parameters, the velocity peaks agree well with the theoretical curve at v of up to 90% of the asymptotic crack speed, v_R.

Crack propagation in homogeneous medium

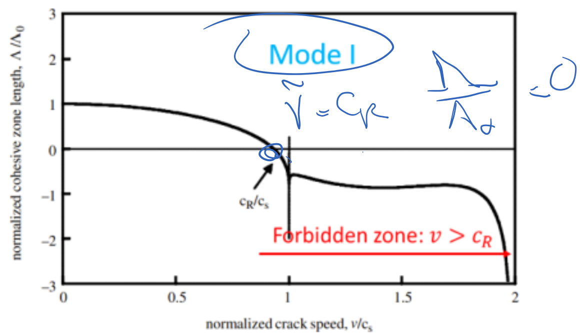


Figure 5. Cohesive zone length as a function of the crack speed for weakly joined identical solids (opening mode).

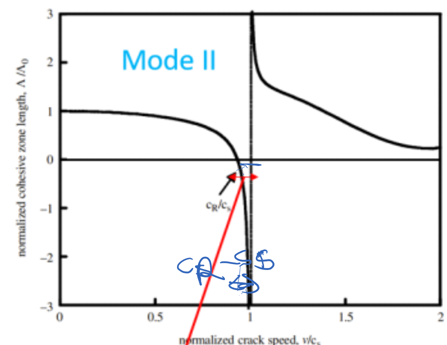
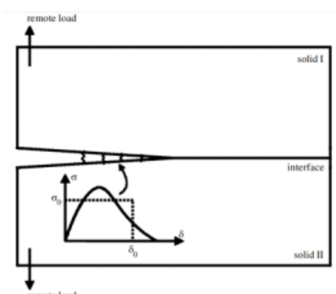


Figure 6. Cohesive zone length as a function of the crack speed for weakly joined identical solids (shear mode).

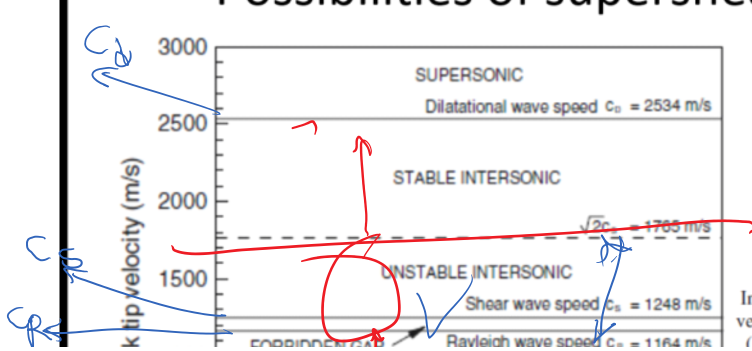
Forbidden zone: $c_R < v < c_s$

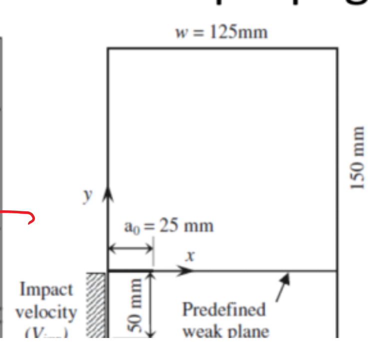


Yu, H.H., Suo, Z., 2000b. Intersonic crack growth on an interface. Proceedings of the Royal Society of London, Series A (Mathematical, Physical and Engineering Sciences) 456, 223–46.

So, for mode II there is the super shear possibility

Possibilities of supershear crack propagation





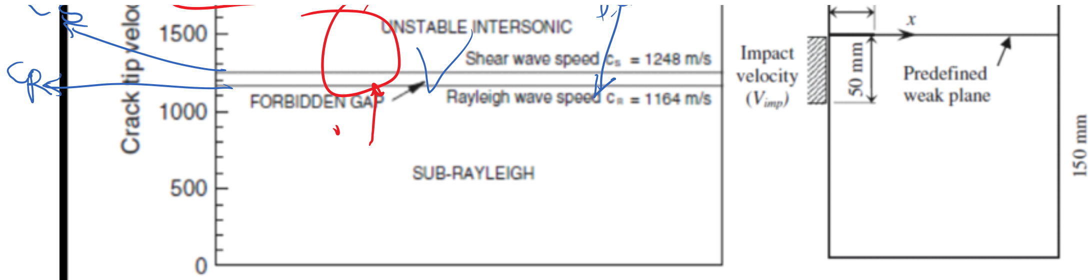


Fig. 5 Representative plot showing wave speeds for Homalite-100 and nomenclature of various regions

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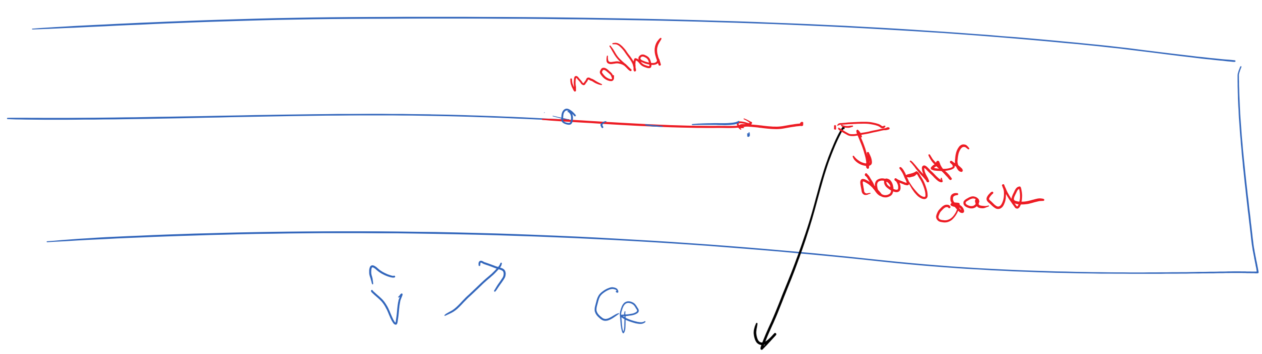
ORIGINAL PAPER

Simulation of dynamic crack growth using the generalized interpolation material point (GIMP) method

Nitin P. Daphalapurkar · Hongbing Lu ·
Demir Coker · Ranga Komanduri



by joing daughter & mother cracks crack can go super shear