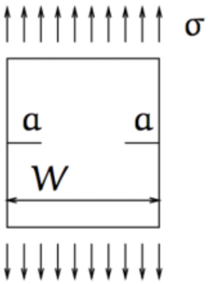
FM20240930

Monday, September 30, 2024 9:42 AM

Double Edge Notch Tensi $\underline{a \ll W}$ $K_I \approx 1.12\sigma\sqrt{\pi a}$



$$K_I = \sigma\sqrt{a}\left[1.12\sqrt{\pi} + 0.76\frac{a}{W} - 8.48\left(\frac{a}{W}\right)^2 + 27.36\left(\frac{a}{W}\right)^3\right]$$

$$\approx 1.12\sigma\sqrt{\pi a} \quad \text{as} \quad \frac{a}{W} \to 0$$

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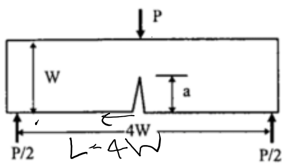
9. Single-edge notch bend (SENB), thickness B



$L = 4W$

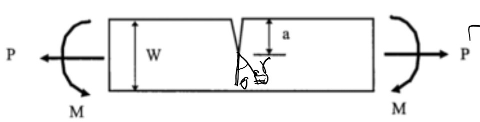
$B = W/2$

$$K_I = Y\frac{4P\sqrt{\pi}}{B\sqrt{W}}$$

$$Y = 1.63\left(\frac{a}{W}\right)^{1/2} - 2.6\left(\frac{a}{W}\right)^{3/2} + 12.3\left(\frac{a}{W}\right)^{5/2} - 21.3\left(\frac{a}{W}\right)^{7/2} + 21.9\left(\frac{a}{W}\right)^{9/2}$$

Since LEFM is a linear theory, we can do superposition

Superposition method

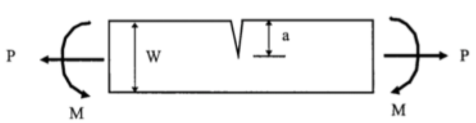


→ tensile

$$\sigma_{ij}(r,\theta) = \sigma_{ij}^{tensile}(r,\theta) + \sigma_{ij}^{bending}(r,\theta) = K_I^{tensile}\frac{1}{\sqrt{2\pi r}}f_{ij}^{I}(\theta) + K_I^{bending}\frac{1}{\sqrt{2\pi r}}f_{ij}^{I}(\theta) \Rightarrow$$

$$\left.\begin{aligned}\sigma_{ij}(r,\theta) &= \left(K_I^{tensile} + K_I^{bending}\right)\frac{1}{\sqrt{2\pi r}}f_{ij}^{I}(\theta) \\ &= K_I^{equi.}\frac{1}{\sqrt{2\pi r}}f_{ij}^{I}(\theta)\end{aligned}\right\} \Rightarrow \boxed{K_I = K_I^{tensile} + K_I^{bending}}$$

we just add K's of different loadings

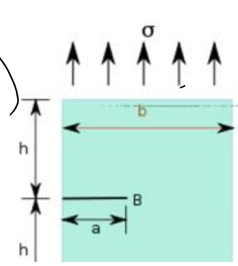


$\frac{a}{W} = .2$

$$K_I^{tensile} = \sigma\sqrt{\pi a}\left(1.12 - .23(.2) + \ldots\right)$$

$$K_I^{tensile} \approx \sigma\sqrt{\pi a} \times 1.12\left(\frac{P}{BW}\right) \rightsquigarrow \sigma_{tensile}$$

$K_I^{bending}$ ... $(6M)$ $\sqrt{\pi}$ ... .055

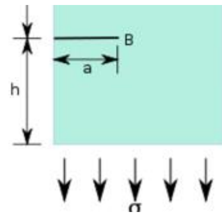


0.4	1.257
0.5	1.500
0.6	1.915

$h/b \geq 1$ and $a/b \leq 0.6$

$$K_I \simeq \sigma\sqrt{\pi a}\times 1.12 (BW)$$

$$K_I^{bending} \simeq \underbrace{\left(\frac{6M}{BW}\right)}_{\sigma_{bending}} \sqrt{\pi a} \times 1.055$$



$h/b \geq 1$ and $a/b \leq 0.6$

$$K_I = \sigma\sqrt{\pi a}\left[1.12 - 0.23\left(\frac{a}{b}\right) + 10.6\left(\frac{a}{b}\right)^2 - 21.7\left(\frac{a}{b}\right)^3 + 30.4\left(\frac{a}{b}\right)^4\right]$$

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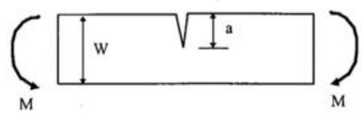
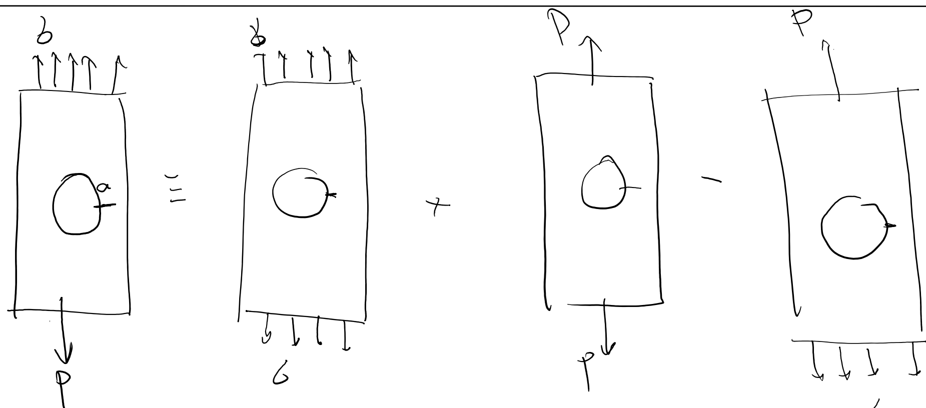
5. Edge crack in a beam of width B subjected to bending



$$K_I = f\left(\frac{a}{W}\right)\sigma\sqrt{\pi a} \text{ where } \sigma = \frac{6M}{BW^2}$$

a/W	$f(a/W)$
0.1	1.044
0.2	1.055
0.3	1.125

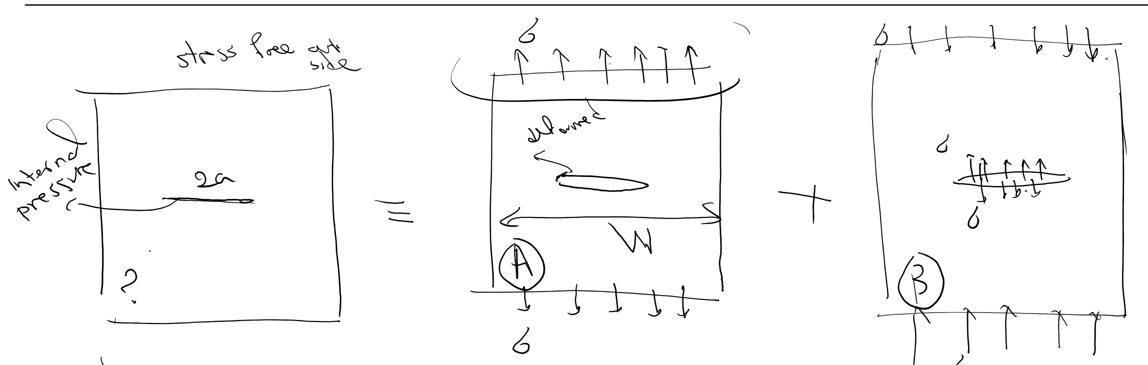
$$K_I \simeq \sqrt{\pi a}\left(1.12\left(\frac{P}{BW}\right) + 1.055\left(\frac{6M}{BW}\right)\right)$$



$$K_I^A = \underbrace{K_I^\sigma + K_I^P}_{\text{we'll get these from SIF table.}} - K_I^B$$

because of sym. $K_I^B = K_I^A \Rightarrow$

$$\boxed{K_I^A = \frac{K_I^\sigma + K_I^P}{2}}$$

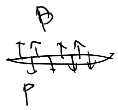


deformed shape

$\sigma_{xx} = 0$

$\sigma_{xy} = 0$

$\sigma_{yy} \to \sigma$



$\sigma_{xx} = 0$

$\sigma_{yy} = 0$

$\sigma_{yy} = -\sigma$

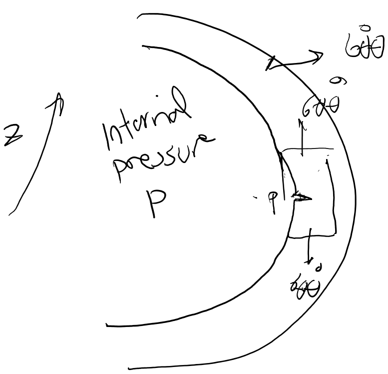
$K_I^B = 0$

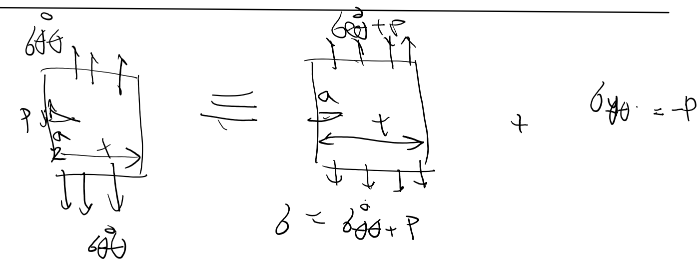
Examples of internal pressure:
Pressure vessel, hydraulic fracturing, porous media

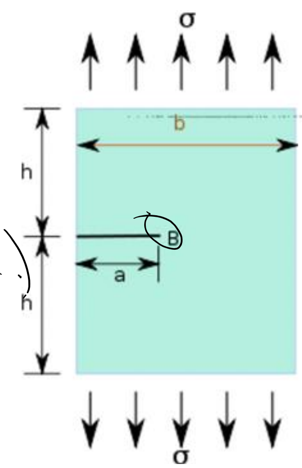
$$K_I = K_I^A + \cancel{K_I^B}^{\,0}$$

$$\boxed{K_I = K_I^B = f\left(\frac{a}{W}, \dots\right)\sigma\sqrt{\pi a}}$$

We generally can move internal pressure to far field load in calculating SIF







0.4	1.257
0.5	1.500
0.6	1.915

$h/b \geq 1$ and $a/b \leq 0.6$

$$K_1 = \sigma\sqrt{\pi a}\left[1.12 - 0.23\left(\frac{a}{b}\right) + 10.6\left(\frac{a}{b}\right)^2 - 21.7\left(\frac{a}{b}\right)^3 + 30.4\left(\frac{a}{b}\right)^4\right]$$

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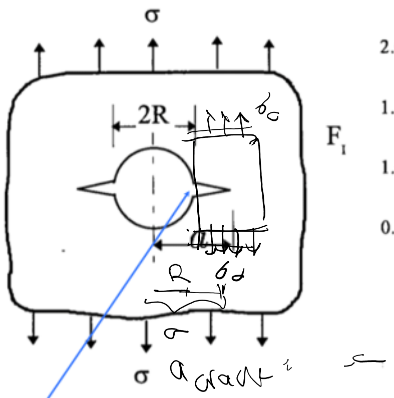
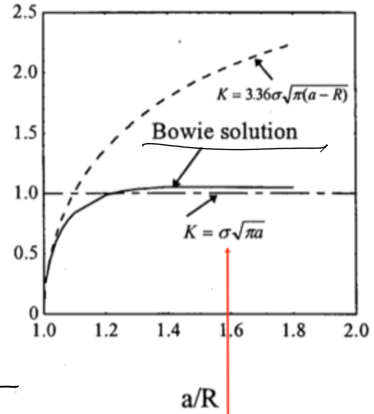
$$K = (\sigma_{\theta\theta}^0 + p)\sqrt{a\pi}\,(1.12 + \dots)$$

$$\simeq 1.12\left(p\frac{R}{t} + p\right)$$

Two small cracks at a hole





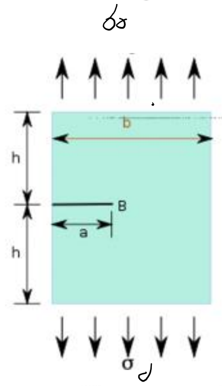
$\boxed{3\sigma}$ edge crack

hole as a part of the crack

$K_I \approx 1.12\, \sigma \sqrt{\pi a_{crack}}$

3σ

$K_I \approx 3.36 \sqrt{\pi c}$



0.5	1.500
0.6	1.915

$h/b \geq 1$ and $a/b \leq 0.6$

$$K_I = \sigma\sqrt{\pi a}\left[1.12 - 0.23\left(\frac{a}{b}\right) + 10.6\left(\frac{a}{b}\right)^2 - 21.7\left(\frac{a}{b}\right)^3 + 30.4\left(\frac{a}{b}\right)^4\right]$$

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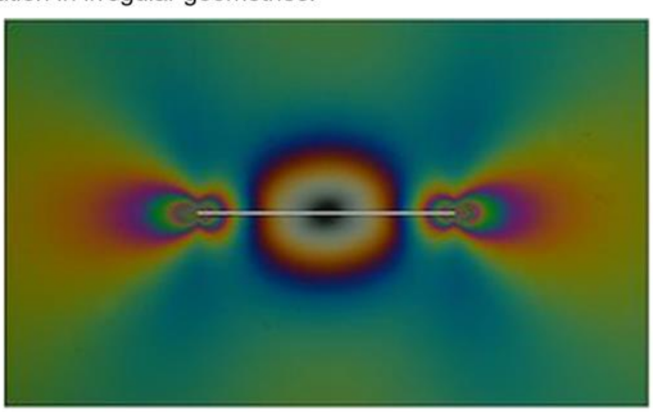
Photoelasticity

Wikipedia

Photoelasticity is an experimental method to determine the stress distribution in a material. The method is mostly used in cases where mathematical methods become quite cumbersome. Unlike the analytical methods of stress determination, photoelasticity gives a fairly accurate picture of stress distribution, even around abrupt discontinuities in a material. The method is an important tool for determining critical stress points in a material, and is used for determining stress concentration in irregular geometries.



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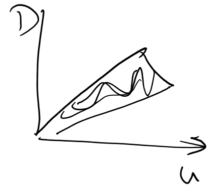
K-G relationship

G



$G = \dfrac{\text{shaded area}}{\Delta a\, B}$ $\qquad$ $G = \dfrac{P^2}{2B}\dfrac{dC}{da}$

$$G = \frac{K_I^2 + K_{II}^2}{E} + \frac{K_{III}^2}{2\mu} \qquad \text{(today)}$$

G = Energy release rate (how much energy is released per unit area of crack creation)

G is a **global energy** measure

$$G = -\frac{d\Pi}{dA}$$

potential energy

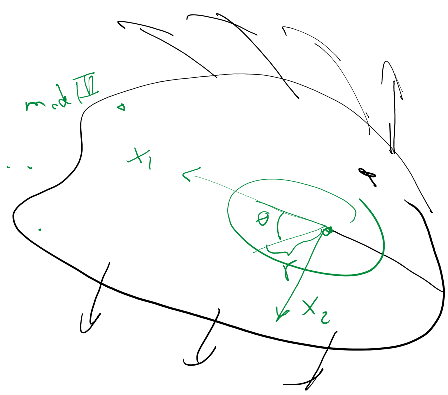
$\Pi = U - W^{ext}$

internal

crack area

K = Stress Intensity Factor

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f^I_{ij}(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} f^{II}_{ij}(\theta) \ldots$$



K is a **local stress** (strain, displacement) measure

How are the units of energy release rate (G) and SIF (K) related.

$$[G] = \frac{[E]}{[A]} = \frac{[F][L]}{[L]^2}$$

energy, area, force

$$\boxed{\begin{aligned}[G] &= [\sigma][L] \quad \text{e.g. MPa m} \\ &= [F]/[L] \quad \frac{N}{m}\end{aligned}}$$

$[K] = ?$

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} \underbrace{f^I_{ij}(\theta)}_{\text{function of } \theta,\ \text{dimensionless}} \Rightarrow [\sigma] = \frac{[K]}{[L^{1/2}]}$$

$[K^2] = [\sigma]^2[L]$

$[G] = [\sigma][L]$

$$\boxed{[K] = [\sigma][L]^{1/2} \quad \text{Pa}\sqrt{m} \quad \text{MPa}\sqrt{m}}$$

$[K^2] = [\sigma]^2[L]$

$[G] = [\sigma][L]$

$$\boxed{[K] = [\sigma][L]^{1/2} \quad \text{i.e.} \quad Pa\sqrt{m} \quad MPa\sqrt{m}}$$

$\frac{[G]}{[K]^2} = \frac{1}{[\sigma]} \rightarrow$ of strength
Young's modulus E

$$\boxed{G \propto \frac{K^2}{E}}$$

dimensional analysis observation

K-G relationship

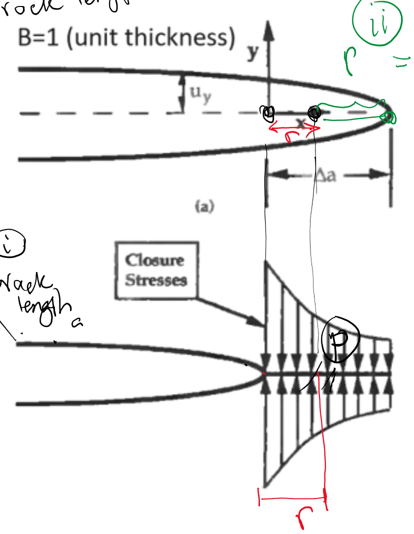
stage (ii)

$\sigma_{yy}^{P\,(ii)} = 0$

$u_y^{(ii)} = \frac{K^{(ii)}}{2\mu}\sqrt{\frac{r^{(ii)}}{2\pi}}\,(\kappa+1)$ from (A)

stage (ii)
crack length $a+\Delta a$

$r^{(ii)} = \Delta a - r$



Stage (i)
before crack propagation

$\sigma_{yy}^{P\,(i)} = \frac{K_I^{(i)}}{\sqrt{2\pi r}}$

$u_y^{(i)} = 0$
vertical displacement

stage (i)
crack length a

Displacement field (A)

$$u_x = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(\kappa - 1 + 2\sin^2\frac{\theta}{2}\right)$$

$$u_y = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(\kappa + 1 - 2\cos^2\frac{\theta}{2}\right)$$

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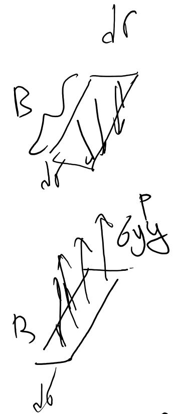
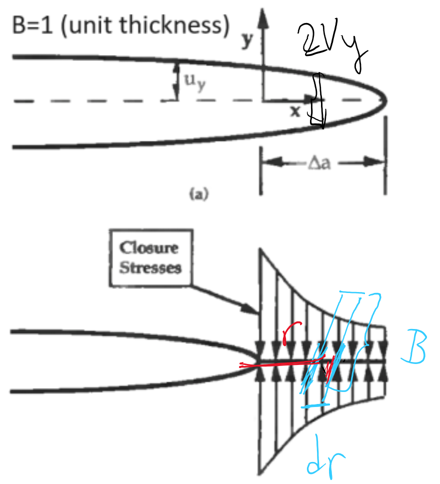
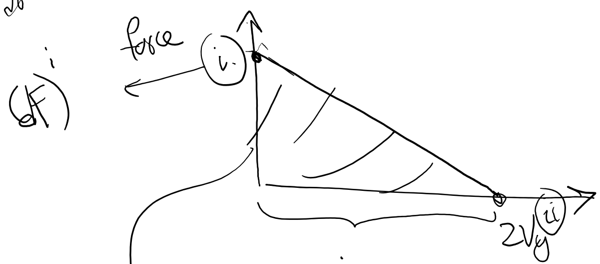


$$dF = \underbrace{dA}_{\text{surface area}}\, \sigma_{yy}^{P'}$$

$$= B\,dr\,\sigma_{yy}^{B}$$

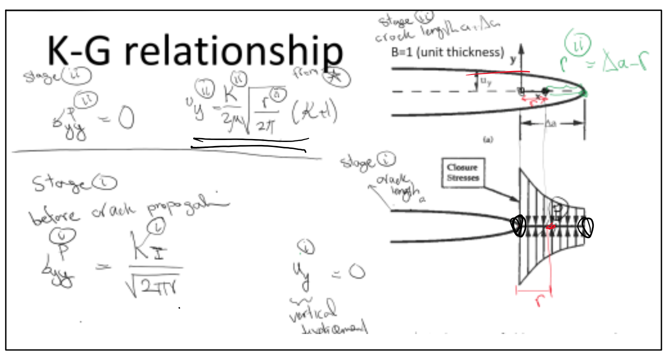




$$\underbrace{dW}_{\text{work}} = \frac{(dF)^{i}\,(2V_y^{\,ii})}{2} = (dF)^{i}\,V_y = \sigma_{yy}^{(i)}\,u_y^{(ii)}\,B\,dr$$

work by opening the crack

$$dW = \sigma_{yy}^{(i)}\,\underline{u_y}^{(ii)}\,B\,dr$$



$$dW = \frac{K_I^{(i)}}{\sqrt{2\pi r}}\,\frac{K_I^{(ii)}}{2\mu}\sqrt{\frac{\Delta a - r}{2\pi}}\,(\kappa+1) \times B\,dr$$

$$G = \frac{W}{B\Delta a} = \frac{\int dW}{B\Delta a} = \int_{r=0}^{\Delta a} \frac{K_I(a)\,K_{II}(a+\Delta a)}{\sqrt{2\pi r}\;2\mu}\sqrt{\frac{\Delta a - r}{2\pi}}\,(\kappa+1)\,dr$$

$$\boxed{G = \frac{(\kappa+1)\,K_I^2}{8\mu}}$$

shear $\mu = \dfrac{E}{2(1+\nu)}$

shear modulus $\mu = \frac{E}{2(1+\nu)}$

κ Kolosov coefficient $\kappa = \begin{cases} 3-4\nu & \text{plane strain} \\ \dfrac{3-\nu}{1+\nu} & \text{plane stress} \end{cases}$

K-G relationship (cont.)

Mode I

$$\boxed{G_I = \begin{cases} \dfrac{K_I^2}{E} & \text{plane stress} \\ (1-v^2)\dfrac{K_I^2}{E} & \text{plane strain} \end{cases}}$$

Mixed mode

$$\boxed{G = \frac{K_I^2}{E'} + \frac{K_{II}^2}{E'} + \frac{K_{III}^2}{2\mu}} \qquad E' = \begin{cases} \dfrac{E}{1-\nu^2} & \text{for plane strain} \\ E & \text{for plane stress} \end{cases}$$

- Equivalence of the strain energy release rate and SIF approach
- Mixed mode: G is scalar => mode contributions are additive
- Assumption: self-similar crack growth!!!

Self-similar crack growth: planar crack remains planar (da same direction as a)

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