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K-G relationship (cont.)

Mode I

$$G_I = \begin{cases} \dfrac{K_I^2}{E} & \text{plane stress} \\ (1-v^2)\dfrac{K_I^2}{E} & \text{plane strain} \end{cases}$$

Mixed mode

$$G = \frac{K_I^2}{E'} + \frac{K_{II}^2}{E'} + \frac{K_{III}^2}{2\mu}$$

$$E' = \begin{cases} \dfrac{E}{1-\nu^2} & \text{for plane strain} \\ E & \text{for plane stress} \end{cases}$$

- Equivalence of the strain energy release rate and SIF approach
- Mixed mode: G is scalar => mode contributions are additive
- Assumption: self-similar crack growth!!!

Self-similar crack growth: planar crack remains planar (da same direction as a)

Key use: by having K's we can determine if the crack grows or not.

Imagine we just have mode I fracture:

$G = \dfrac{K_I^2}{E'}$ does the crack grow?

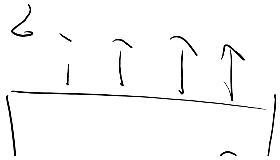
$G \geq R$ crack grows

$\dfrac{K_I^2}{E'} = R \rightarrow$ Pa-m

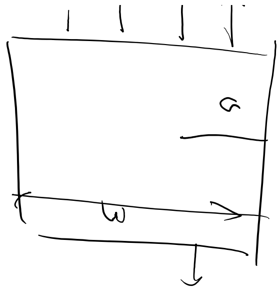
$$\boxed{\begin{array}{l} K_I = \sqrt{RE'} = K_{Ic} \\ K_{Ic}: \text{critical stress intensity factor (toughness)} \quad \text{Pa}\sqrt{\text{m}} \end{array}}$$

crack growth potential

σ



$K = f(a,w)\,\sigma\sqrt{\pi a} = K_{Ic}$ crack

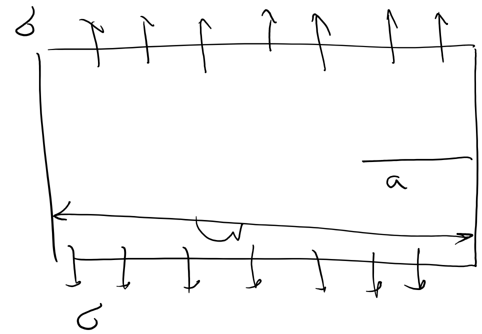


$K_I = f(a/w)\,\sigma\sqrt{\pi a} = K_{IC}$ crack can grow

Similarly for mode II $\dfrac{K_{II}^2}{E'} = R$ $K_{II} = \sqrt{RE'} = K_{II_C} = K_{I_C}$

from this perspective

Sample problems



$$K_I = f\left(\frac{a}{w}\right)\sigma\sqrt{\pi a} = K_{IC}$$

- a
- σ
- K_{IC}

3 kinds of problem (2 given , find the 3rd):

(i) a given . material (K_{IC}) given $\Longrightarrow$ Find σ_{max}

eg measurement resolution in the absence of observable crack

$$\sigma_{max} = \frac{K_{IC}}{\sqrt{\pi a}\, f(a/w)}$$

(ii) σ given, material (K_{IC}) give $\Longrightarrow$ Find a_{max}

$$f\left(\frac{a_{max}}{w}\right)\sqrt{\pi a} = \frac{K_{IC}}{\sigma}$$

in general this is a non linear equation

(iii) σ given , a given $\Longrightarrow$ Find the appropriate material

$$K_I = \left(f\left(\frac{a}{w}\right)\sqrt{\pi a}\right)\sigma$$

Not very common

$$K_{I_c} = \left(f\left(\frac{a}{W}\right)\sqrt{\pi a}\right)\sigma$$

material
Not very common

Side note related to last time

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Dynamic fragmentation of brittle solids: a multi-scale model

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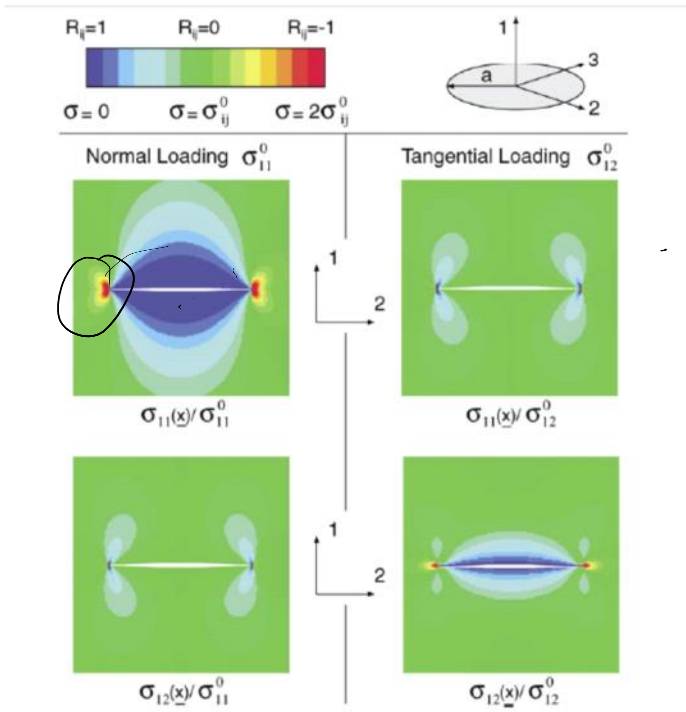

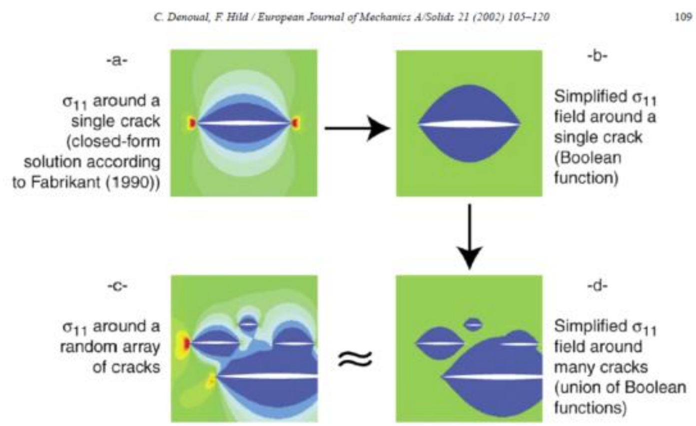

Plastic Fracture Mechanics (PFM)
- When can we use LEFM

5.2. Plastic zone models

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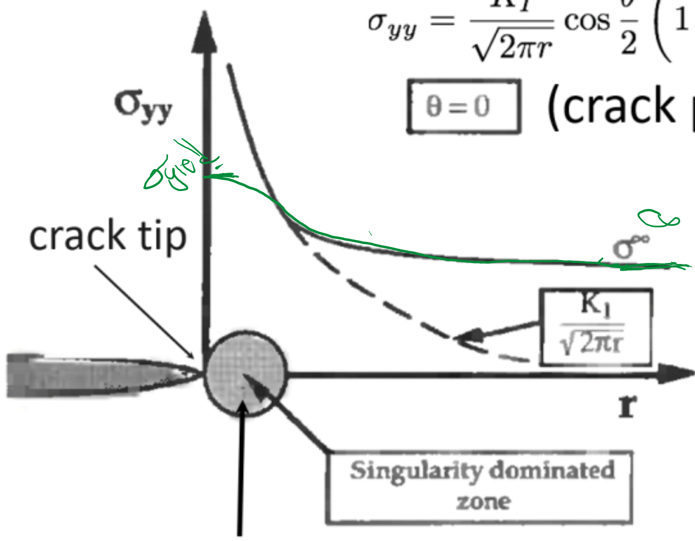
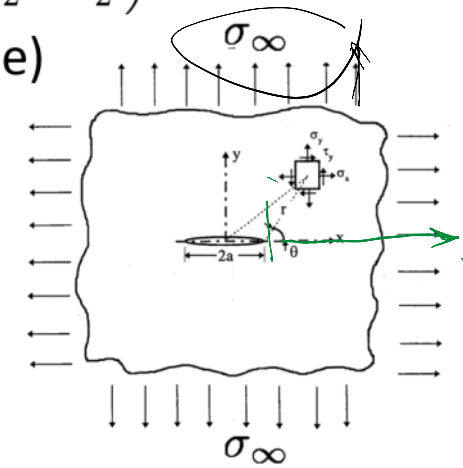
- 1D Models: Irwin, Dugdale, and Barenbolt models

Singular dominated zone

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$$

$\boxed{\theta = 0}$ (crack plane)



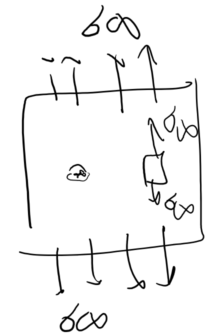


$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1 - \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1 + \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)$$

K-dominated zone



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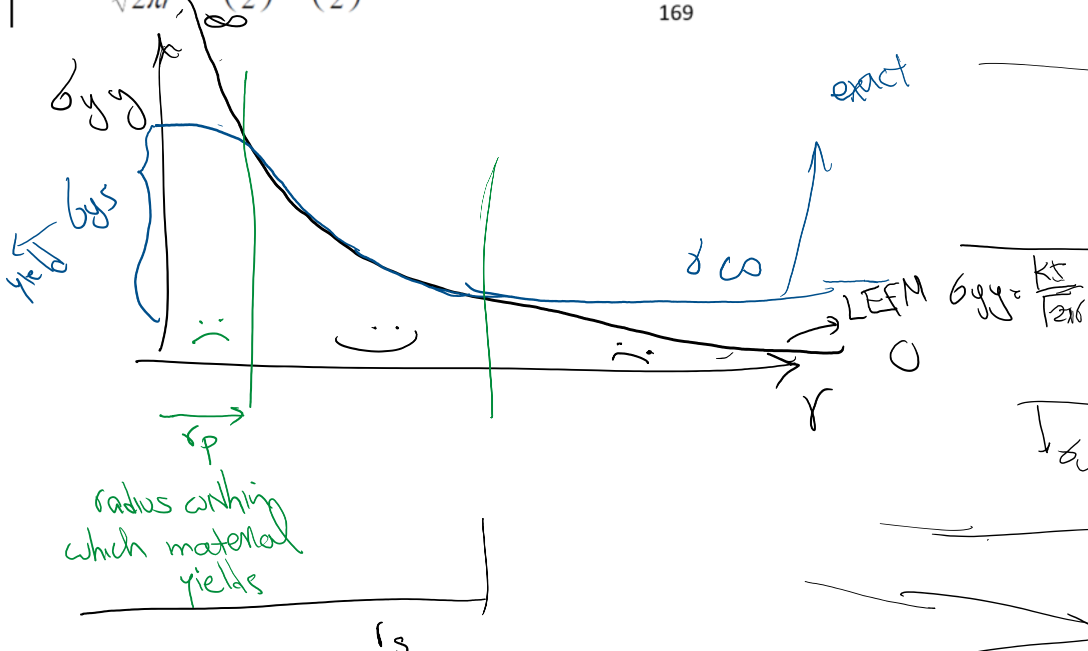
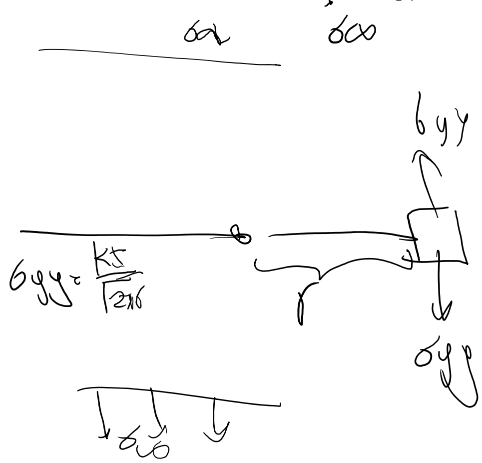
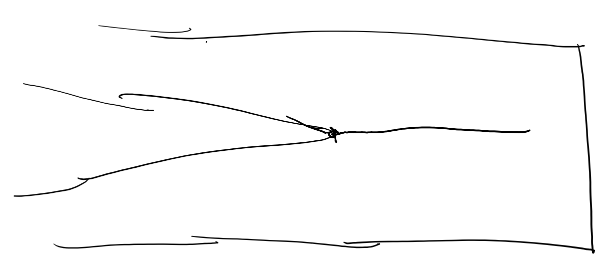


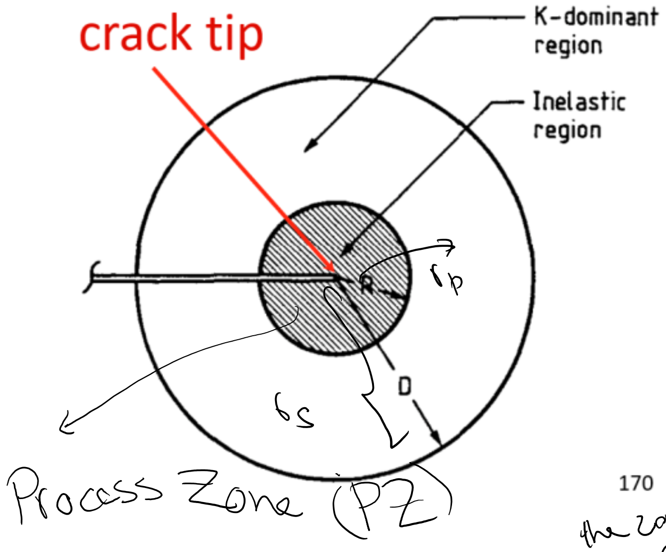


singular dominant zone



LEFM is a good model when $r_s >> r_p$

LEFM is a good model when ... $\gg r_p$



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(SSY)

Small-scale yielding: LEFM still applies with minor modifications done by G. R. Irwin

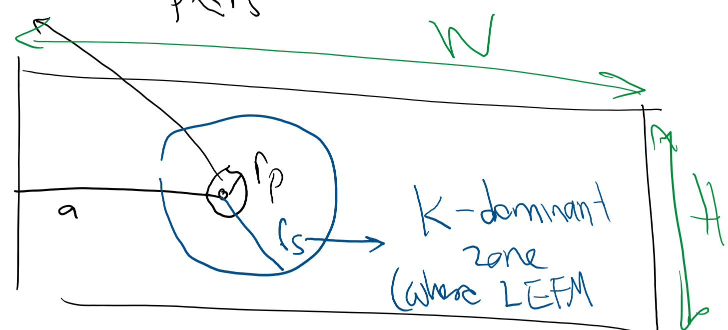
$$\boxed{R \ll D}$$

process zone size (PZS)

$r_p \ll r_s$

SSY

the zone that we have significant non linear response (e.g. plasticity)



K-dominant zone (where LEFM is close to the exact solution)

SSY: $r_p \ll r_s, a, W, H, \ldots$

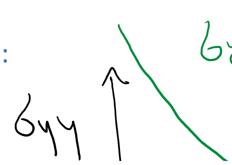
SSY $\longrightarrow$ we can use LEFM

$r_p = ?$

$r_s = ?$

today

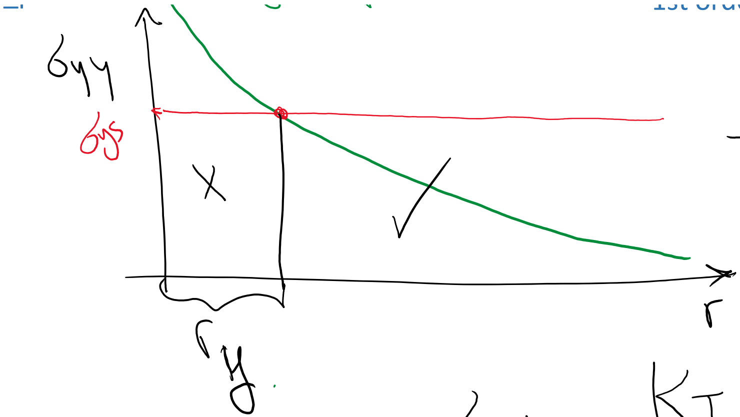
Find r_p:

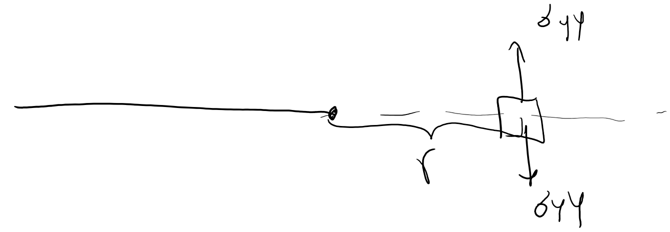


$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$

Plastic correction:
1st order approximation

σ_{yy}

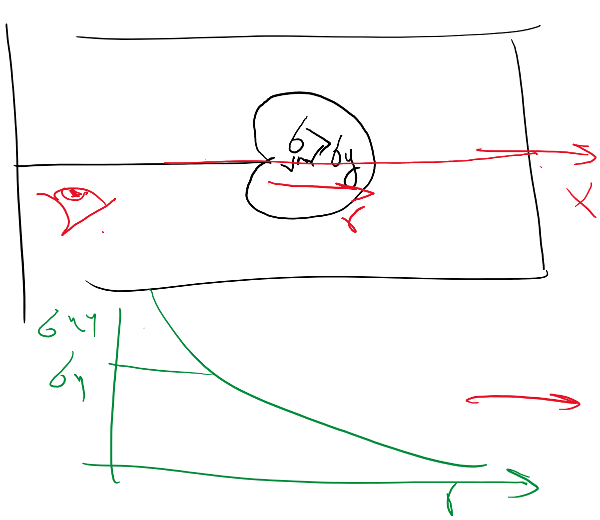


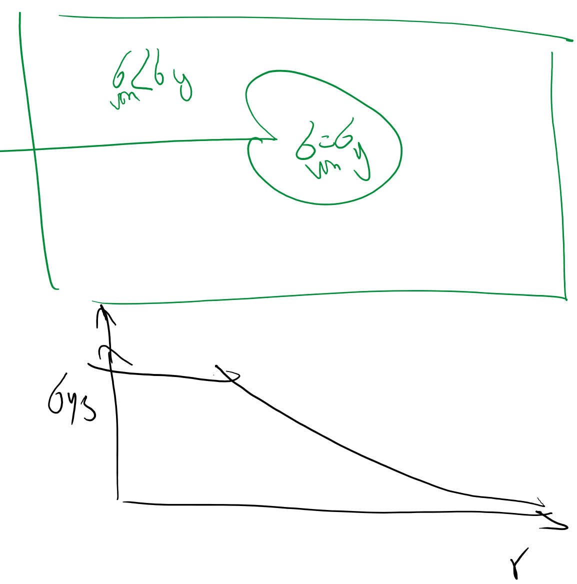


$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$ and @ r_y : $\sigma_{yy} = \sigma_{ys}$

$\sigma_{ys} = \frac{K_I}{\sqrt{2\pi r_y}} \Longrightarrow$ (1) $\boxed{r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2}$

$K_I : [\sigma][L]^{1/2}$
$\sigma_{ys} = [\sigma]$





Constitutive equation

X breaks down

Hook's law $\sigma_{ij} = D_{ijkl}\varepsilon_{kl} \qquad i,j,k,l = 1,2,3$

Isotropic

3D $T_{ij} = \lambda\delta_{ij}E_{kk} + 2\mu E_{ij} \qquad \text{or} \qquad \mathbf{T} = \lambda\mathbf{I}_E + 2\mu\mathbf{E}$

2D (plane strain)

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & 0 \\ \nu & (1-\nu) & 0 \\ \nu & \nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}$$

2D (plane stress)

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \frac{1}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}$$

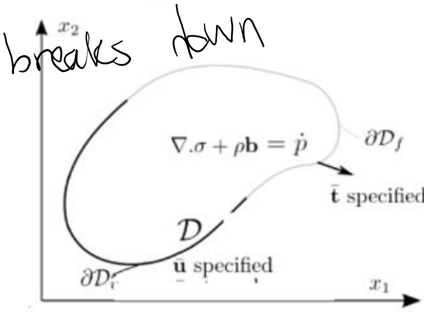
2D (plane stress) $\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{Bmatrix} = \frac{1}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}$

$$p = \rho \mathbf{v}$$

Balance of linear momentum → breaks down

$$\nabla.\sigma + \rho \mathbf{b} = \dot{p} \quad \text{or} \quad \sigma_{ij,j} + \rho b_j = \rho \ddot{u}_i$$

- Static: $\dot{\mathbf{p}} = 0$
- No body force $\mathbf{b} = 0$



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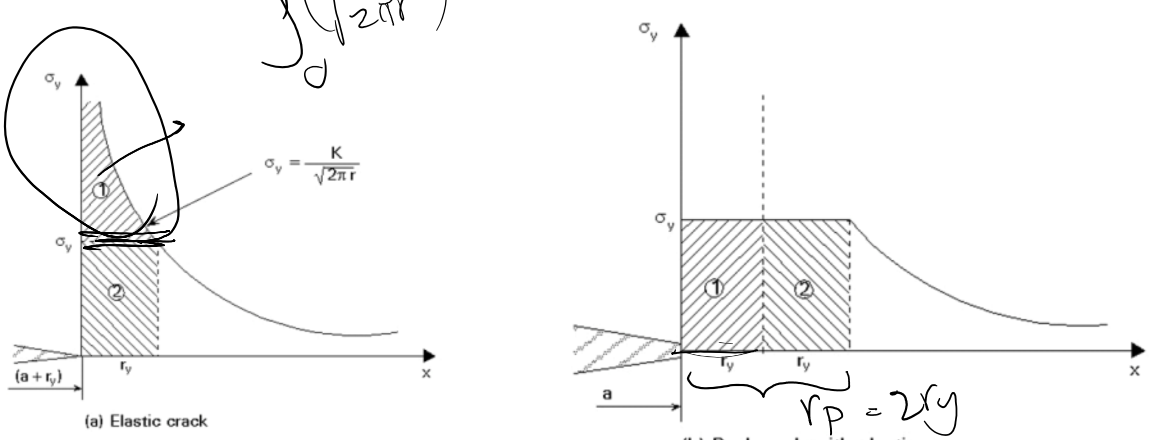
To get the actual Process zone size (PZS) = r_y or r_p, we need to solve the problem (BC + PDE + constitutive equations) from the beginning to find the correct solution. Where around the crack the material yields or undergoes large inelastic deformation is called the PZS.

Rather than doing this difficult problem, we can do a bit better than the 1st order approximation as shown below:

2.Irwin's plastic correction

$$\int_0^{r_y} \sigma_{yy} \left(\frac{K_I}{\sqrt{2\pi r}}\right) dr$$



(a) Elastic crack

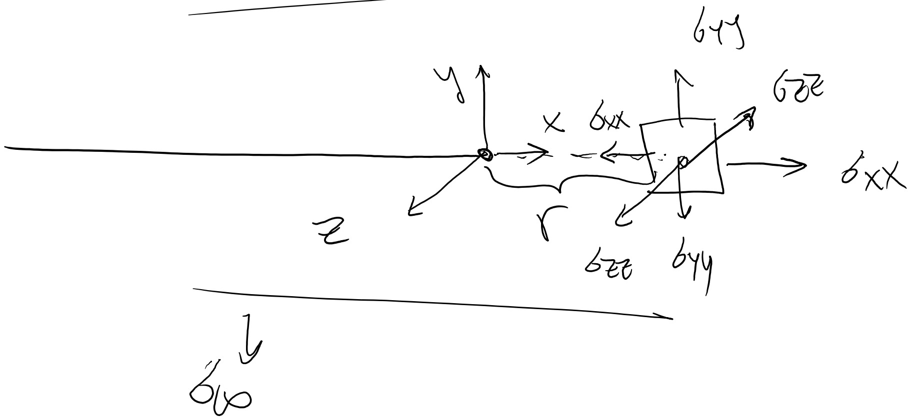
(b) Real crack with plastic zone

$$\boxed{r_p = 2r_y = \frac{1}{\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2}$$

This is better than r_y, but still is not doing any stress redistribution.

$\sigma_{ys} = ?$ how is it related to σ_Y : yield strength

σ_∞



$\sigma_{xy} = 0$ (mode I)

$\left\{ \begin{array}{l} \sigma_{xz} = 0 \\ \sigma_{yz} = 0 \end{array} \right.$ we're solving an in-plane problem

$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ are principal stresses

In 2D & 3D a common stress measure to evaluate plasticity is the von Mises stress

$$\sigma_{vm} = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}} \le \sigma_Y$$

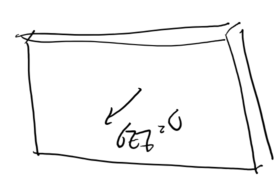
yield stress

$\sigma_{vm} = \sigma_Y$ material is on the yield surface

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$$

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}}$$

$$\sigma_{zz} \left\{ \begin{array}{ll} = 0 & \text{p. stress} \\ \neq 0 & \text{p. strain} \end{array} \right.$$



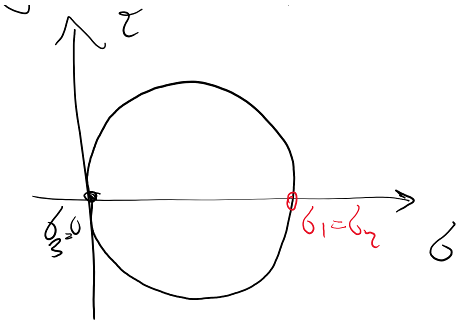
p. stress $\quad \sigma_1 = \sigma_2 = \dfrac{K_I}{\sqrt{2\pi r}} \quad \sigma_3 = 0$

Mohr circle

P. stress $\sigma_1 = \sigma_2 = \frac{K}{\sqrt{2\pi r}}$ $\sigma_3 = 0$

$$\sigma_{vm} = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}} = \sigma_1$$

$$= \sigma_{yy} = \sigma_Y$$



3-a $\boxed{\text{P. stress yielding condition is } \sigma_{yy} = \sigma_Y}$

p. strain $\sigma_1 = \sigma_2 = \sigma_{xx} = \sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$

$\sigma_{zz} \neq 0$ but we know $\epsilon_{zz} = 0$ but

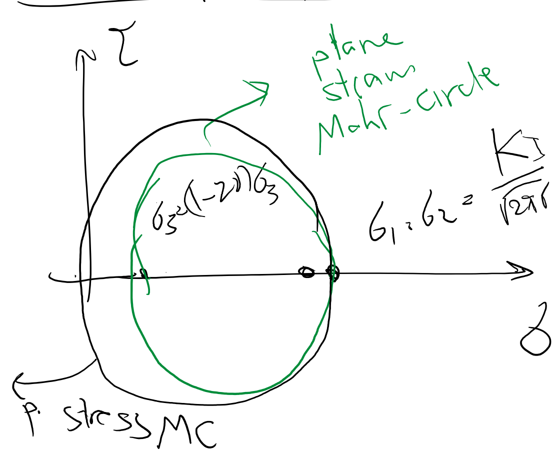
$$\epsilon_{zz} = \frac{1}{E}(\sigma_{zz} - \nu\sigma_{xx} - \nu\sigma_{yy}) = 0 \rightarrow \sigma_{zz} - 2\nu\sigma_{yy} = 0 \rightarrow \sigma_{zz} = 2\nu\sigma_{xx}$$

because $\sigma_{xx} = \sigma_{yy}$

$$\sigma_{vm} = \sqrt{\frac{(\underbrace{\sigma_1 - \sigma_2}_{0})^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}{2}}$$

$$= \sqrt{\frac{(\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{yy} - \sigma_{zz})^2}{2}} =$$

$\sigma_2 = \sigma_{xx} = \sigma_{yy}$

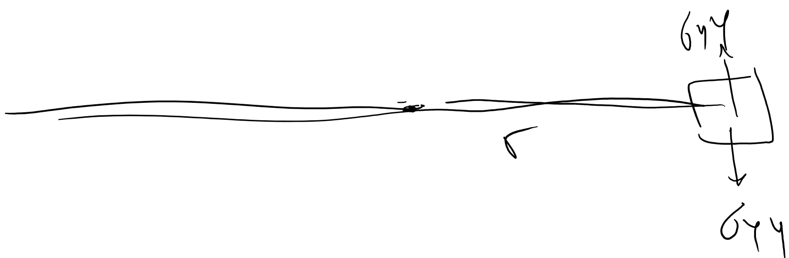


$$\sigma_{vm} = \sigma_{yy} - \sigma_{zz} = (1-2\nu)\sigma_{yy} = \sigma_Y$$

(3b) $\boxed{\text{P. strain} \quad \sigma_{yy} = \frac{\sigma_Y}{1-2\nu} \text{ Correspons to yielding}}$

$\sigma_y \qquad \dfrac{\qquad}{1-2\nu}$ ... to yielding

3-a $\boxed{\begin{array}{l}\text{p. stress} \\ \text{yielding condition is } \sigma_{yy} = \sigma_Y\end{array}}$



$\sigma_{yy} = \sigma_{ys}$ yielding condition

where $\sigma_{ys} = \begin{cases} \sigma_Y & \text{p. stress} \\ \dfrac{\sigma_Y}{1-2\nu} & \text{p. strain} \end{cases}$

$$r_p = \frac{1}{\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2 = \begin{cases} \dfrac{1}{\pi}\left(\dfrac{K_I}{\sigma_Y}\right)^2 & \text{p. strain} \\ \dfrac{(1-2\nu)^2}{\pi}\left(\dfrac{K_I}{\sigma_Y}\right)^2 & \text{p. stress} \end{cases}$$

p. stress

$\overline{\text{e.g}}\ \nu = .2 \qquad r_p \approx \begin{cases} \dfrac{1}{\pi}\left(\dfrac{K_I}{\sigma_Y}\right)^2 & \text{p. stress} \\ \dfrac{1}{3\pi}\left(\dfrac{K_I}{\sigma_Y}\right)^2 & \text{p. strain} \end{cases}$