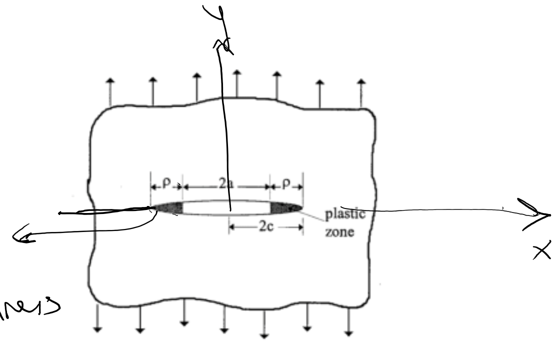
FM20241009

Wednesday, October 9, 2024 9:51 AM

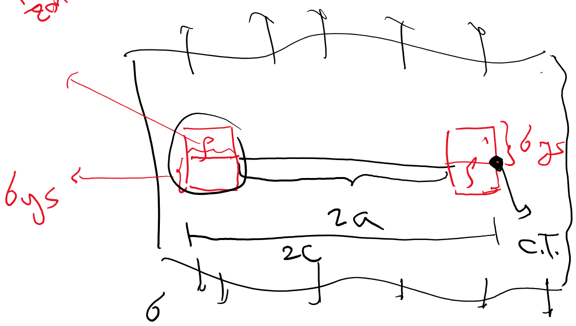
3. Strip Yield Model

proposed by Dugdale and Barrenblatt



* we only limit stress by σ_{ys} on $y=0$ (crack line)
* we do not consider plasticity in the bulk

plastic zone size (similar to r_y & r_p)



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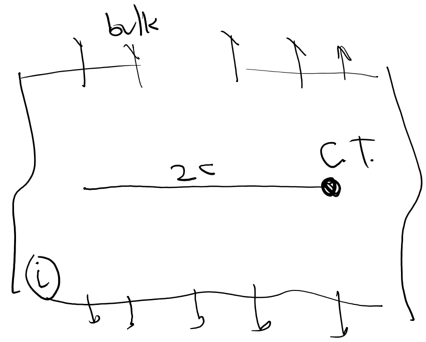

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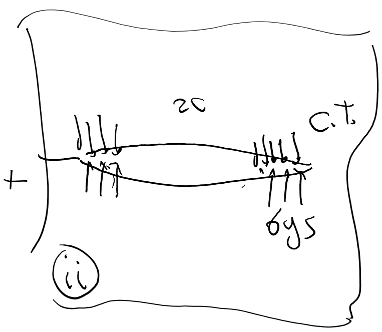


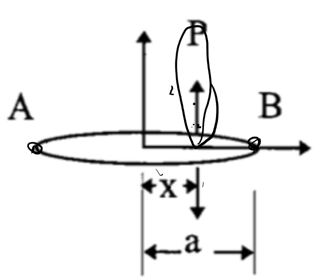
$c = a + \rho$

$$K = \begin{cases} \sigma\sqrt{\pi a} \; \times \\ 0 \; \checkmark \end{cases}$$

because $\sigma = \sigma_{ys}$ @ C.T. (finite)

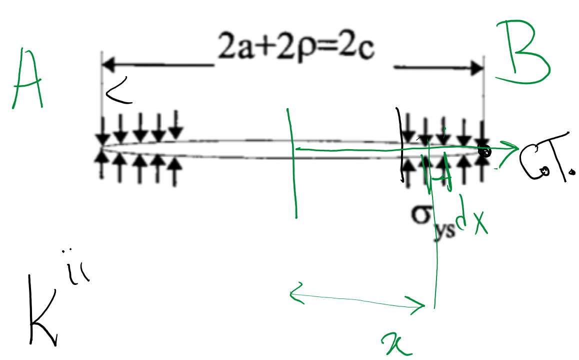
$$K^{i} = \sigma\sqrt{\pi c} \qquad K^{ii} = ?$$

$$K = K^{i} + K^{ii} = 0 \qquad K^{ii} = -K^{i}$$



$$K_A = \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a+x}{a-x}}$$

$$K_B = \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a-x}{a+x}}$$



$dP = \sigma_{ys}\, dx$

K^{ii}

$$K^{ii} = \int_a^c \frac{-\sigma_{ys}\,dx}{\sqrt{\pi c}}\sqrt{\frac{c-x}{c+x}}\,dx + \int_a^c \frac{-\sigma_{ys}\,dx}{\sqrt{\pi c}}\sqrt{\frac{c+x}{c-x}}$$

$K_B = \frac{\sigma}{\sqrt{\pi a}}\sqrt{\frac{a+x}{a-x}}$

Anderson, p64

$\int_a \sqrt{\pi c} \quad \underbrace{\sqrt{c+x}}_{\text{for the forces on the right}} \qquad \int_a \sqrt{\pi c}$

$$K^{ii} = -2\sigma_{ys}\sqrt{\frac{a+\rho}{\pi}}\cos^{-1}\left(\frac{a}{a+\rho}\right)$$

$$K^{i} = \sigma\sqrt{\pi c}$$

$K^{i} + K^{ii} = 0 \Rightarrow$

$$2\sigma_{ys}\sqrt{\frac{a+\rho}{\pi}}\cos^{-1}\left(\frac{a}{a+\rho}\right) = \sigma\sqrt{\pi(a+\rho)}$$

c

$$\boxed{\cos\Big(\underbrace{\frac{\pi\sigma}{2\sigma_{ys}}}_{\varepsilon}\Big) = \frac{a}{a+\rho} \qquad \rho = ?}$$

for field stress

if SSY holds (as we'll see today $\Rightarrow \frac{\sigma}{\sigma_{ys}} \ll 1$)

$\cos\varepsilon \simeq 1 - \frac{\varepsilon^2}{2}$ where $\varepsilon = \frac{\pi\sigma}{2\sigma_{ys}}$

$\varepsilon \ll 1$

we'll obtain

$$\rho = \frac{\pi^2\sigma^2 a}{8\sigma_{ys}^2} = \frac{\pi}{8}\left(\frac{K_I}{\sigma_{ys}}\right)^2$$

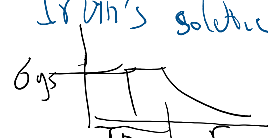
0.392

Strip yield model

$$r_p = \left(\frac{1}{\pi}\right)\left(\frac{K_I}{\sigma_{ys}}\right)^2$$

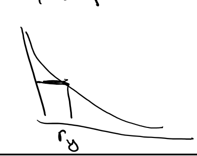
.318

Irwin's solution

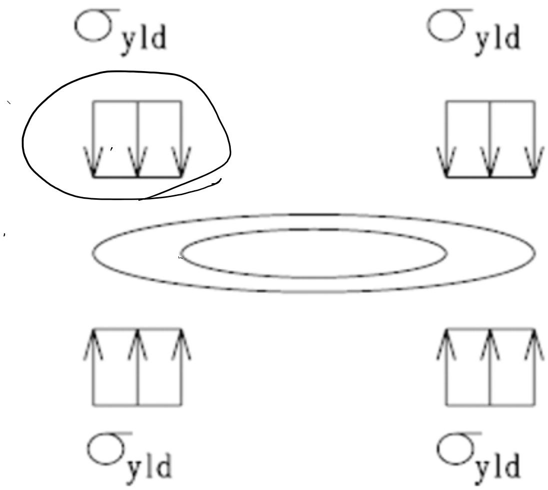


$$r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2$$

.159



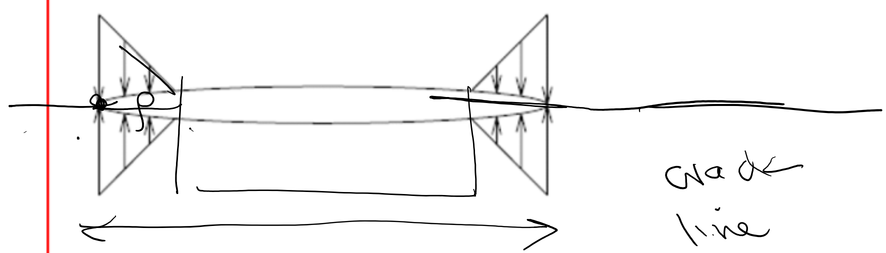
Dugdale: Uniform stress



More appropriate for polymers

Barenblatt: Linear stress

bulk doesn't



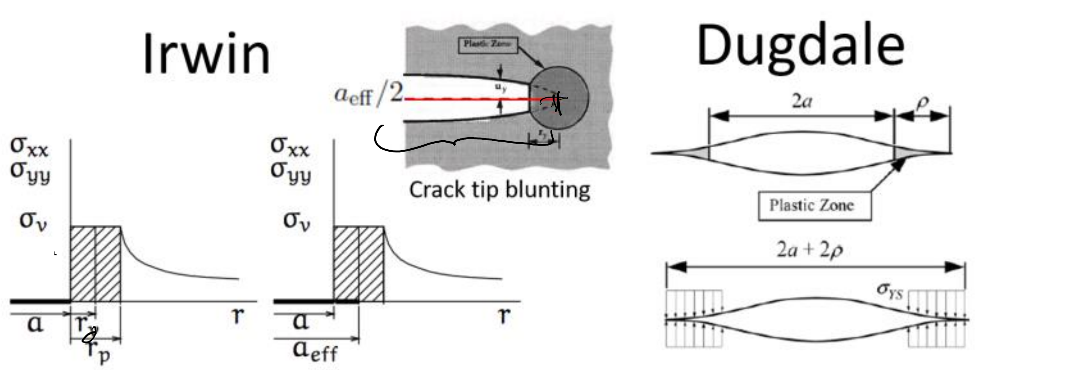
crack line

More appropriate for metals

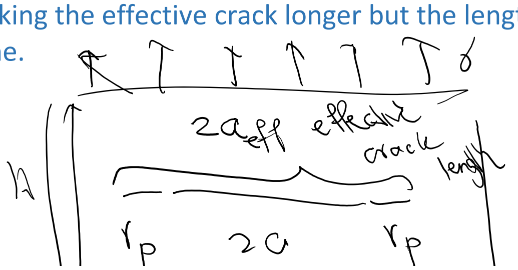
can have nonlinear response

We'll see that when SSY condition is satisfied we can use LEFM
Still we want to push the boundaries a bit and continue using LEFM by doing some "tricks" until LEFM completely breaks down.

Effective crack length

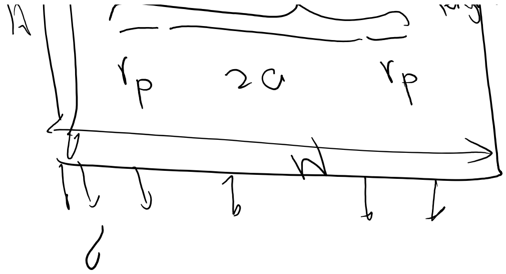


Key idea:
As LEFM starts to break down, we can still use it a bit longer but making the effective crack longer but the length of process zone.



This is an iterative solution process in general:

1) $K_{eff} = f(a_{eff}, W, H, \dots)\sqrt{\pi a_{eff}}\ \sigma$



1) $K_{eff} = f(a_{eff}, W, H, \dots)\sqrt{\pi a_{eff}}\ \sigma$

2) $a_{eff} \simeq a + r_p \simeq a + \frac{1}{\pi}\left(\frac{K_{eff}}{\sigma_{ys}}\right)^2$

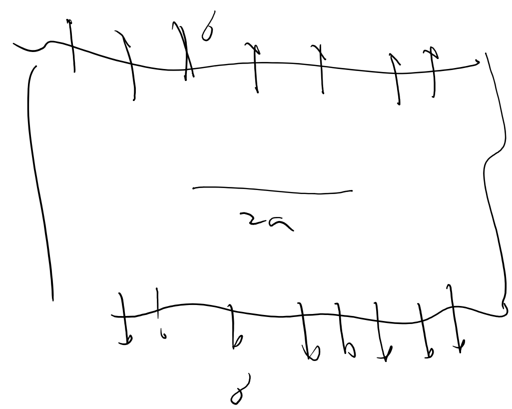
or r_y / or ρ

Iterative Solution:

- choose $a_{eff} = a$ in 1) $\Rightarrow K_{eff}$
- in 2) get a_{eff}

Repeat unless $\left| a_{eff}^{(n)} - a_{eff}^{(n-1)} \right| < \varepsilon$

Example: Effective crack length for an infinite domain:



1) $K_{eff} = \sqrt{\pi a_{eff}}\ \sigma$

2) $a_{eff} = a + r_y = a + \frac{1}{2\pi}\left(\frac{K_{eff}}{\sigma_{ys}}\right)^2$

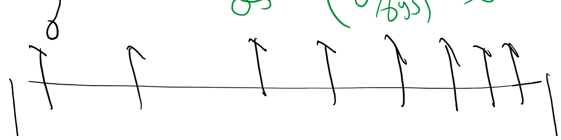
r_y rather r_p is used

$$a_{eff} = a + \frac{1}{2\pi}\left(\frac{\sqrt{\pi a_{eff}}\ \sigma}{\sigma_{ys}}\right)^2 \Rightarrow a_{eff} \Rightarrow \text{in 1 get } K_{eff}$$

$$K_{eff} = \frac{\sqrt{\pi a}\ \sigma}{\sqrt{1 - \frac{1}{2}\left(\frac{\sigma}{\sigma_{ys}}\right)^2}}$$

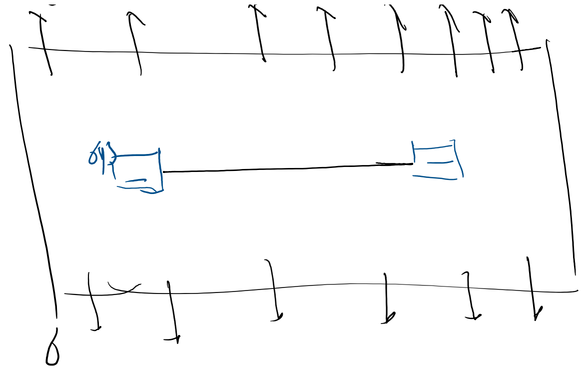
→ K we would have gotten for crack length a

correction → 0 as $(\sigma/\sigma_{ys})^2 \to 0$



$$\left(\frac{\sigma}{\sigma_{ys}}\right)^2 \ll 1 \Rightarrow$$

$K_{eff} \approx K$



$\frac{\sigma}{\sigma_{ys}} = 0.1 \qquad \left(\frac{\sigma}{\sigma_{ys}}\right)^2 = 0.01$

$\frac{\sigma}{\sigma_{ys}} = \frac{1}{\sqrt{10}} = .316 \qquad \frac{\sigma}{\sigma_{ys}} = 0.1$

$\left(\frac{\sigma}{\sigma_{ys}}\right)^2 \ll 1$	$\left(\frac{\sigma}{\sigma_{ys}}\right)^2 \lesssim 1$	$\left(\frac{\sigma}{\sigma_{ys}}\right)^2 \sim 1$ O(1)
LEFM ✓	getting close to 1 can use effective crack length	PFM

Consider a large central cracked plate subjected to a uniform stress of 130 MPa. The fracture toughness K_c=50MPa√m, the yield strength σ_{ys}=420MPa.

(a) What is the maximum allowable crack length? LEFM — no effective crack length

(b) What is the maximum crack length if plastic correction is taken into account. Plane stress and ~~Irwin~~'s correction. 1st order

$K_c = 50\ \text{MPa}\sqrt{m}$

$\sigma_{ys} = 420\ \text{MPa}$

(a) $K = \sigma\sqrt{\pi a^{max}_{LEFM}} = K_c$

$$a^{max}_{LEFM} = \frac{1}{\pi}\left(\frac{K_c}{\sigma}\right)^2 = \frac{1}{\pi}\left(\frac{50\ \text{MPa}\sqrt{m}}{130\ \text{MPa}}\right)^2$$

$$a^{max}_{LEFM} = \frac{1}{\pi}\left(\frac{K_c}{\sigma}\right) = \frac{1}{\pi}\left(\frac{50\,MPa\sqrt{m}}{130\,MPa}\right)^2$$

$$2a = 94.2e{-3}$$

$$\boxed{2a^{max}_{LEFM} = 94.2\text{ mm}}$$

(b)

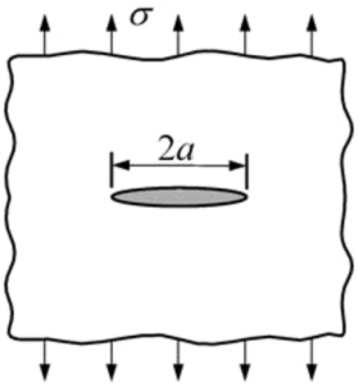
$$K = K_{eff} = \frac{\sigma\sqrt{\pi a^{max}_{eff}}}{\sqrt{1 - \frac{1}{2}\left(\frac{\sigma}{\sigma_{ys}}\right)^2}} = \frac{50\sqrt{\pi a^{max}_{eff}}}{\sqrt{1 - \frac{1}{2}\left(\frac{130}{420}\right)^2}} = K_c = 50\ MPa\sqrt{m}$$

$$\boxed{2a^{max}_{eff} = 89.7\text{ mm}}$$

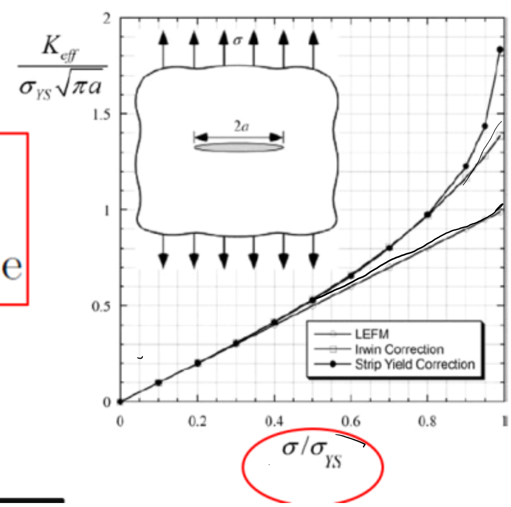
$$\left(\frac{\sigma}{\sigma_{ys}}\right)^2 = \left(\frac{130}{420}\right)^2 \sim 0.1$$

LEFM is accurate enough

Effective crack length



$$K_{eff} = \frac{\sigma\sqrt{\pi a}}{\sqrt{1-\frac{1}{2}\left(\frac{\sigma}{\sigma_{YS}}\right)^2}}$$



$$\boxed{\text{As } \frac{\sigma}{\sigma_{ys}} \text{ increases} \Rightarrow \text{LEFM becomes less accurate}}$$

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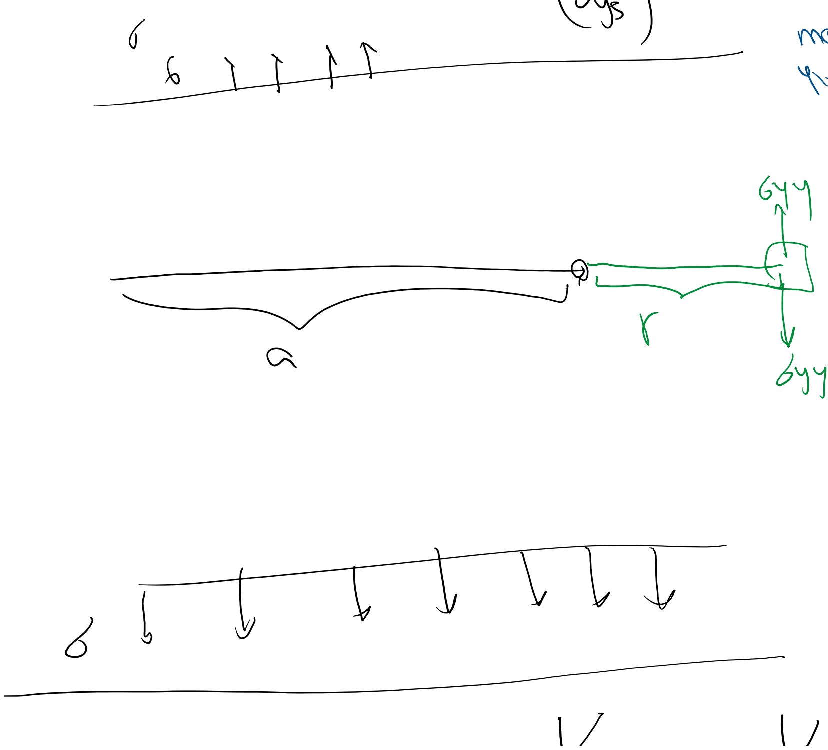
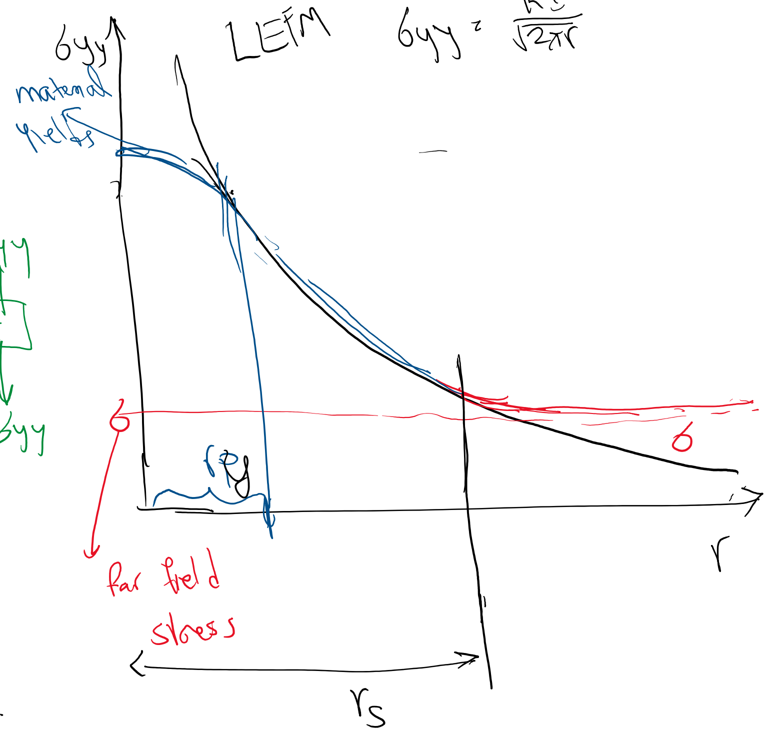
Relation of effective crack length and $\left(\frac{\sigma}{\sigma_{ys}}\right)^2$ to LEFM

LEFM $\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$





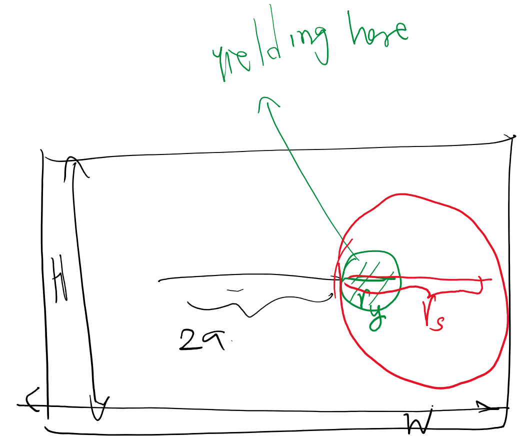
's

$$\sigma_{yy}(r_s) = \sigma = \frac{K_I}{\sqrt{2\pi r_s}}$$

K-dominant zone or singular dominant zone

for field stress

$$\boxed{\begin{aligned} r_s &= \frac{1}{2\pi}\left(\frac{K_I}{\sigma}\right)^2 \\ \text{we had } r_y &= \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2 \end{aligned}}$$



SSY $\quad r_y \ll \underbrace{\text{all other lengths}}_{H, W, a, r_y}$ including r_s

$$r_y \ll r_s \longrightarrow \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2 \ll \frac{1}{2\pi}\left(\frac{K_I}{\sigma}\right)^2$$

$$\Longrightarrow \boxed{\left(\frac{\sigma}{\sigma_{ys}}\right)^2 \ll 1}$$