FM20241014

Monday, October 14, 2024 9:35 AM

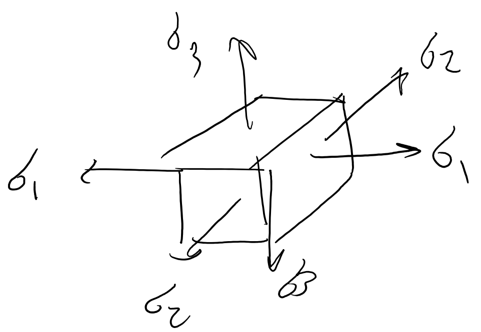
2D models for plastic zone size

5.2.2 Plastic zone shape: 2D models

- 2D models
- plane stress versus plane strain plastic zones

More popular

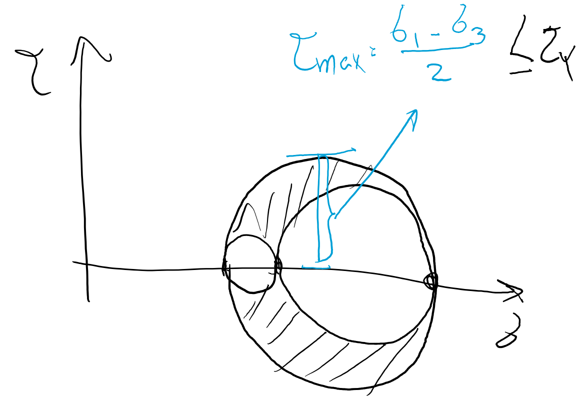
J2 (von Mises plasticity)

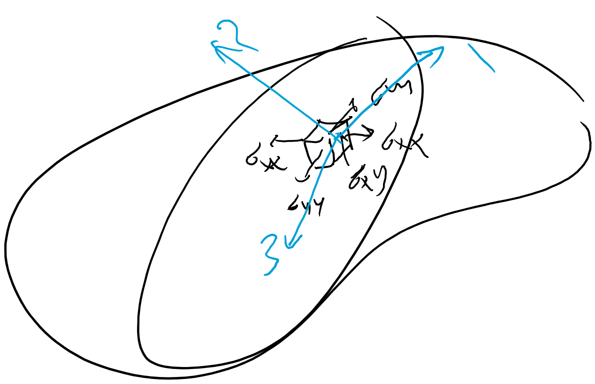


$$\sigma_{vm} = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}} \leq \sigma_Y$$

Tresca's model

$$\tau_{max} = \frac{\sigma_1-\sigma_3}{2} \leq \tau_Y$$





von-Mises criterion

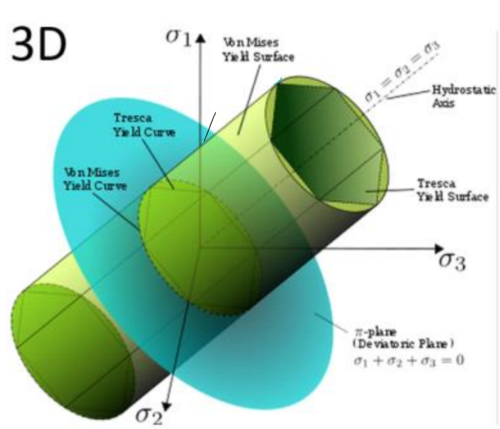
$$\sigma_v = \sqrt{3J_2}$$
$$= \sqrt{\frac{(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2+6(\sigma_{12}^2+\sigma_{23}^2+\sigma_{31}^2)}{2}}$$
$$= \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}{2}}$$
$$= \sqrt{\frac{3}{2}\, s_{ij}s_{ij}}$$

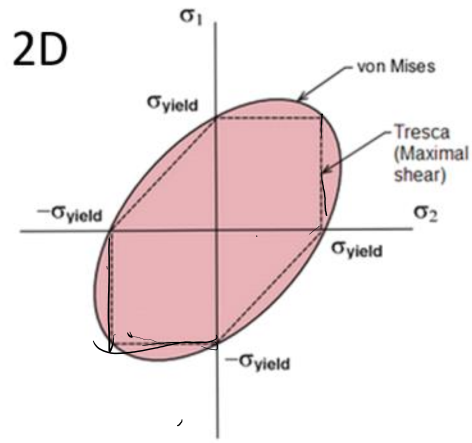
s is stress deviator tensor

$$\boldsymbol{\sigma}^{dev} = \boldsymbol{\sigma} - \frac{1}{3}(\mathrm{tr}\ \boldsymbol{\sigma})\,\mathbf{I}$$

Tresca criterion
Maximum shear stress

$$\sigma_{tresca} = \sigma_1 - \sigma_3 > \sigma_{max}$$





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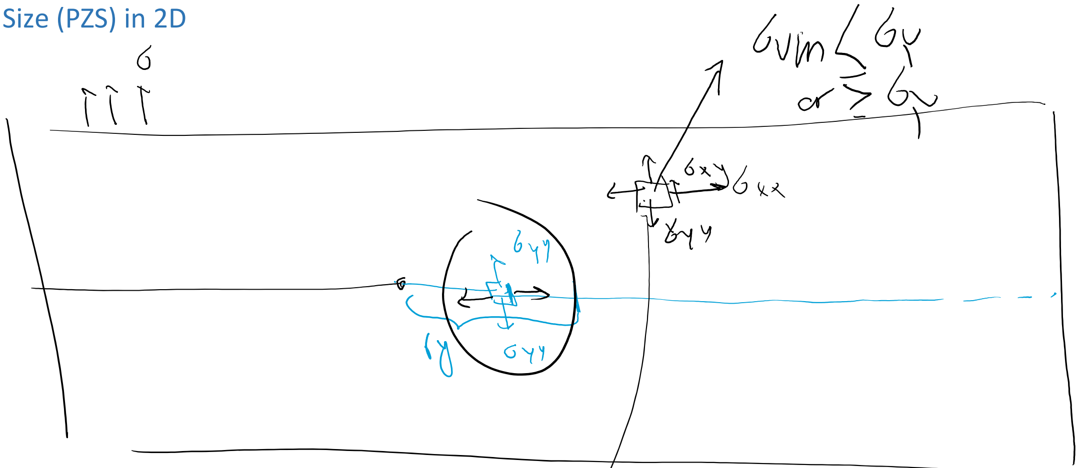
$\sigma_3 = 0$ plane stress

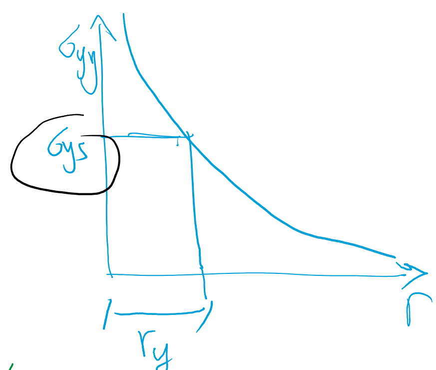
We want to use von Mises or Tresca conditions to determine the Plastic Zone Size (PZS) in 2D



$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} f_{xx}(\theta)$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} f_{yy}(\theta)$$

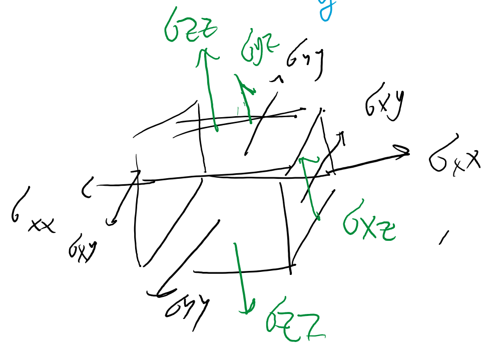
$$\sigma_{xy} = \frac{K_I}{\sqrt{2\pi r}} f_{xy}(\theta)$$



in-plane σ_{xx} ✓ σ_{yy} ✓ σ_{xy} ✓

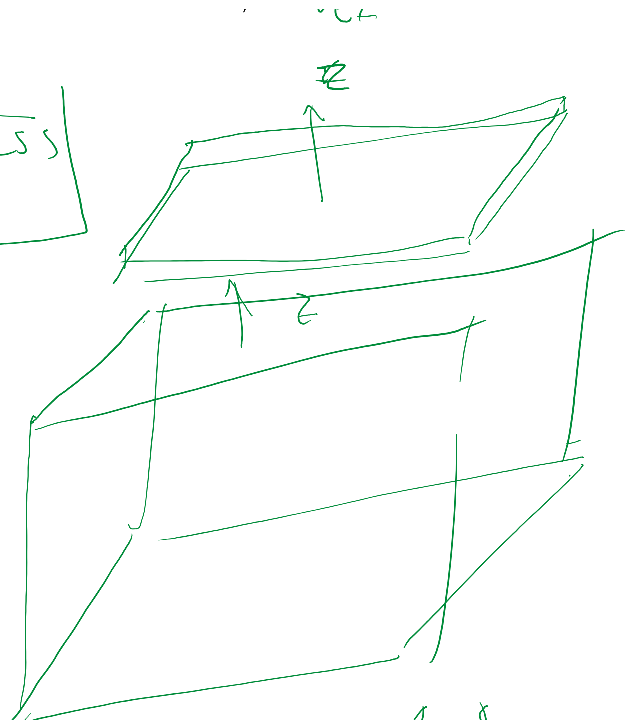
out-of plane σ_{zz}, $\sigma_{xz} = 0$, $\sigma_{yz} = 0$

?



y ?

$$\boxed{\sigma_{zz} = 0} \quad \text{plane stress}$$

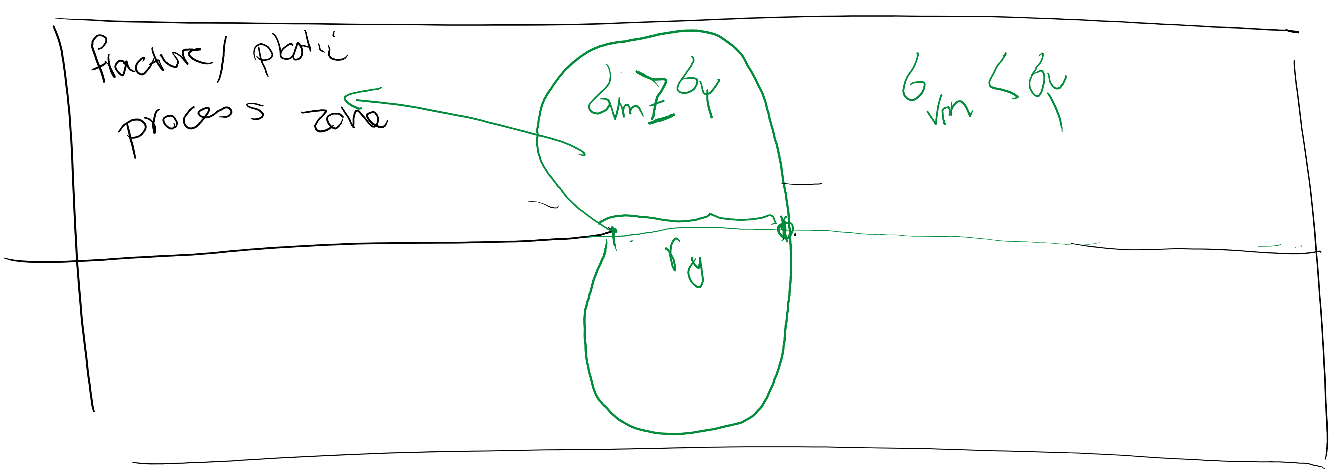


$$\epsilon_{zz} = 0 \quad \text{plane strain}$$

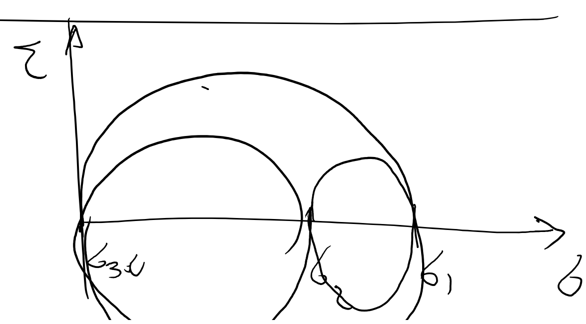
$$\rightarrow \epsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu(\sigma_{xx}+\sigma_{yy})\right) = 0$$

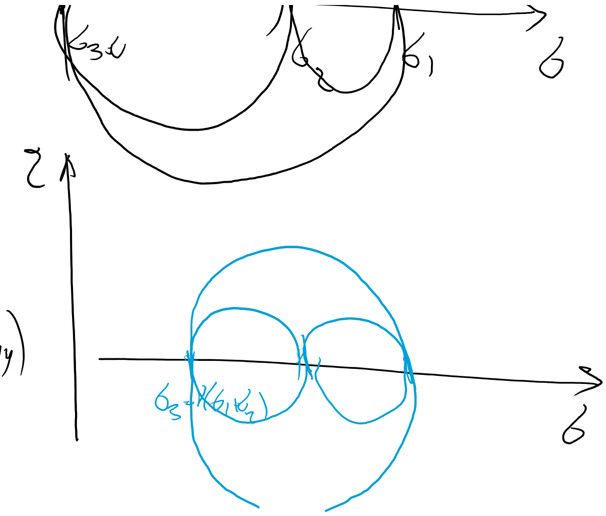
$$\Rightarrow \boxed{\sigma_{zz} = \nu(\sigma_{xx}+\sigma_{yy})} \quad \text{p. strain}$$

$$\sigma_{vm} = \sqrt{\frac{(\sigma_{xx}-\sigma_{yy})^2+(\sigma_{yy}-\sigma_{zz})^2+(\sigma_{zz}-\sigma_{xx})^2+6(\sigma_{xy}^2+\cancel{\sigma_{yz}^2}+\cancel{\sigma_{zx}^2})}{2}} = \sigma_Y$$



plane stress $\sigma_3 = \sigma_{zz} = 0$





p. Strain

$\sigma_3 = \sigma_{zz} = \nu(\sigma_{xx}+\sigma_{yy})$
$= \nu(\sigma_1+\sigma_2)$

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von-Mises criterion

$$\sigma_e = \sigma_{ys}$$

$$\sigma_e = \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2\right]^{1/2}$$

Principal stresses:

$$\sigma_1,\sigma_2 = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \left[\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2+\tau_{xy^2}\right]^{1/2}$$

Mode I, principal stresses

$$\sigma_1 = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\right)$$

$$\sigma_2 = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\right)$$

$$\sigma_{zz} = \sigma_3 = \begin{cases} 0 & \text{plane stress} \\ \dfrac{2\nu K_I}{\sqrt{2\pi r}}\cos\dfrac{\theta}{2} & \text{plane strain} \end{cases}$$

$$\boxed{r_y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2\left[1+\cos\theta+\frac{3}{2}\sin^2\theta\right]}$$ plane stress

$$r_y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2\left[(1-2\mu)^2(1+\cos\theta)+\frac{3}{2}\sin^2\theta\right]$$ plane strain

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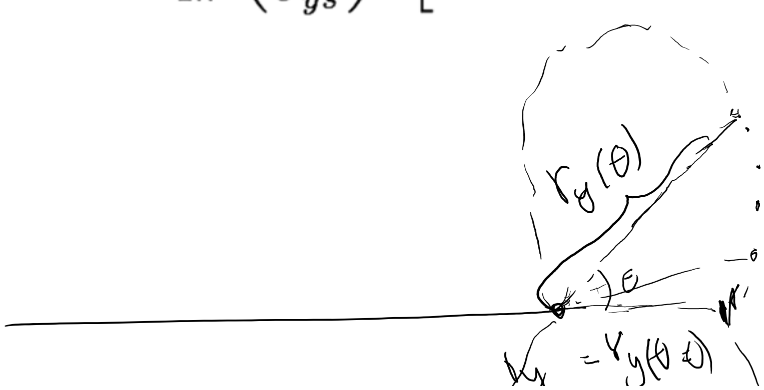
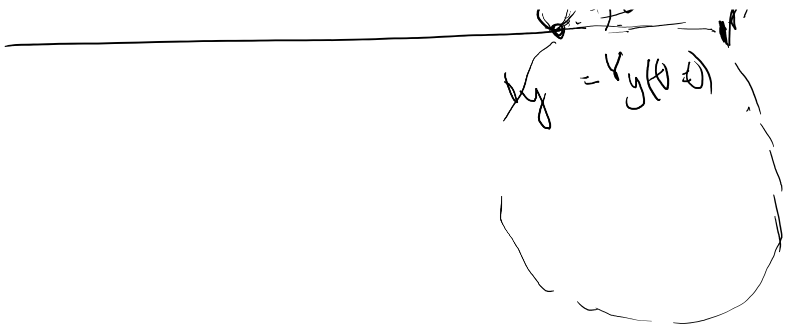
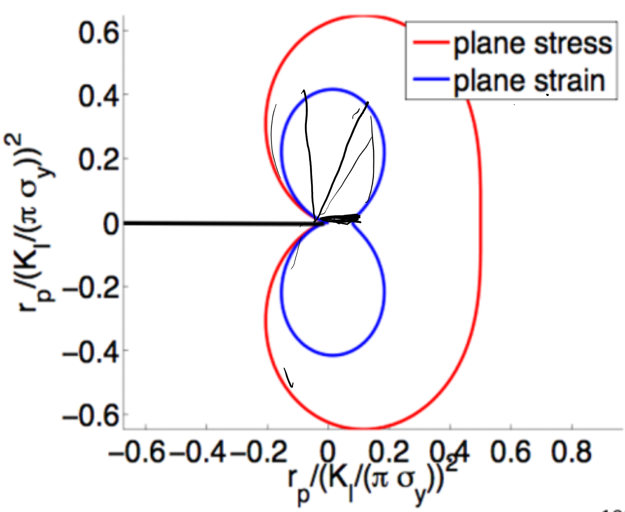




von-Mises criterion

$$\boxed{r_y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2\left[1+\cos\theta+\frac{3}{2}\sin^2\theta\right]}$$

plane stress



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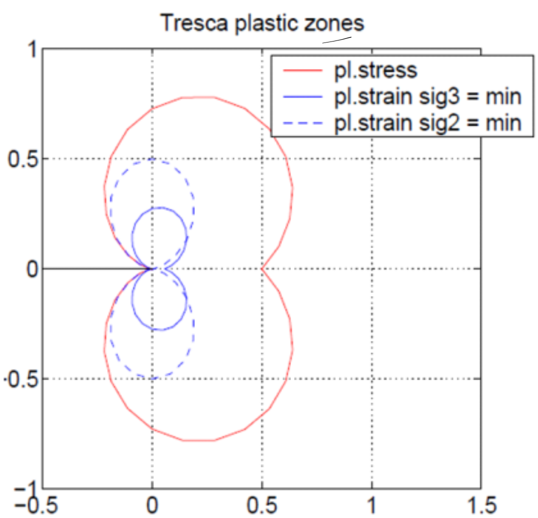
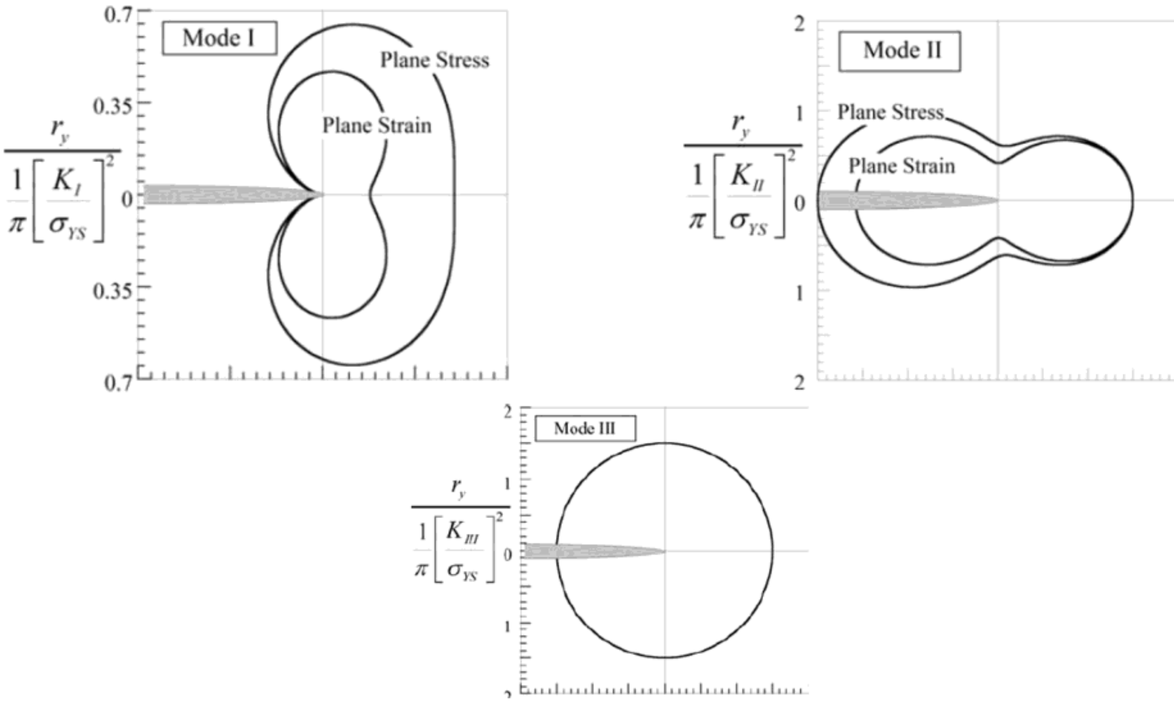
Tresca criterion



Plastic zone shape: Mode I-III



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This is exactly what we are doing now (instead of theta = 0, we look for all theta):

Plastic correction:
1st order approximation

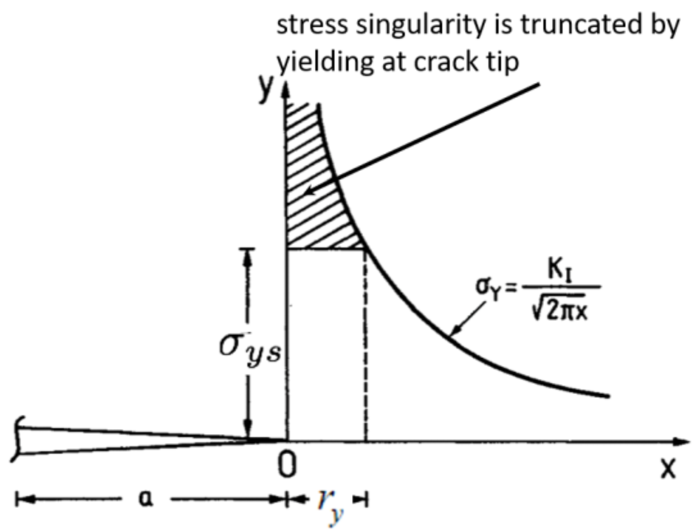
- A cracked body in a **plane stress** condition
- Material: **elastic perfectly plastic** with yield stress σ_{ys}

On the crack plane $\theta = 0$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$$

$\sigma_{yy} = \sigma_{ys}$ (yield occurs)

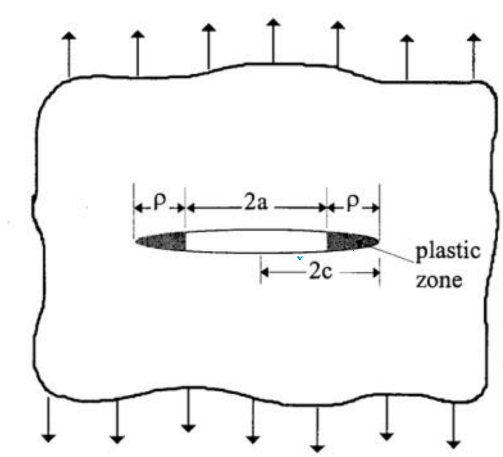
$$\boxed{r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2}$$

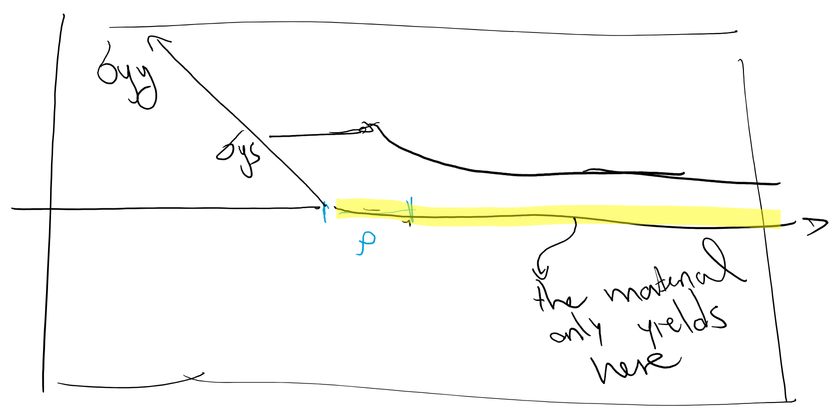


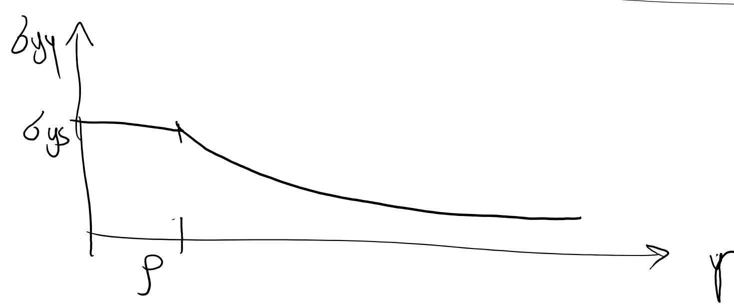
first order **approximation** of plastic zone size: equilibrium is not satisfied

Can we rebalance the forces

Strip yield model does that







To extend this idea, by getting an exact solution, but now allowing the yielding not only in the strip but also everywhere in the bulk, we need to use numerical solutions

Stress redistributed for 2D

Dodds, 1991, FEM solutions

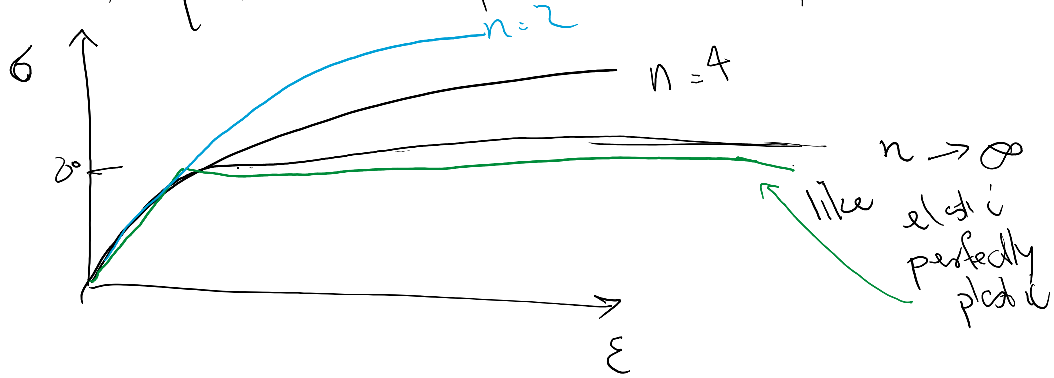
Ramberg-Osgood material model

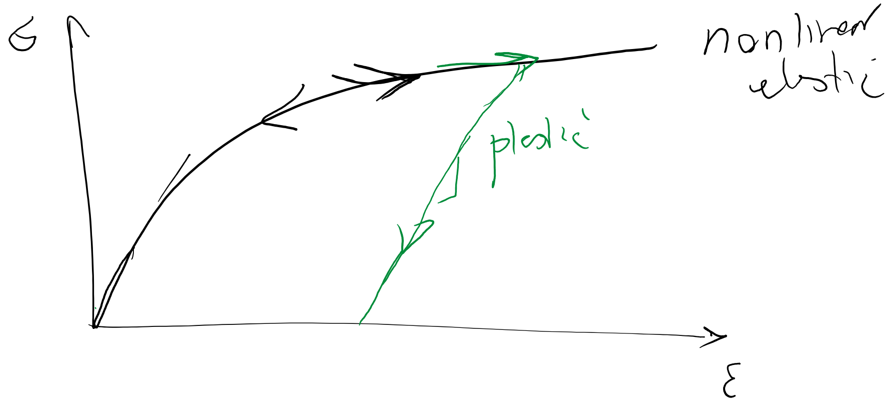
$$\frac{\varepsilon}{\varepsilon_o} = \frac{\sigma}{\sigma_o} + \alpha\left(\frac{\sigma}{\sigma_o}\right)^n \qquad (1.01)^{100}$$

$\varepsilon \approx \sigma$

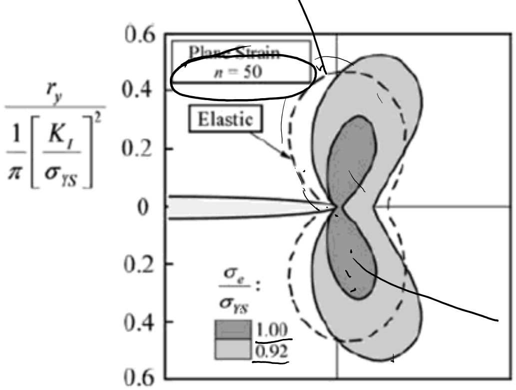
- Low n: High strain-hardening.
- $n \to \infty$: Similar to elastic perfectly plastic.

Nonlinear elasticity (Not a plastic model)

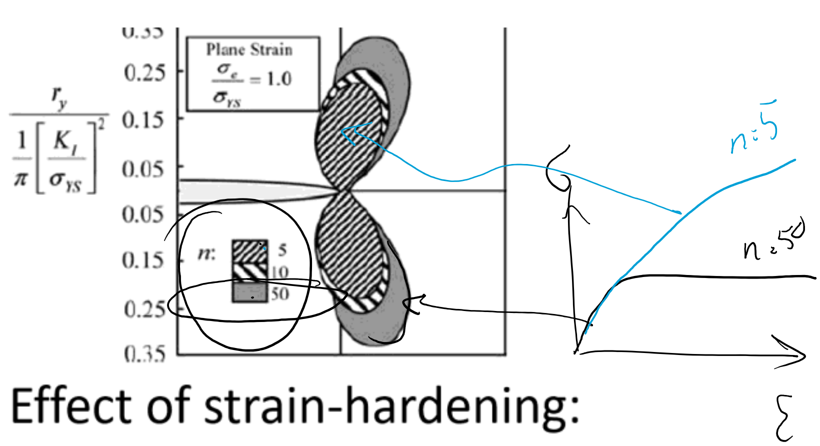




solution we looked at with no load balancing

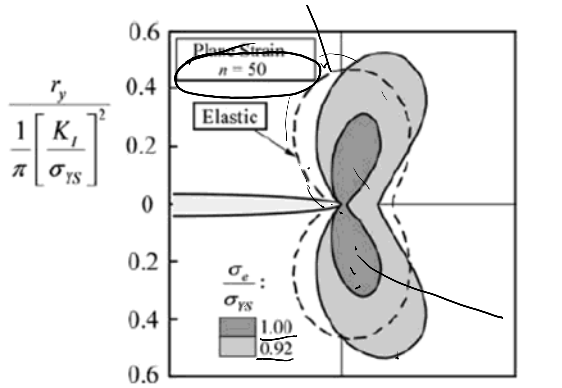


Effect of definition of yield

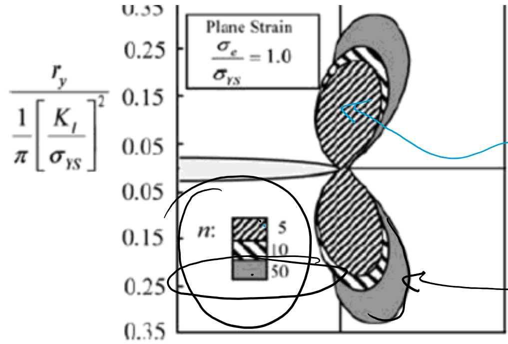


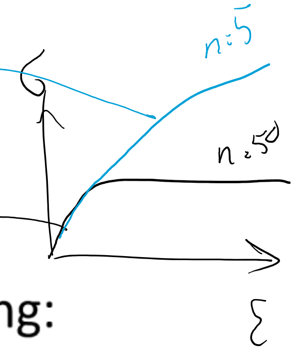
Effect of strain-hardening:

Higher hardening (lower n) =>



Effect of definition of yield
(some level of ambiguity)





Effect of strain-hardening:
Higher hardening (lower n) =>
smaller zone

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Key finding

whether a 1D model r_y, r_p, ρ

or 2D " with or without load balancing

or K_{II} or K_{III}

$$r_p = (\alpha)\left(\frac{K_I}{\sigma_Y}\right)^2$$

Plastic zone sizes:Summary

criterion	state	r_y or r_p	$\dfrac{r_y \vert r_p}{(K_I/\sigma_y)^2}$
Von Mises	plane stress	$\frac{1}{2\pi}\left(\frac{K_I}{\sigma_y}\right)^2$	0.1592
Von Mises	plane strain	$\frac{1}{18\pi}\left(\frac{K_I}{\sigma_y}\right)^2$	0.0177
Tresca	plane stress	$\frac{1}{2\pi}\left(\frac{K_I}{\sigma_y}\right)^2$	0.1592
Tresca	plane strain $\sigma_1 > \sigma_2 > \sigma_3$	$\frac{1}{18\pi}\left(\frac{K_I}{\sigma_y}\right)^2$	0.0177
Tresca	plane strain $\sigma_1 > \sigma_3 > \sigma_2$	0	0
Irwin	plane stress	$\frac{1}{\pi}\left(\frac{K_I}{\sigma_y}\right)^2$	0.3183
Irwin	plane strain (pcf = 3)	$\frac{1}{\pi}\left(\frac{K_I}{3\sigma_y}\right)^2$	0.0354
Dugdale	plane stress	$\frac{\pi}{8}\left(\frac{K_I}{\sigma_y}\right)^2$	0.3927
Dugdale	plane strain (pcf = 3)	$\frac{\pi}{8}\left(\frac{K_I}{3\sigma_y}\right)^2$	0.0436

Source: Schreurs (2012)

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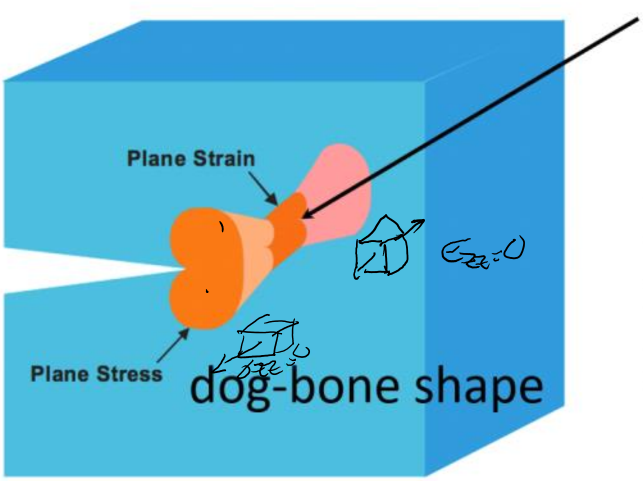
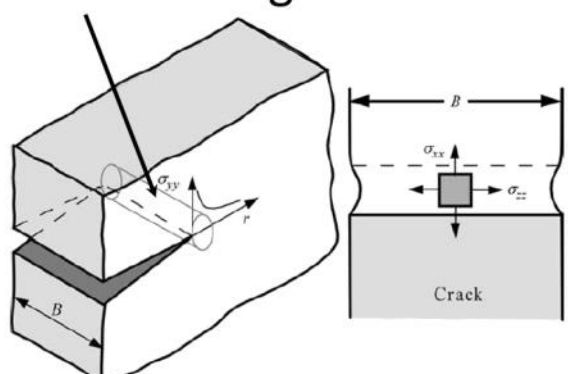
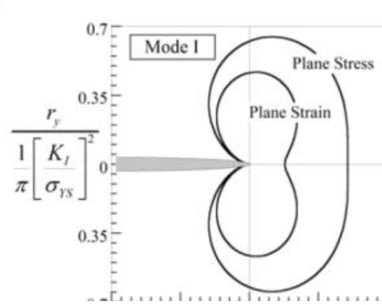
Plane stress vs plane strain conditions

constrained by the surrounding material





- Plane stress failure: more ductile
- Plane strain failure: mode brittle



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- Plane strain failure: mode brittle

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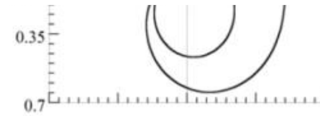
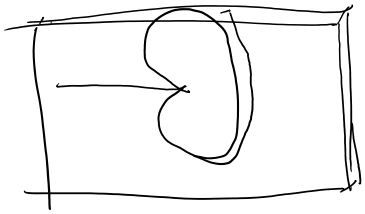


all like a plane stress shape



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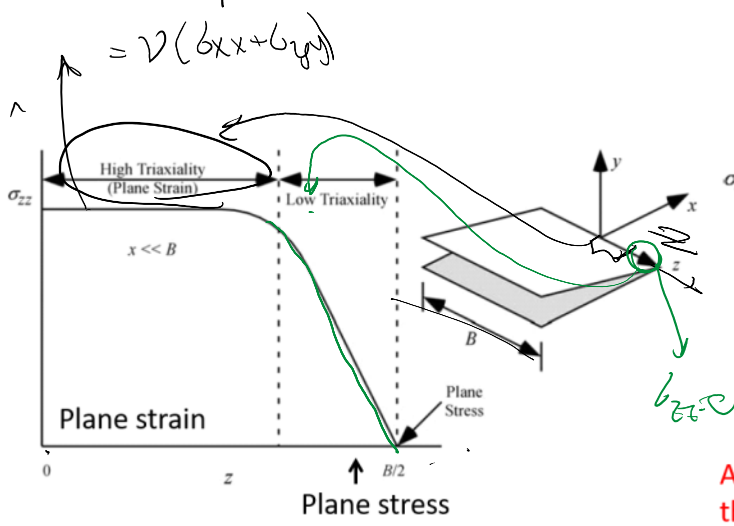

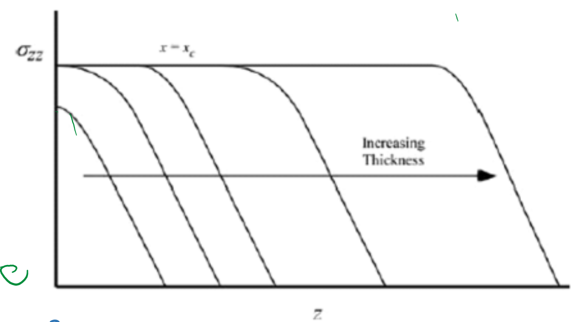

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As the thickness increases more through the thickness bahaves as plane strain

To decide whether the plate is in plane stress or strain condition we compare

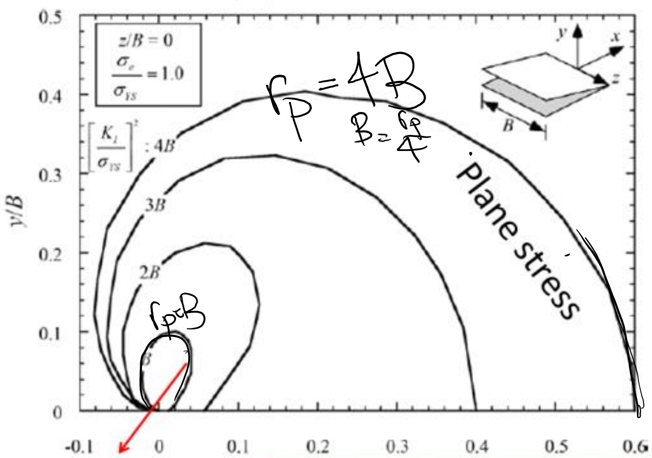
B and r_p

$B >> r_p$ plane strain

$B < r_p$ p stress

What thicknesses are plane stress?

- As $\frac{\left(\frac{K}{\sigma_{ys}}\right)^2}{B}$ increases:
  1. The plastic zone expands (load is increasing)
  2. Plastic zone transitions from plane strain to plain stress
- Note that $r_p \propto \left(\frac{K}{\sigma_{ys}}\right)^2$



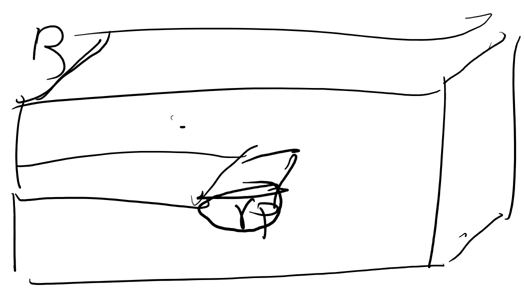
$$\frac{\left(\frac{K}{\sigma_{ys}}\right)^2}{B} \propto \frac{r_p}{B} : \begin{cases} \text{low (high } B) & \text{plane strain} \\ \text{high (low } B) & \text{plane stress} \end{cases}$$

Change of plastic loci to plane stress mode as "relative B decreases". Nakamura & Park, ASME 1988

For plane strain condition we must have $B > \left(\frac{K}{\sigma_{ys}}\right)^2 \propto r_p$

$r_p = \left(\frac{K}{\sigma_y}\right)^2$

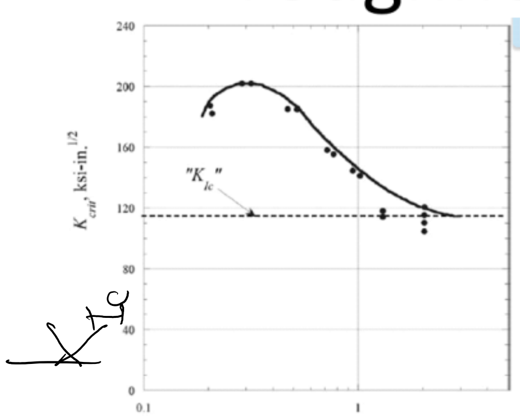
1D model

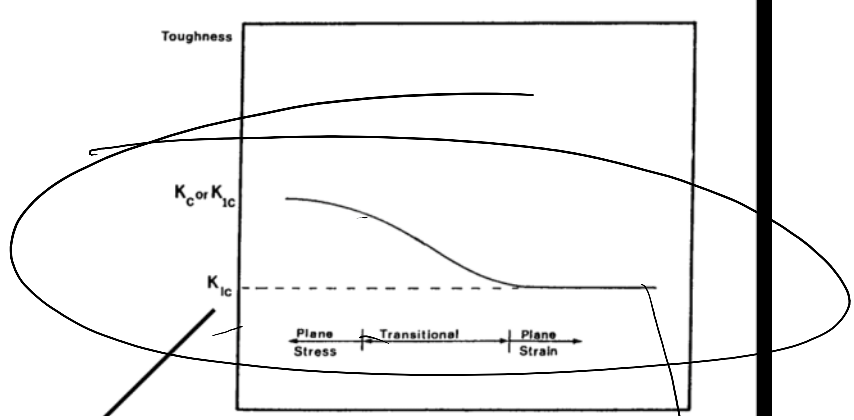


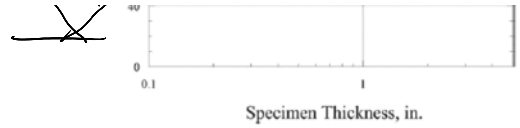
Bigger fracture process zone -> Larger energy dissipation per unit area of crack advance (that is the material is tougher)

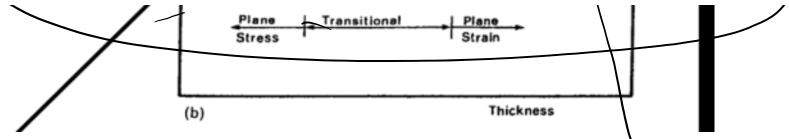
=> plane stresss should have a higher toughness (resistance)

Toughness vs. thickness









Converged toughness versus thickness

we generally report plane strain toughness

Plane strain fracture toughness **lowest K (safe)**

(Irwin) $K_c(B) = K_{Ic}\left(1 + \frac{1.4}{B^2}\left[\frac{K_{Ic}}{\sigma_{ys}}\right]^4\right)^{1/2}$ Note that $\frac{1}{B^2}\left[\frac{K}{\sigma_{ys}}\right]^4 \propto \left(\frac{r_p}{B}\right)^2$

$\left(\frac{K_I}{\sigma_y}\right)^2 \propto r_p$

$\left(\frac{r_p}{B}\right)^2$

as $B \to \infty$ $K_c \to K_{Ic}$ of plane strain