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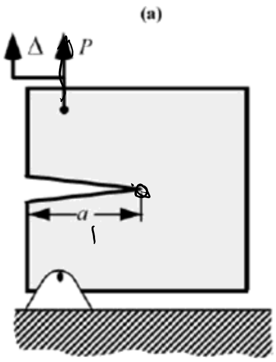
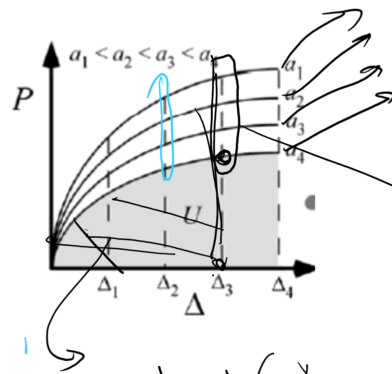
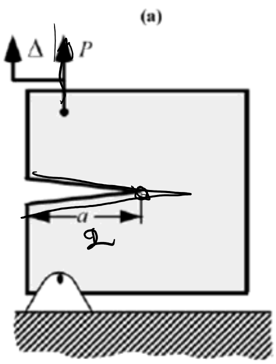
Monday, October 28, 2024 9:29 AM





for experiments for different a sizes

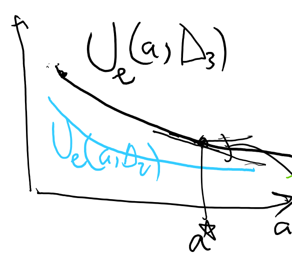
$U_e(\Delta_3, a_4)$



For a fixed grip Δ is constant

results of these four points resemble a fixed grip

$$G = -\frac{1}{B}\frac{dU_e}{da}$$



because Δ is fixed

$-$ slope $= -\frac{dU_e}{da}$

this G will be equal to R

$$\boxed{G(a^*, \Delta_3) = -\frac{1}{B}\frac{dU_e}{da}(a^*, \Delta_3)}$$

- Rice proposes a method where to obtain the J integral (G), one experiment is needed for certain geometries

  cf. Anderson 3.2.5 for details

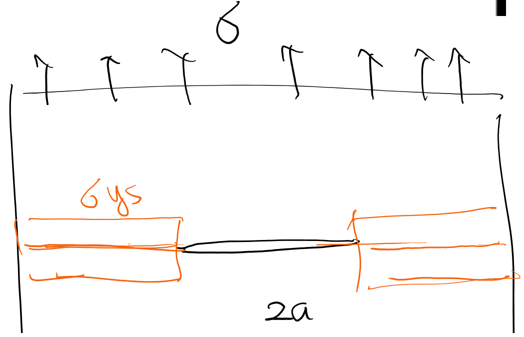
5.3. 7. Fracture mechanics versus material (plastic strength

For fracture we have

$$K_I = f\left(\frac{a}{W}\right)\sqrt{\pi a}\,\sigma$$

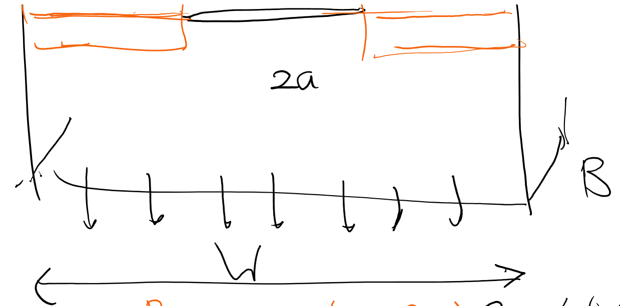
for crack propagation

$$K_{Ic} = f\left(\frac{a}{W}\right)\sqrt{\pi a}\,\sigma_{max}$$

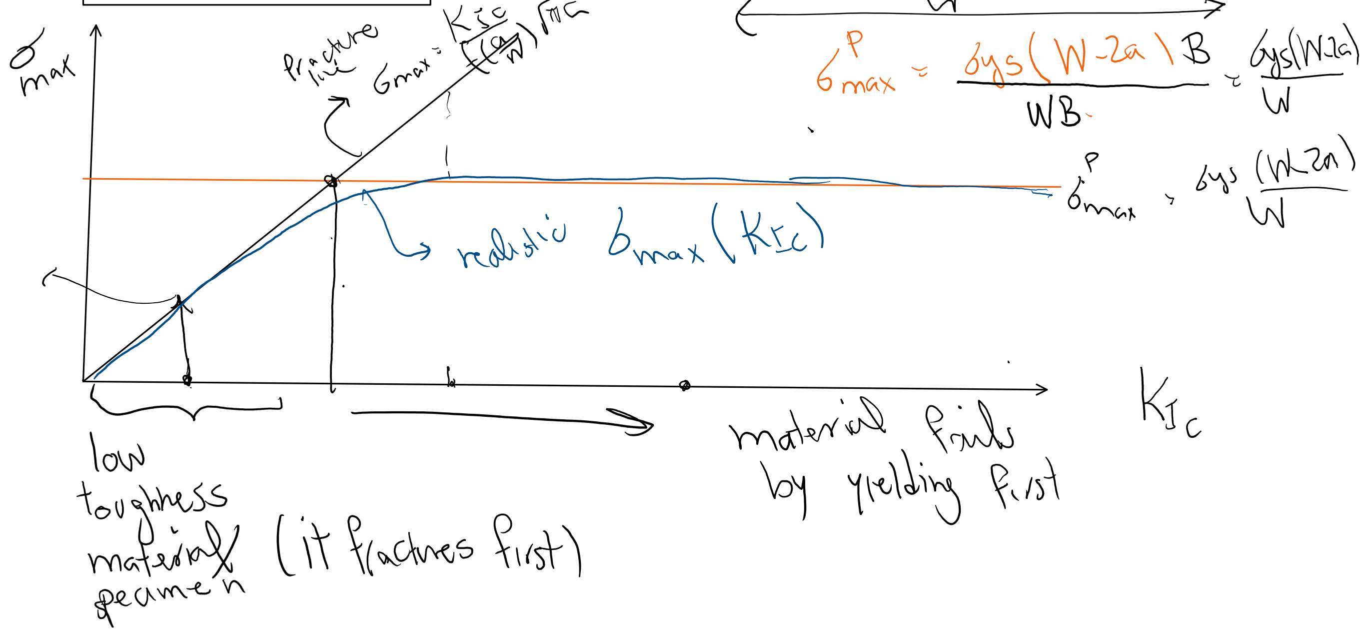


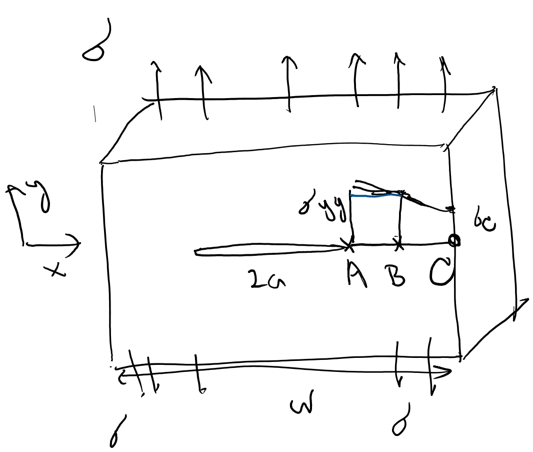
$$K_{Ic} = f\left(\frac{a}{W}\right)\sqrt{\pi a}\;\sigma^f_{max}$$

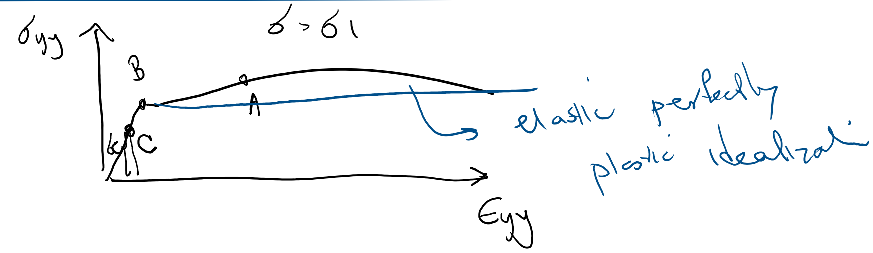
$$\Rightarrow \boxed{\sigma^f_{max} = \frac{K_{Ic}}{f\left(\frac{a}{W}\right)\sqrt{\pi a}}} \quad (1)$$

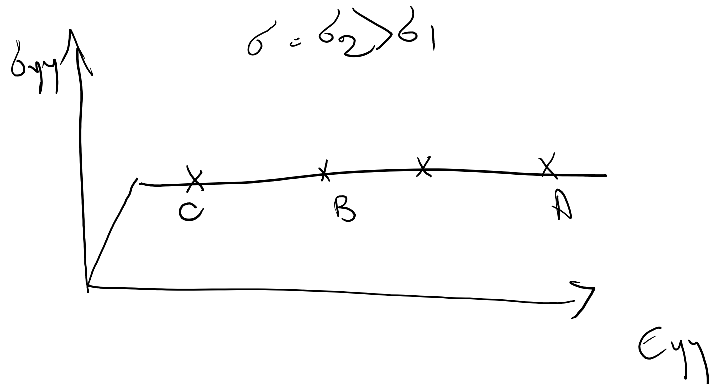


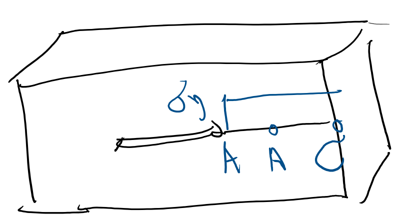
$$\sigma^P_{max} = \frac{\sigma_{ys}(W-2a)B}{WB} = \frac{\sigma_{ys}(W-2a)}{W}$$



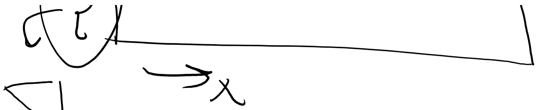




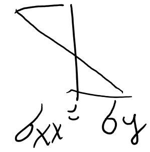




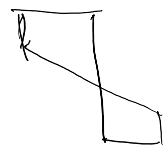
Another example HW3

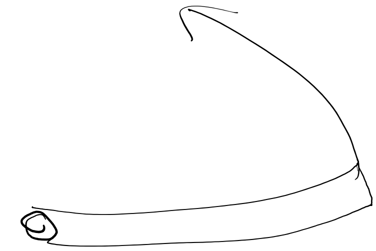


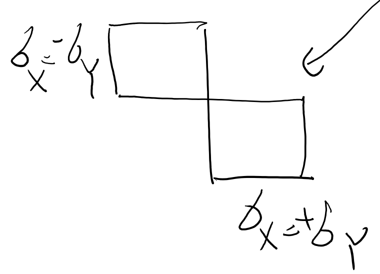
$P = P_{NL-ini}$



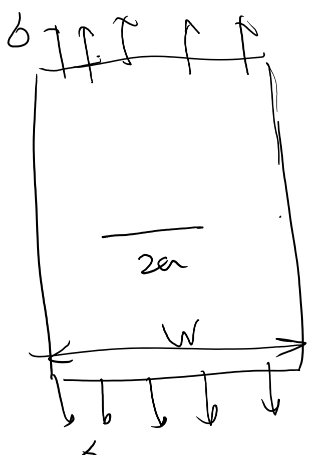
$P > P_{NL-ini}$







at this point this becomes a hinge



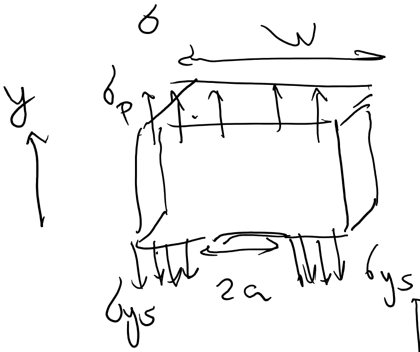
$K_I = f\left(\frac{a}{W}\right)\sqrt{\pi a}\,\sigma$

$K_{IC} = f\left(\frac{a}{W}\right)\sqrt{\pi a}\,\sigma_f$

$\Rightarrow \sigma_f = \dfrac{K_{IC}}{f\left(\frac{a}{W}\right)\sqrt{\pi a}}$

$f\left(\frac{a}{W}\right) \to \sec\left(\frac{\pi a}{W}\right)$

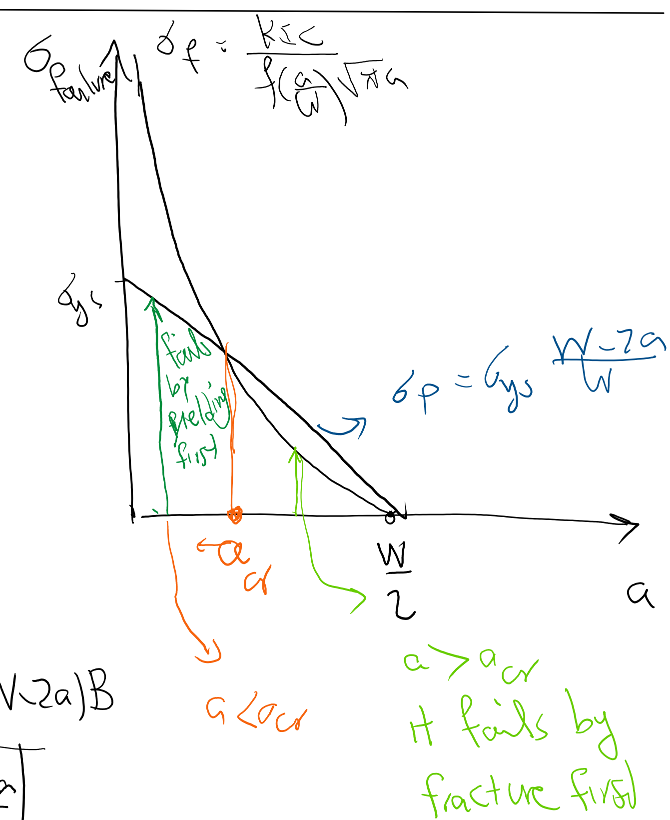
$a = \frac{W}{2}$



$\Sigma F_y = 0$

$\sigma_p B W = \sigma_{ys}(W-2a)B$

$$\boxed{\sigma_p = \sigma_{ys}\,\frac{W-2a}{W}}$$

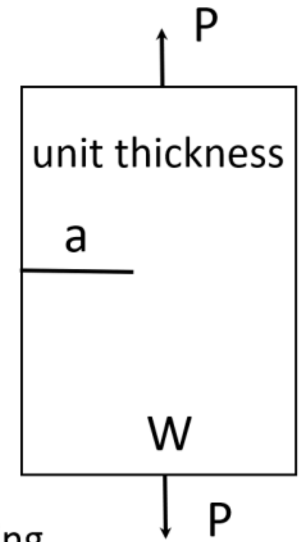


Fracture vs. Plastic collapse

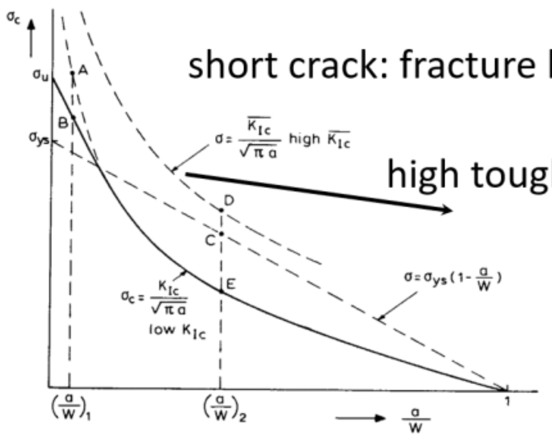
$$\sigma_{\text{net}} = \frac{P}{W-a} = \sigma\frac{W}{W-a}$$

(cracked section)

$$\sigma = \frac{P}{W}$$



Yield: $\sigma\dfrac{W}{W-a} = \sigma_{ys} \longrightarrow \sigma = \sigma_{ys}\left(1-\dfrac{a}{W}\right)$



short crack: fracture by plastic collapse!!!

high toughness materials:yielding before fracture

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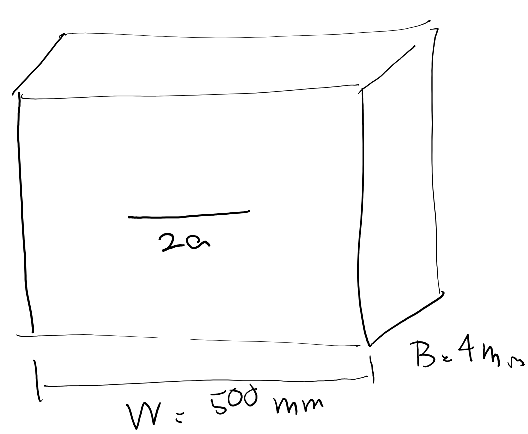
Example

<u>Example 4.11</u> Estimate the failure load under uniaxial tension for a centre-cracked panel of aluminium alloy of width W=500 mm, and thickness B=4 mm, for the following values of crack length $2a$ = 20 mm and $2a$ = 100 mm. Yield stress σ_y =350 MPa and fracture toughness K_{Ic}=70 MPa $\sqrt{m}$

$2a = 20$ mm
OR
$2a = 100$ mm

$K_{Ic} = 70$ MPa $\sqrt{m}$
$\sigma_{ys} = 350$ MPa



Does it fail by yielding or fracture in each case?

Case 1) 2a = 20 mm $\Longrightarrow a = 10$ mm

Yielding $\quad \sigma_p = \sigma_{ys}\left(\dfrac{W-2a}{W}\right) = 350\text{MPa}\left(\dfrac{500-20}{500}\right)$

$$\boxed{F_p = \sigma_p B W = 672\,kN \qquad a = 10\,mm} \quad i$$

Fracture $\quad K_{Ic} = f\left(\frac{a}{W}\right)\sigma_f\sqrt{\pi a} \Rightarrow \sigma_f = \frac{K_{Ic}}{f\left(\frac{a}{W}\right)\sqrt{\pi a}} = \frac{70\times10^6\,Pa\sqrt{m}}{\sqrt{\sec\left(\underbrace{\frac{\pi\times10}{500}}_{\frac{\pi a}{W}}\right)}\;\underbrace{\sqrt{\pi\times10e\text{-}3}}_{a}}$

$\Rightarrow \sigma_f = 394.6\,MPa$

$$\boxed{F_f = \sigma_f B W = 790\,kN \qquad a = 10\,mm} \quad ii$$

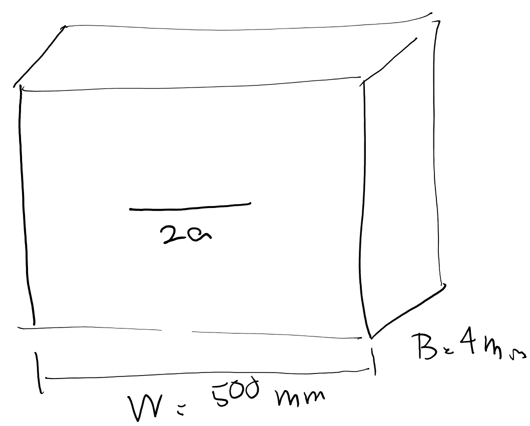
The specimen will fail by yielding for a = 10 mm.

$2a = 100\,mm$

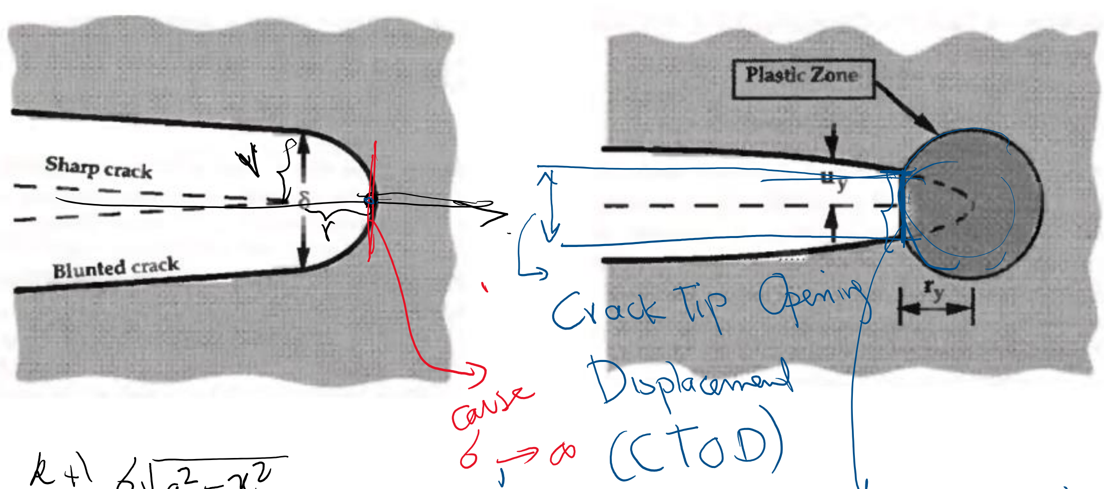
$F_p = 560\,kN$

$F_f = 172.2\,kN$ ✓

It fractures first for a = 50 mm



5.4. Crack tip opening displacement (CTOD), relations with J and G



$v = \frac{k+1}{\ldots}\sigma\sqrt{a^2 - x^2}$

$$V = \underbrace{\frac{\kappa+1}{4\mu}\,\sigma\sqrt{a^2 - x^2}}_{\text{mid crack in an infinite domain}} \qquad \text{ellipse}$$

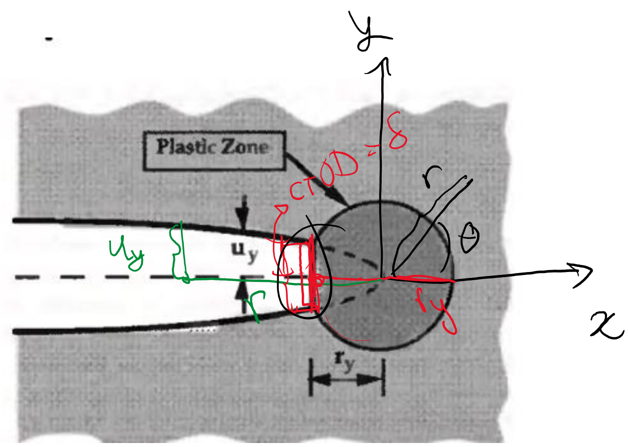
$\sigma \to \infty$ (CTOD)

$\downarrow$ crack tip blunting

$$V = \frac{(\kappa+1)}{2\mu} K_I \sqrt{\frac{r}{2\pi}} \qquad \text{parabola}$$

Parallel to Rice's work in the USE, Wells in the UK looked at CTOD as a measure of nonlinear material response in the FPZ.

$$\underset{\text{Rice}}{J} \;\propto\; \underbrace{\text{CTOD}}_{\text{Wells}}$$



Estimate for the CTOD:

$$u_y(r) = \frac{(\kappa+1)K_I}{2\mu}\sqrt{\frac{r}{2\pi}}$$

$$r \to r_y \qquad u_y = \frac{\delta}{2}$$

$$\frac{\delta}{2} = \frac{(\kappa+1)\,K_I}{2\mu}\sqrt{\frac{r_y}{2\pi}}$$

$$r_y = \frac{1}{\pi}\left(\frac{K_I}{\sigma_{ys}}\right)$$

$$\kappa = \frac{3-\nu}{1+\nu} \quad \text{p. stain}$$

$$\mu = \frac{E}{2(1+\nu)}$$

$$\Longrightarrow \boxed{\delta \approx \frac{4}{\pi}\,\frac{K_I^2}{\sigma_{ys} E}}$$

estimate for CTOD

Back of envelope relation between δ & J

K_I^2 K_I^2

$$G = J = \frac{K_I^2}{E'} \quad \text{eg} \quad = \frac{K_I^2}{E} \qquad \text{mode I}$$

$$K_I^2 = JE \quad , \quad \delta = \underbrace{A}_{\text{constant}} \frac{K_I^2}{\sigma_{ys} E} = A \frac{JE}{\sigma_{ys} E} \longrightarrow$$

$$J = C\,\delta\,\sigma_{ys}$$

$$\underbrace{[\sigma][L] \qquad [L] \quad [\sigma]}_{}$$

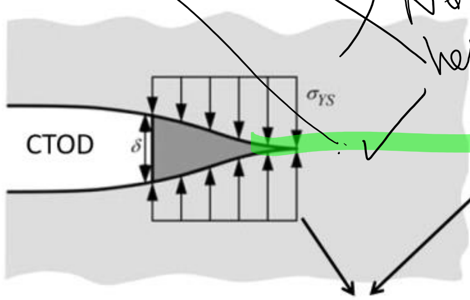
holds even in Nonlinear fracture $J \propto \delta$

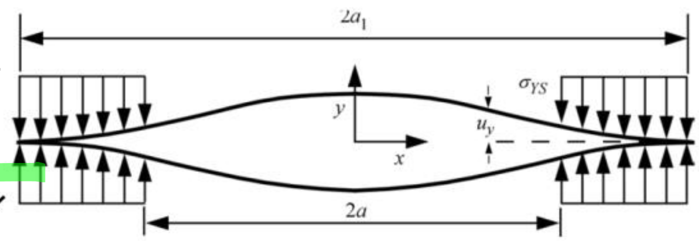
NL $\nearrow$ J & δ go up proportionally

Crack Tip Opening Displacement: Strip yield model

yielding can happen on crack line

Not here





Stresses that yielded K = 0

K = 0 ➡ $Z = \frac{2\sigma_{YS}}{\pi}\left[\frac{k}{z}\sqrt{\frac{z^2-a_1^2}{1-k^2}}\right]$ ➡ $\tilde{Z} = \frac{2\sigma_{YS}}{\pi}[z\omega_1 - a\omega_2]$

➡ $u_y = \frac{2}{E}\mathrm{Im}\tilde{Z} = \frac{4\sigma_{YS}}{\pi E}\left[a\coth^{-1}\left(\frac{1}{a_1}\sqrt{\frac{a_1^2-z^2}{1-k^2}}\right) - z\coth^{-1}\left(\frac{k}{z}\sqrt{\frac{a_1^2-z^2}{1-k^2}}\right)\right]$

$z = a$ ➡ $\delta = 2u_y = \frac{8\sigma_{YS}a}{\pi E}\ln\left(\frac{1}{k}\right) = \frac{8\sigma_{YS}a}{\pi E}\left[\frac{1}{2}\left(\frac{\pi}{2}\frac{\sigma}{\sigma_{YS}}\right)^2 + \frac{1}{12}\left(\frac{\pi}{2}\frac{\sigma}{\sigma_{YS}}\right)^4 + \cdots\right]$ ➡

$$\boxed{\text{For } \sigma/\sigma_{YS} \to 0 \qquad \delta = \frac{K_I^2}{\sigma_{YS}E}}$$

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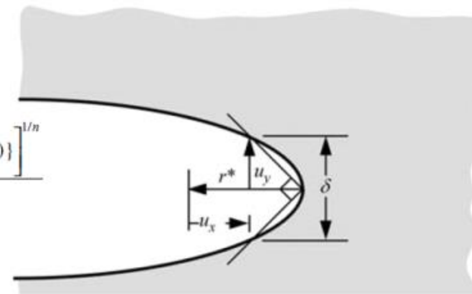
CTOD-J relation

- When SSY is satisfied G = J so we expect:

$$G = m\sigma_y\delta \quad \Rightarrow \quad \boxed{J = m\sigma_y\delta}$$

- In fact this equation is valid well beyond validity of LEFM and SSY
- E.g. for HRR solution Shih showed that:



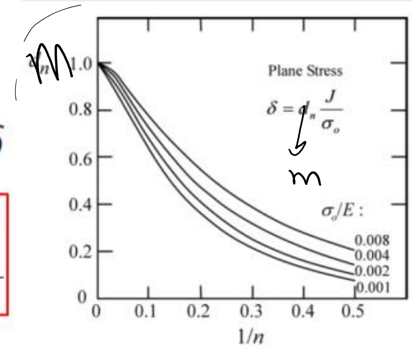
$$u_i = \frac{\alpha\sigma_o}{E}\left(\frac{EJ}{\alpha\sigma_o^2 I_n r}\right)^{\frac{n}{n+1}} r\tilde{u}_i(\theta, n) \qquad d_n = \frac{2\tilde{u}_y(\pi,n)\left[\frac{\alpha\sigma_o}{E}\{\tilde{u}_x(\pi,n)+\tilde{u}_y(\pi,n)\}\right]^{1/n}}{I_n}$$

- δ is obtained by 90 degree method:
  Deformed position corresponding to $r^* = r$ and $\varphi = -\pi$ forms 45 degree w.r.t crack tip)

$$\frac{\delta}{2} = u_y(r^*, \pi) = r^* - u_x(r^*, \pi)$$

$$\boxed{r^* = \left(\frac{\alpha\sigma_o}{E}\right)^{1/n}\{\tilde{u}_x(\pi,n)+\tilde{u}_y(\pi,n)\}^{\frac{n+1}{n}}\frac{J}{\sigma_o I_n}} \quad \Rightarrow \quad J = m\sigma_0\delta$$

for $\boxed{m = \frac{1}{d_n},\; d_n = \frac{2\tilde{u}_y(\pi,n)\left[\frac{\alpha\sigma_o}{E}\{\tilde{u}_x(\pi,n)+\tilde{u}_y(\pi,n)\}\right]^{1/n}}{I_n}}$



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$$\boxed{J = \delta\,\sigma_o\,m}$$

RO model σ_o (similar to σ_{ys})