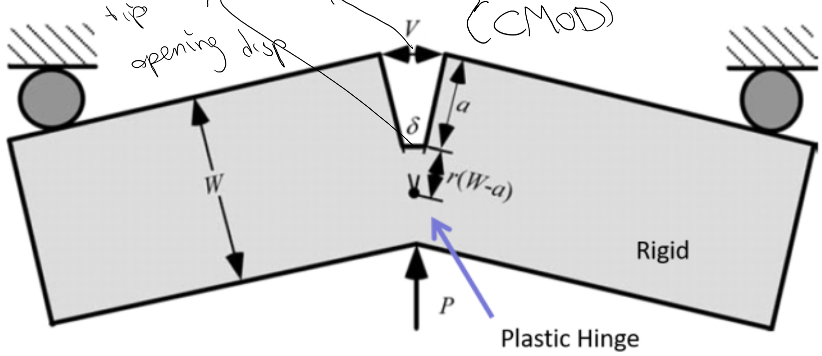
CTOD experimental determination

(δ, CTOD) crack tip opening disp

crack mouth opening displacement (CMOD)



$$\frac{\delta_t}{CMOD} = \frac{r(W-a)}{r(W-a)+a}$$

similarity of triangles

r : rotational factor [-], between 0 and 1

For high elastic deformation contribution, elastic corrections should be added

factor e.g. $\frac{\pi}{4}$, 1, ...

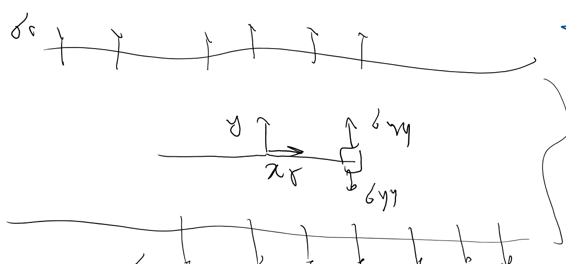
$$J = (\ )\,\delta\,\sigma_{ys}$$

fracture process zone

FPZ

Singular dominant zone

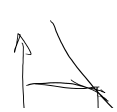
recall how r_p's were determined?



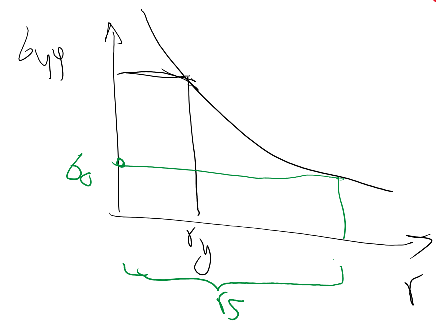
$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$$

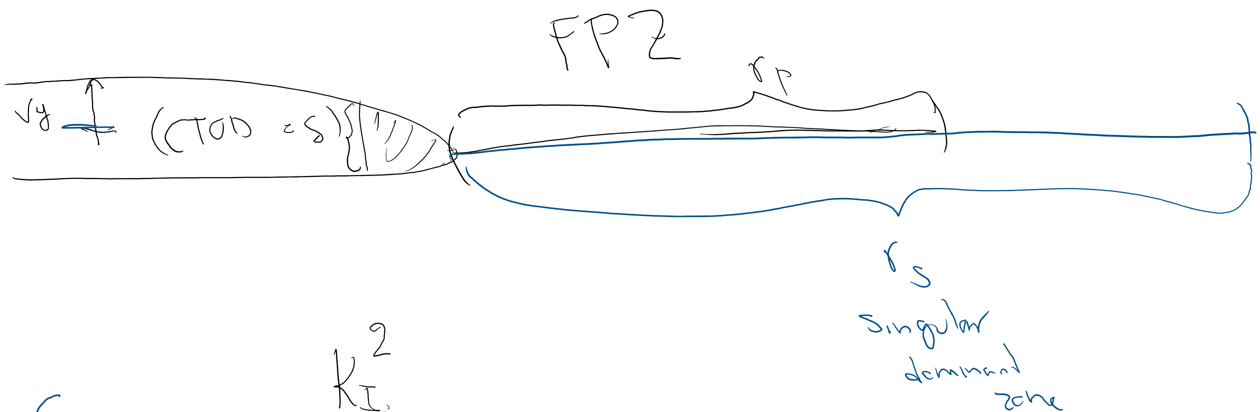
σ_{ys} r_y $\Rightarrow \sigma_{ys} = \frac{K_I}{\sqrt{2\pi r_y}} \Rightarrow \boxed{r_y = \left(\frac{K_I}{\sigma_{ys}}\right)^2 \frac{1}{2\pi}}$

$r_p = 2r_y$ $\quad r_p = \frac{1}{\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2$



$b \qquad \sigma_0 \qquad r_s \implies r_s = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_0}\right)^2$





$$\delta \propto \frac{K_I^2}{E\sigma_{ys}}$$

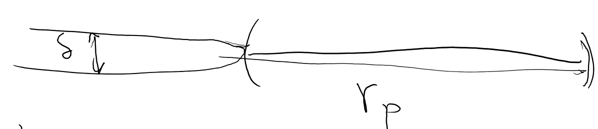
δ ↓ or any measure of crack opening disp

if SSY holds

$$\delta \sim \frac{K_I^2}{E\,\sigma_{ys}} \ll r_p \sim \frac{K_I^2}{\sigma_{ys}^2} \ll r_s \sim \frac{K_I^2}{\sigma_0^2}$$

"vertical size" of PZ → δ; "horizontal" size of PZ ← r_p

$$\frac{\delta}{r_p} \simeq \frac{\sigma_{ys}}{E}$$



$\delta \ll r_p \qquad \frac{\delta}{r_p} \simeq \frac{\sigma_{ys}}{E}$

☆ this concept holds in all fracture models: (Cohesive models, phase field, LEFM, ...)

& all applications

& all applications
mode I, mode II (earthquake)

FPZ size

$$r_p \propto \underbrace{\tilde{\Delta}}_{\text{disp scale}} \left(\frac{E}{\underbrace{\tilde{\sigma}}_{\text{strength}}} \right)$$

Disp, length

$$\underbrace{\frac{\tilde{\Delta}}{r_p}}_{\text{strain}} \approx \frac{\tilde{\sigma}}{E}$$

5.3. 6. Small scale yielding (SSY) versus large scale yielding (LSY)

We already observed 3 length scales for the fracture:

- Singular dominant zone size (LENGTH)
- Fracture Process zone size (LENGTH)
- Crack opening displacement (DISPLACEMENT)

$$r_s \propto \frac{K_I^2}{\sigma_0^2}$$

$$r_p \propto \frac{K_I^2}{\sigma_{ys}^2}$$

$$\delta \propto \frac{K_I^2}{\sigma_{ys} E}$$

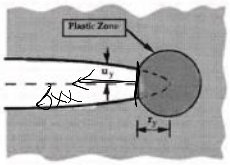
We want to discuss the interaction of these length scales:

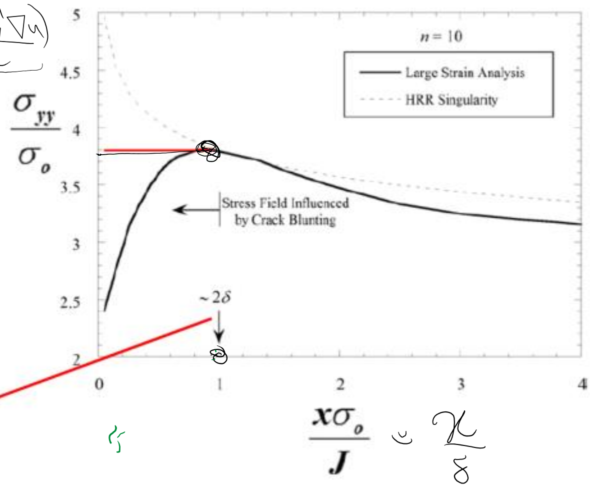
First, we examine the accuracy of the HRR solution very close to the crack tip:

Limitations of HRR solution

Limitations of HRR analysis

- Small strain: $\epsilon = \frac{1}{2}\left(\nabla \mathbf{u} + \nabla^T \mathbf{u}\right)$ $+\left(\frac{\nabla u^T \nabla u}{2}\right)$ finite strain
  (accurate for $\epsilon \lessapprox 0.1$)
- Small deformation theory (*e.g.*, not using PK stresses, etc)
- Elastic HRR model instead of plastic model
- Crack tip blunting: $\Rightarrow \sigma_{xx} = 0$



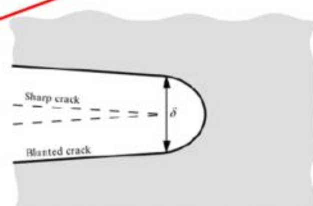


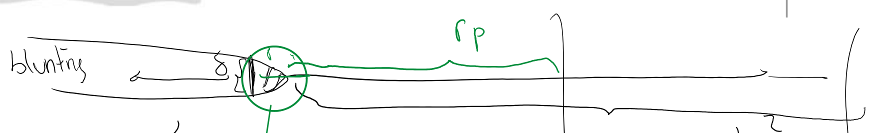
McMeeking and Parks, ASTM STP 668, ASTM 1979

$J \propto \sigma_o \delta$

δ: Crack tip opening

$\delta \propto \frac{K^2}{\sigma_y E}$





$r_s = \frac{K_I^2}{\sigma_o^2}$

$\delta \approx \frac{K_I^2}{\sigma_{ys} E} \ll r_p \propto \frac{K_I^2}{\sigma_{ys}^2}$

HRR is completely off

Complete nonlinear plastic large deformation solution

HRR good

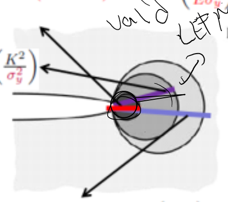
LEFM good

From SSY to LSY

Large strain radius $r_n \propto \delta$ (CTOD): $\delta = \mathcal{O}\left(\frac{K^2}{E\sigma_y}\right)$

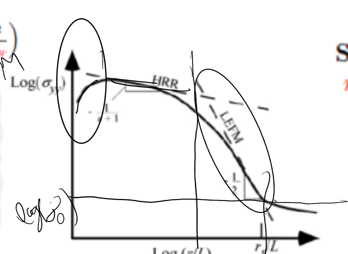
valid LEFM

plastic radius: $r_p = \mathcal{O}\left(\frac{K^2}{\sigma_y^2}\right)$



K-dominant radius: $r_s = \mathcal{O}\left(\frac{K^2}{\sigma^2}\right)$

σ = far field stress



r_p

r_s

SSY (Small Scale Yielding)

$r_n \ll r_p \ll r_s \quad \Rightarrow$

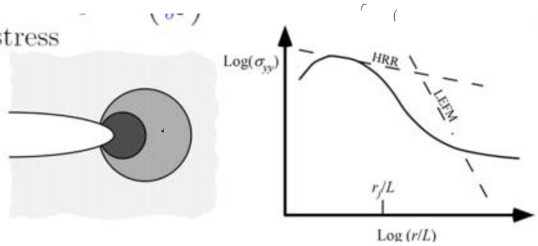
$$\boxed{\frac{r_p}{r_s} \propto \left(\frac{\bar{\sigma}}{\sigma_y}\right)^2 \ll 1}$$

LEFM X

LEFM valid $\frac{r_p}{L} \ll 1$ Remember $\bar{\sigma} \lessapprox 0.1\sigma_y$

LEFM valid $\quad \frac{r_p}{r_s} \ll 1$ (eg $0.01 \equiv \bar{\sigma} \approx 0.1\sigma_y$)

$\bar{\sigma}$: applied stress



Elastic plastic condition

$r_n \ll r_p \approx r_s \quad \Rightarrow$

$$\boxed{\frac{r_n}{r_p} \ll 1, \quad \underbrace{\frac{r_p}{r_s} \propto \left(\frac{\bar{\sigma}}{\sigma_y}\right)^2 \approx 1}_{}}$$

SSY no longer holds

$\Longrightarrow$ LEFM is invalid

K or G (LEFM)

In this case we can use J to get local stress solution:

- Final form of HRR solution:

$$\sigma_{ij} = \sigma_0 \left(\frac{EJ}{\alpha\sigma_0^2 I_n r}\right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}(n,\theta)$$

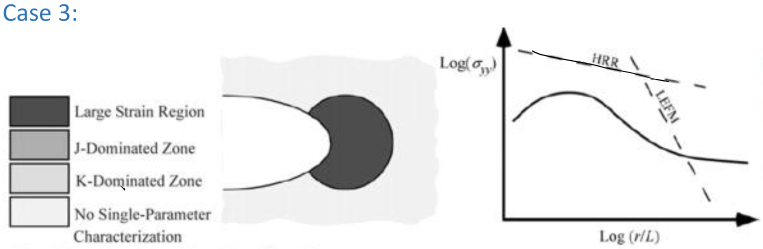
$$\epsilon_{ij} = \frac{\alpha\sigma_0}{E}\left(\frac{EJ}{\alpha\sigma_0^2 I_n r}\right)^{\frac{n}{n+1}} \tilde{\epsilon}_{ij}(n,\theta)$$

J

J plays the role of K for local σ, ε, u fields

AND J itself is still the energy release rate

Case 3:



LSY(Large Scale Yielding)

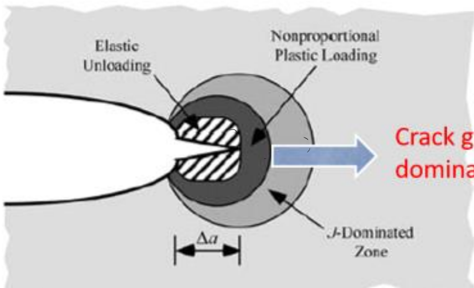
$r_n \approx r_p$

Note that $\frac{\delta}{r_p} \propto \left(\frac{\sigma_y}{E}\right)$

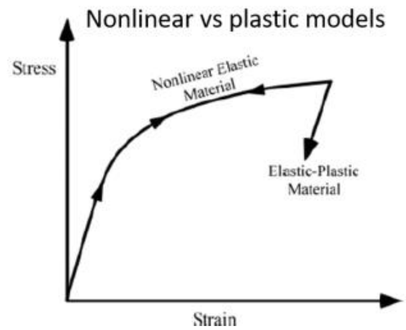
When we end up with case 3 (Large scale yielding that cannot be characterized with J alone)?

LSY: When a single parameter (G, K, J, CTOD) is not enough?

- Under considerable plastic deformation and crack propagation when unloading and non-proportional zones grow out of J dominant zone with crack propagation. Reasons are:
  - Unloading: In J integral analysis plastic model was replaced by a nonlinear solid
  - Single-parameter identification not valid since various stress components increase at different rates

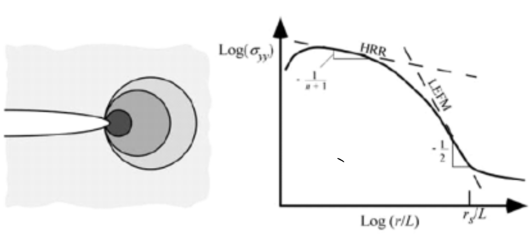


Crack growing out of J-dominant zone



249

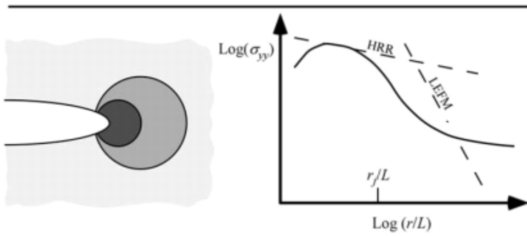
From SSY to LSY



LEFM: SSY satisfied and gener have Generally have

$(\bar{\sigma})^2 \ll (\sigma_y)^2 \Rightarrow r_p \ll r_s$

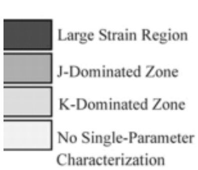
Relevant parameters:
G (energy) K (stress)

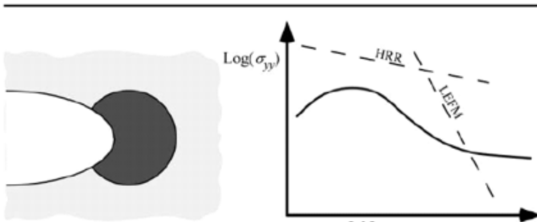


PFM (or NFM): SSY is gradually violated and

$\bar{\sigma} \approx \sigma_y$

Relevant parameters:
J (energy & used for stress)





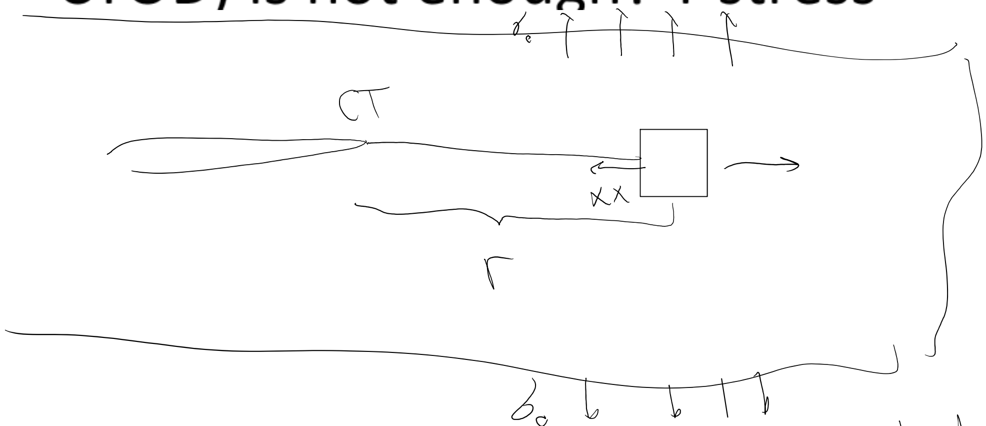
248 248

LSY condition:
No single parameter can characterize fracutre!
J + other parameters (e.g. T stress, Q-J, etc)

Aside from the role of T stress in LSY, T stress is quite important in fracture mechanics.

What's T stress?

LSY: When a single parameter (G, K, J, CTOD) is not enough? T stress



$$\sigma_{xx}(r) = \frac{K_I}{\sqrt{2\pi r}} f_{xx}(\theta) + T$$

constant stress for σ_{xx}

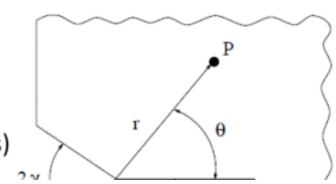
HOT

Crack solution using V-Notch

- Sharp crack

$$\alpha = \pi \Rightarrow \begin{aligned} \sin(2\pi\lambda_n) &= 0 \\ \sin(2\pi\xi_n) &= 0 \end{aligned} \Rightarrow$$

$\lambda_n = \frac{n}{2}$ with $n = 1, 3, 4, \cdots$ (n = 2 constant stress)



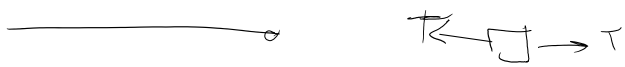
similar to

$$f(x+\Delta x) = f(x) + \Delta x f'(x) + \frac{\Delta x^2}{2} f''(x) + \ldots$$

T stress is the 2nd term is the asymptotic expansion of σ_{xx}

& it's a constant stress

$$\sigma_{xx}(r,\theta) = \frac{K_I}{\sqrt{2\pi r}} f_{xx}(\theta) + T$$

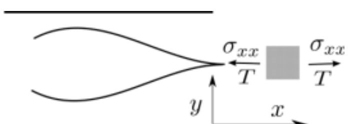


T: (tensile)

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) + \begin{bmatrix} T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \nu T \end{bmatrix}$$

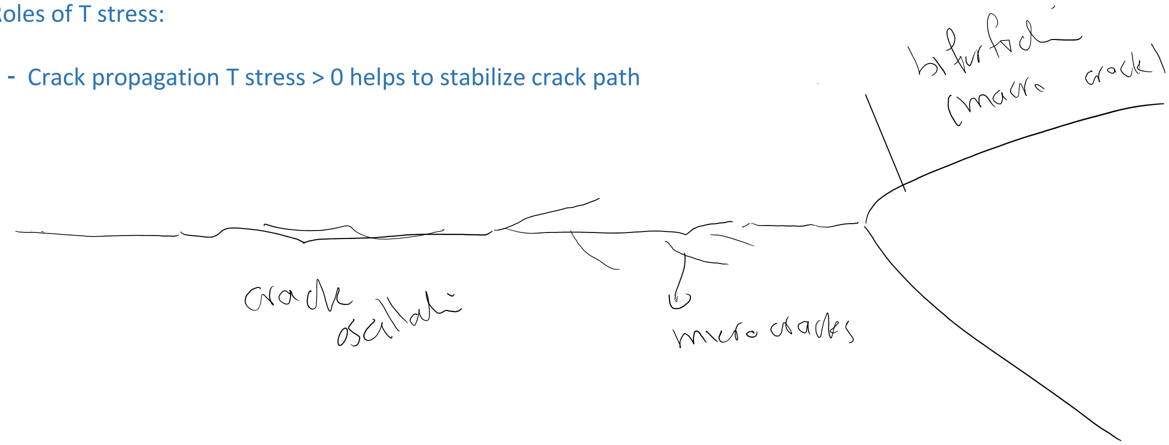
$\nu T \rightarrow$ plane strain

$\cancel{B = \frac{T\sqrt{\pi a}}{K_I}}$



Roles of T stress:

- Crack propagation T stress > 0 helps to stabilize crack path



Crack extends (crack speed increases)

crack becomes more unstable

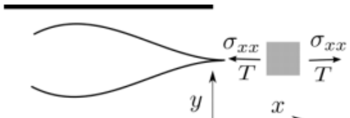
T stress ↗ crack path becomes more stable

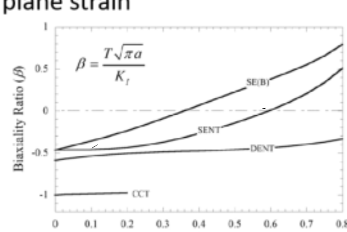
Second role that is relevant to the current discussion:

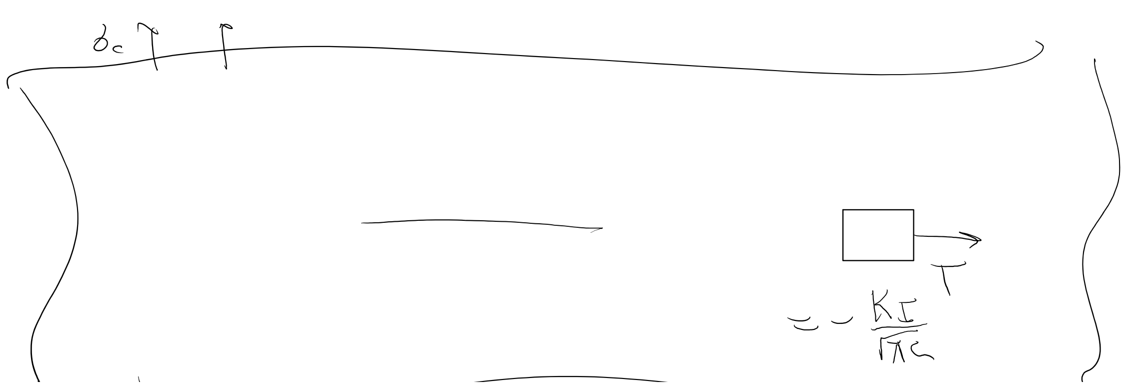
- Higher order terms in stress expansion:
  - T stress (linear analysis)
    $$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) + \begin{bmatrix} T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \nu T \end{bmatrix}$$
    plane strain
    - Constant σ_{xx} in LEFM expansion
    - Nondimensional biaxiality ratio: $\beta = \frac{T\sqrt{\pi a}}{K_I}$
    - Example $\beta = -1$ for mode-I crack in infinite domain.
    - T stress redistributes plastic stress
    - $\beta(T)$ depend on particular geometry/loading configuration
    - Effect of $T(\beta)$ on toughness:

      High (+) $T \Rightarrow$ Constrained (triaxial) stress $\Rightarrow$ Toughness ↘ Ductility ↘

      Low (-) $T \Rightarrow$ Lose constraint $\Rightarrow$ Toughness ↗ Ductility ↗
    - T stress also influnces crack path stability (particulary in dynamic fracture)

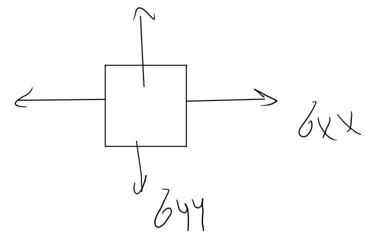






$= -\frac{K_I}{\sqrt{\pi a}}$

T stress
tensile



σxx & σyy are
closer