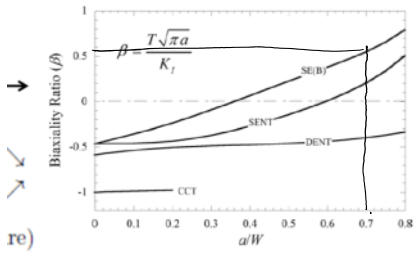
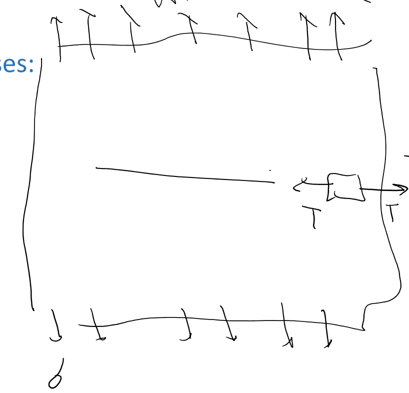
FM20241104

Monday, November 4, 2024 9:52 AM

Different crack geometries have different T stresses:



infinite domain
and crack



$\beta = -1$

$\beta = \frac{T\sqrt{\pi a}}{K_I}$ $\beta = -1$

SEB

for the same K

crack grows easier for this geometry

$\beta \nearrow$ σ_{11} & σ_{22} get closer $\Rightarrow$ more difficult to yield

$\frac{a}{W} = .7 \Rightarrow \beta = +.052$

this has a lower R

$$\boxed{T \text{ (or } \beta) \nearrow \Rightarrow \text{more biaxial loading} \Rightarrow R \text{ (toughness)} \searrow}$$

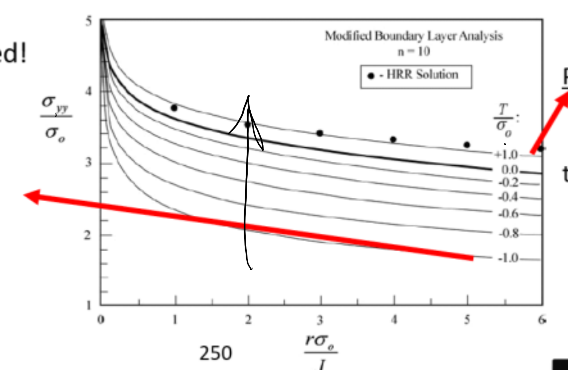
What's the effect of T (beta) in terms of local stress distribution:

Plastic analysis: σ_{yy} is redistributed!

Kirk, Dodds, Anderson

High negative T stress:

- Decreases σ_{yy}
- Decreases triaxiality



Positive T stress:

- Slightly Increases σ_{yy} and increase triaxiality

LSY: When a single parameter (G, K, J, CTOD) is not enough? J-Q theory

- Q parameter (J-Q theory) Valid for nonlinear analysis
  - Added as a hydrostatic shift in front of crack to (HRR) stress fields

$$\sigma_{ij} \approx (\sigma_{ij})_{T=0} + Q\sigma_0\delta_{ij} \qquad \left(|\theta| \le \frac{\pi}{2}\right)$$

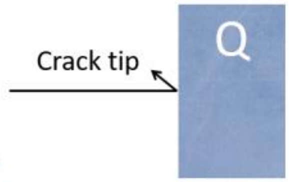
  - Similar to T positive Q increases triaxiality and reduces fracture resistance

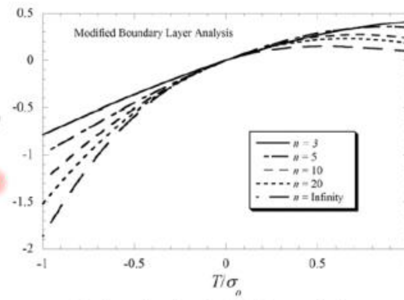
$$J_c = J_c(Q)$$

High (+) $Q \Rightarrow$ Constrained (triaxial) stress $\Rightarrow$ Toughness ↘ Ductility ↘

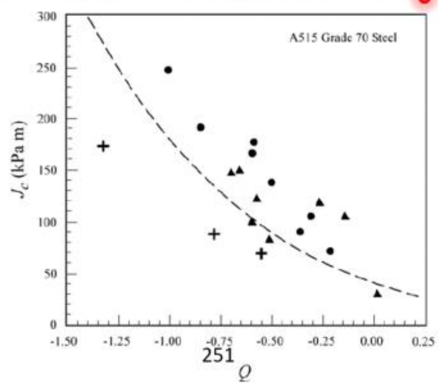
Low (-) $Q \Rightarrow$ Lose constraint $\Rightarrow$ Toughness ↗ Ductility ↗

- More number of parameters: With extensive deformation two-parameter models such as K, T or J, Q evnetully break.





n: strain hardening in HRR analysis



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6.1 Fracture mechanics in Finite Element Methods (FEM)

6.1.1. Introduction to Finite Element method

6.1.2. Singular stress finite elements

6.1.3. Extraction of K (SIF), G

6.1.4. J integral

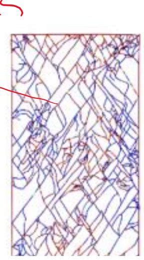
6.1.5. Finite Element mesh design for fracture mechanics

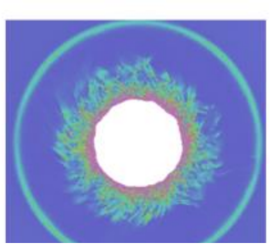
6.1.6. Computational crack growth

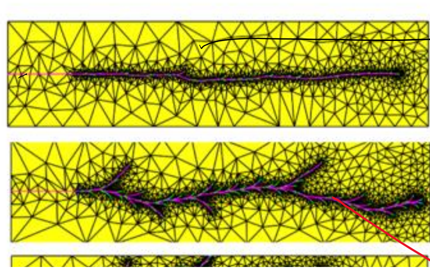
6.1.7. Extended Finite Element Method (XFEM)

Sharp interface models

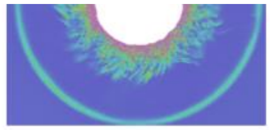
cracks

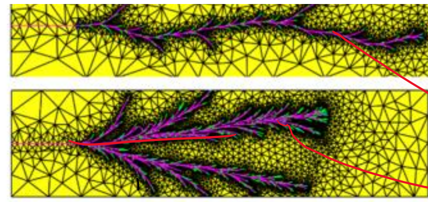




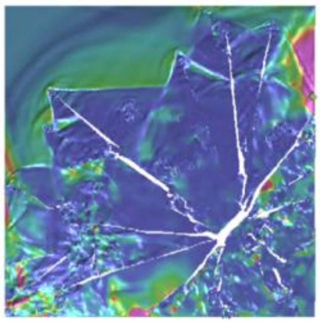


elements





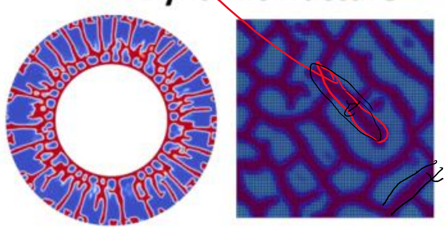
cracks



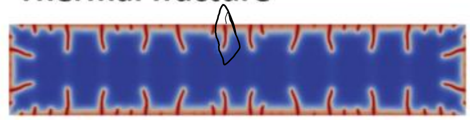
approximate a crack

Bulk models

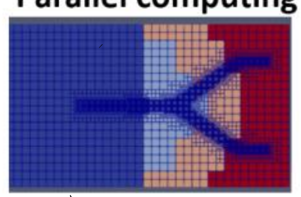
Dynamic fracture



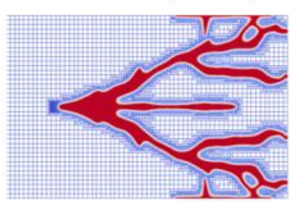
Thermal fracture



Parallel computing

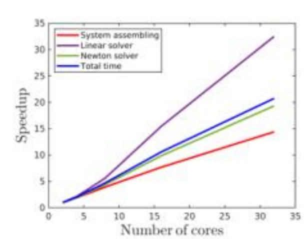


Mesh adaptivity



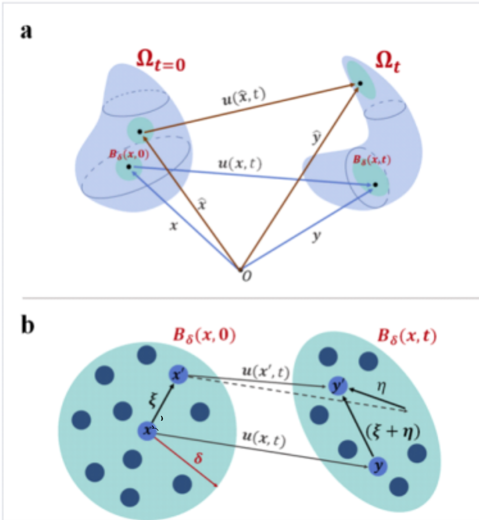
crack has thickness in these models

Credit: Giang Huynh

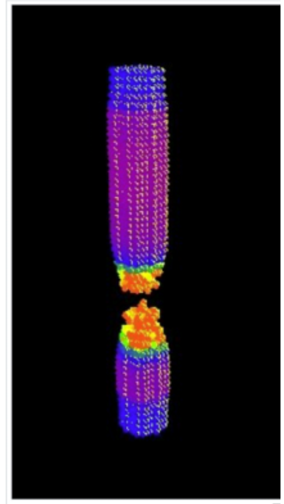


- Continuum (bulk) damage model
- Phase field

Particle models



(a) Kinematics of material body Ω_t within peridynamic theory. (b) Representation of peridynamic horizon of $\mathbf{x}$.



Computer model of the necking of an aluminum rod under tension Colors indicate temperature increase due to plastic heating. Calculation performed with the Emu computer code using peridynamic state-based framework.

https://en.wikipedia.org/wiki/Peridynamicss

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* Peridynamics

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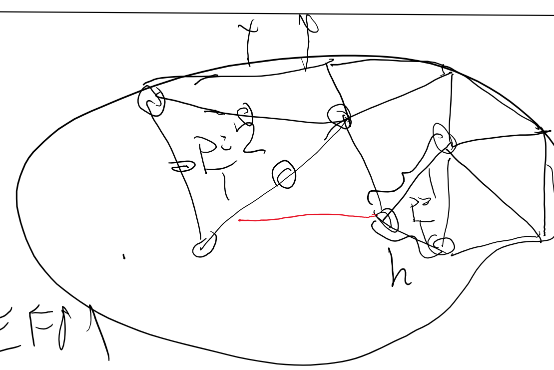
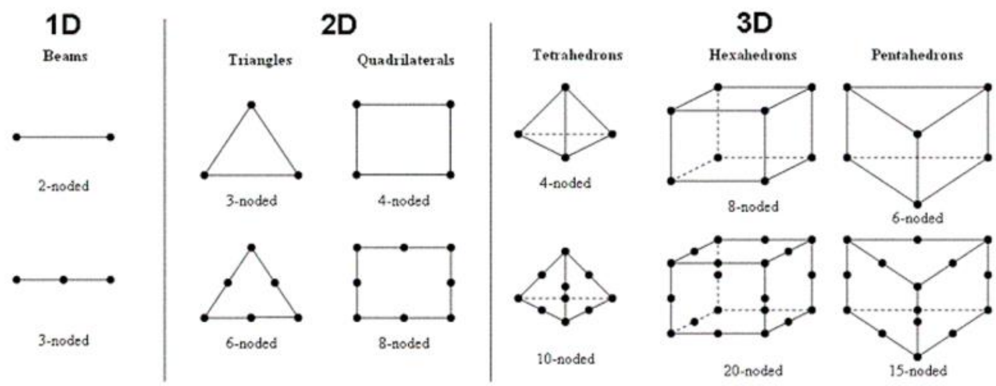
h smaller

<

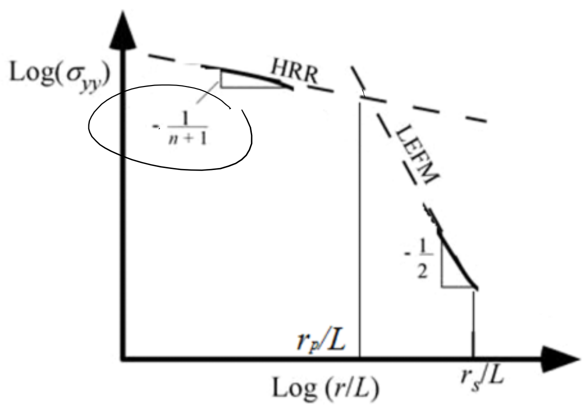
Process length scale relevant

{ r_s LEFM
{ r_p PFM



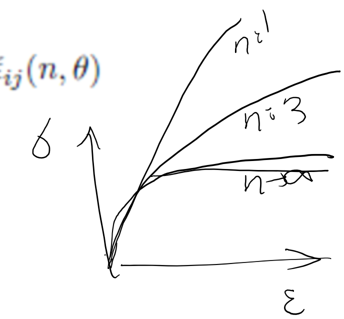


LEFM $\varepsilon, \sigma \propto r^{-\frac{1}{2}}$



$$\sigma_{ij} = \sigma_0 \left(\frac{EJ}{\alpha \sigma_0^2 I_n r} \right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}(n, \theta)$$

$$\epsilon_{ij} = \frac{\alpha \sigma_0}{E} \left(\frac{EJ}{\alpha \sigma_0^2 I_n r} \right)^{\frac{n}{n+1}} \tilde{\epsilon}_{ij}(n, \theta)$$

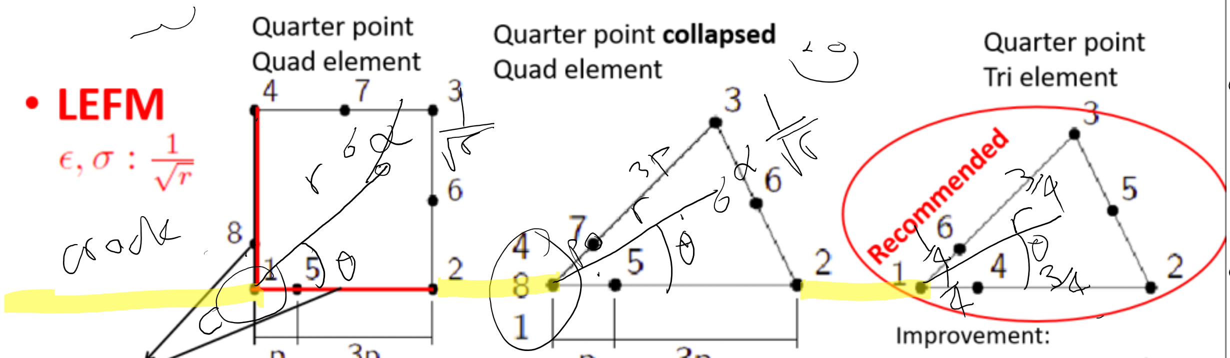


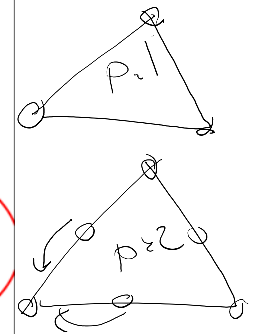
- NLFM (PFM): For HRR solution stress $\frac{1}{r^{\frac{1}{n+1}}}$ and strain $\frac{1}{r^{\frac{n}{n+1}}}$ are still singular $\Rightarrow$
  - for elastic-perfectly plastic ($n \to \infty$) stress is bounded and strain is $\frac{1}{r}$ singular

Isoparametric singular elements

- **LEFM**

$\epsilon, \sigma : \frac{1}{\sqrt{r}}$





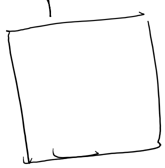
singular form $\frac{1}{\sqrt{r}}$ only along these lines
NOT recommended

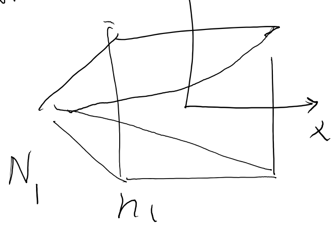
Improvement:
- $\frac{1}{\sqrt{r}}$ from inside all element

Problem
- Solution inaccuracy and sensitivity when opposite edge 3-6-2 is curved

Improvement:
- Better accuracy and less mesh sensitivity

p=1 element



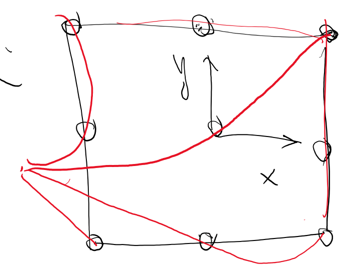


solution is a linear combination of

1

$x \quad y$

xy



1

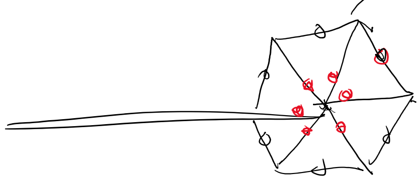
$x \quad y$

$x^2 \quad xy \quad y^2$

$x^2y \quad xy^2$

x^2y^2

LEFM



creates

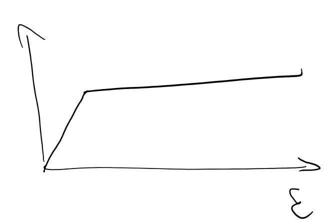
$\sigma, \varepsilon \propto \frac{1}{\sqrt{r}}$

around the crack tip

Elastic Perfectly plastic

$\varepsilon \propto \frac{1}{r}$

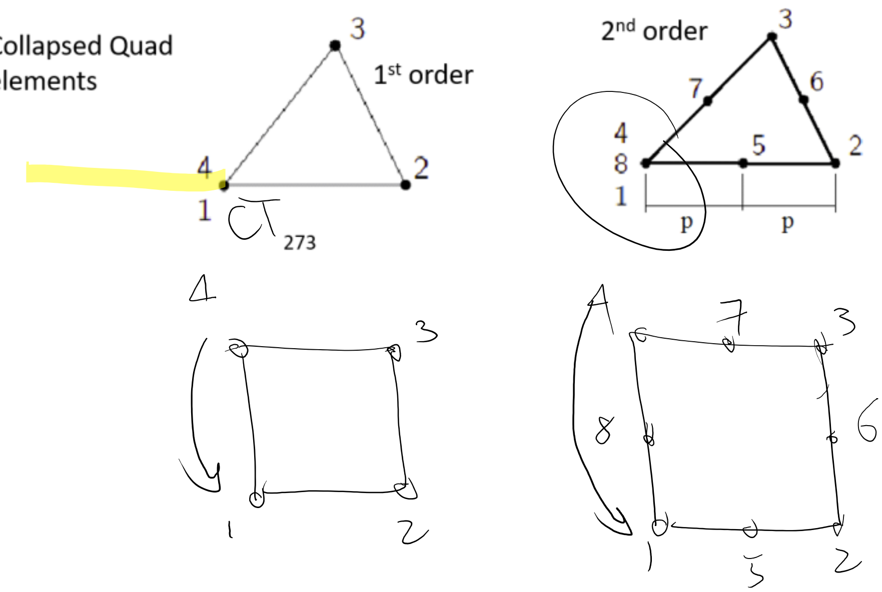
σ bounded



- **Elastic-perfectly plastic**

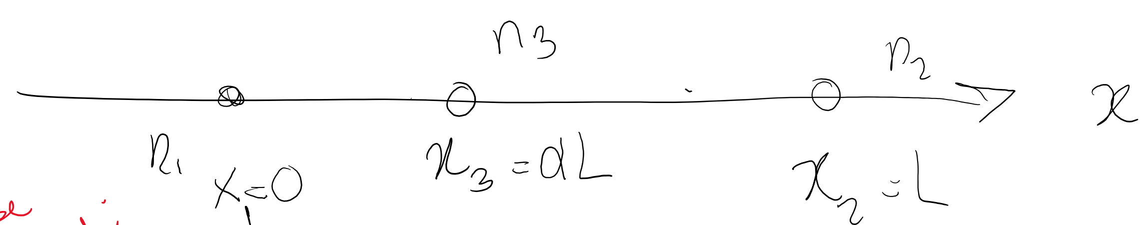
$\epsilon : \frac{1}{r}$

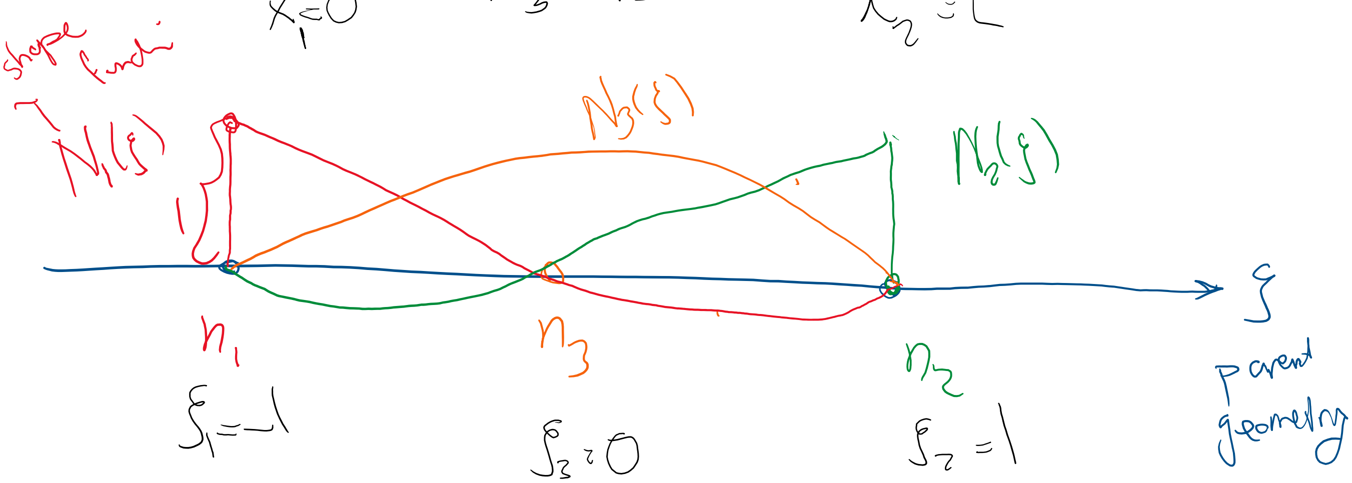
Collapsed Quad elements



Motivation:
Why 1/4, 3/4 positions generate 1/sqrt(r) singularity?

$\alpha =$





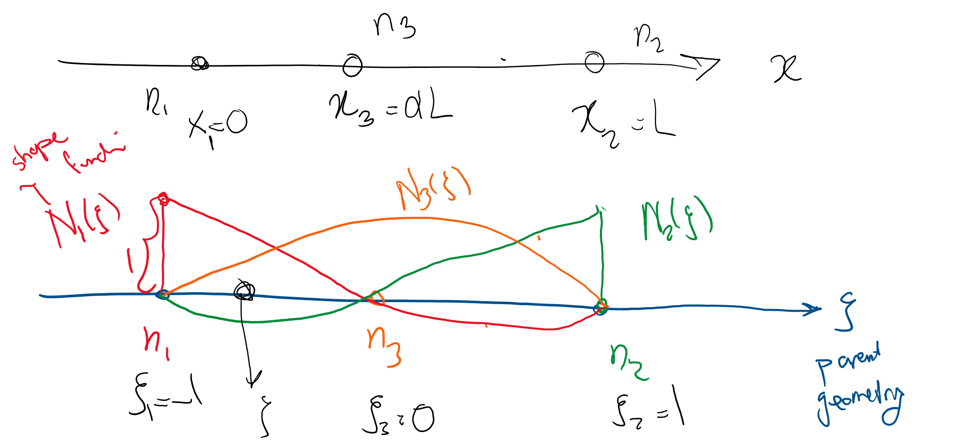
$$N_1(\xi) = \frac{(\xi - \xi_2)(\xi - \xi_3)}{(\xi_1 - \xi_2)(\xi_1 - \xi_3)} = \frac{\xi(\xi - 1)}{2}$$

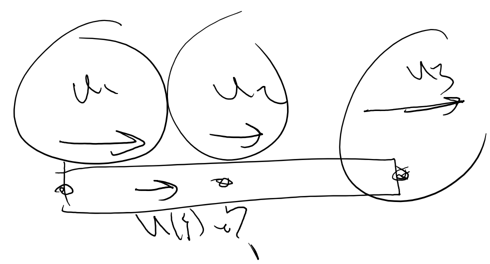
$$\begin{cases} N_1(\xi = -1) = 1 \\ N_1(\xi = 1) = 0 \\ N_1(\xi = 0) = 0 \end{cases}$$

$$N_2(\xi) = \frac{(\xi - \xi_1)(\xi - \xi_3)}{(\xi_2 - \xi_1)(\xi_2 - \xi_3)} = \frac{\xi(\xi + 1)}{2}$$

$$N_2(\xi) = \frac{(\xi-\xi_1)(\xi-\xi_3)}{(\xi_2-\xi_1)(\xi_2-\xi_3)} = \frac{\xi(\xi+1)}{2}$$

$$N_3(\xi) = \frac{(\xi-\xi_1)(\xi-\xi_2)}{(\xi_3-\xi_1)(\xi_3-\xi_2)} = 1-\xi^2$$



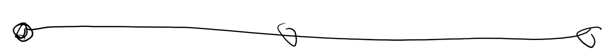


$$u(\xi) = u_1 N_1(\xi) + u_2 N_2(\xi) + u_3 N_3(\xi)$$

displacement

$$\varepsilon = \frac{du(\xi)}{dx} = \frac{du(\xi)}{d\xi}\frac{d\xi}{dx}$$

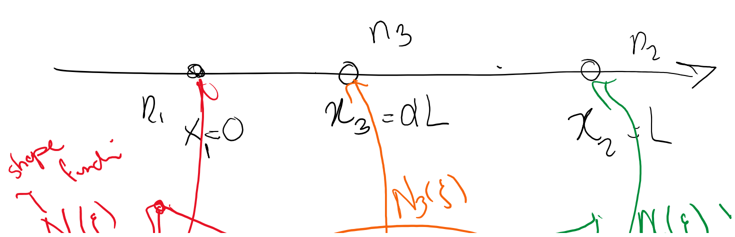
strain



$$\varepsilon(\xi=-1) = \infty$$

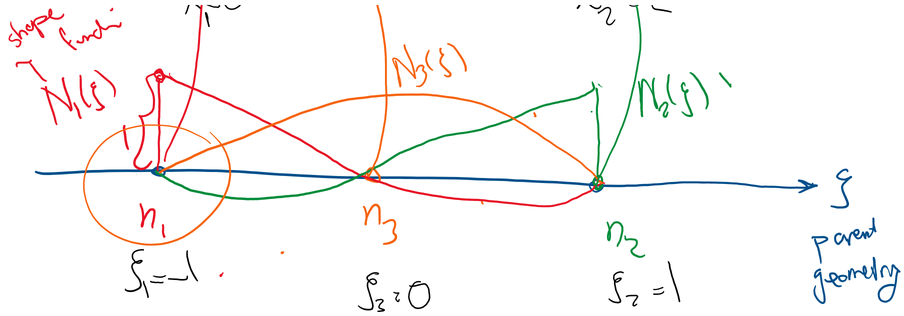
$$\boxed{\varepsilon = \frac{du(\xi)}{d\xi} \Bigg/ \frac{dx(\xi)}{d\xi}}$$

we need to write $x(\xi)$



$\xi = \xi_1 = -1 \Rightarrow x = x_1 = 0$

$\xi = \xi_2 = 1 \Rightarrow x = x_2 = L$



$$\xi = \xi_2 = 1 \Rightarrow x = x_2 = L$$

$$\xi = \xi_3 = 0 \Rightarrow x = x_3 = \alpha L$$

$$x(\xi) = x_1 N_1(\xi) + x_2 N_2(\xi) + x_3 N_3(\xi)$$

check this

$$x(\xi=\xi_1) = x_1 \underbrace{N_1(\xi_1)}_{1} + x_2 \underbrace{N_2(\xi_1)}_{0} + x_3 \underbrace{N_3(\xi_1)}_{0}$$

$$= x_1 \checkmark$$

easily verify $x(\xi=\xi_2) = x_2$

$x(\xi=\xi_3) = x_3$

$$u(\xi) = u_1 N_1(\xi) + u_2 N_2(\xi) + u_3 N_3(\xi)$$

$$x(\xi) = x_1 N_1(\xi) + x_2 N_2(\xi) + x_3 N_3(\xi)$$