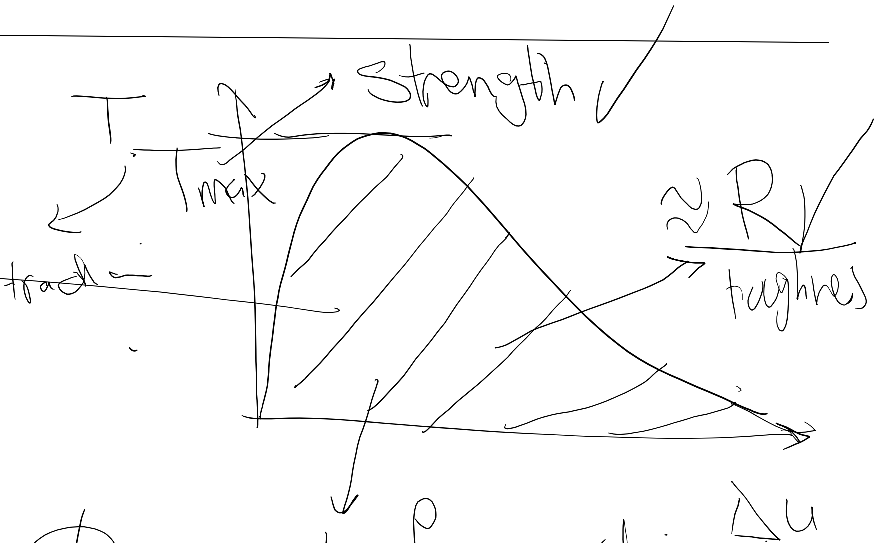
FM20241120

Wednesday, November 20, 2024 9:43 AM

$[\sigma][L]$



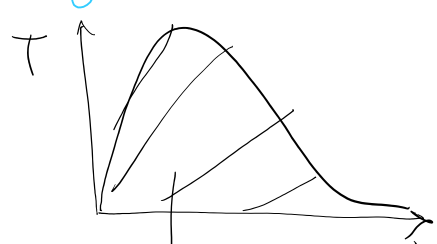
Φ = work of separation

$G = R \rightarrow [R] = [\sigma][L]$ Pa·m

$$\left[\frac{\text{energy}}{\text{Unit area}}\right] = \frac{F \cdot L}{L^2} = \underbrace{\left[\frac{F}{L^2}\right]}_{\sigma}[L]$$

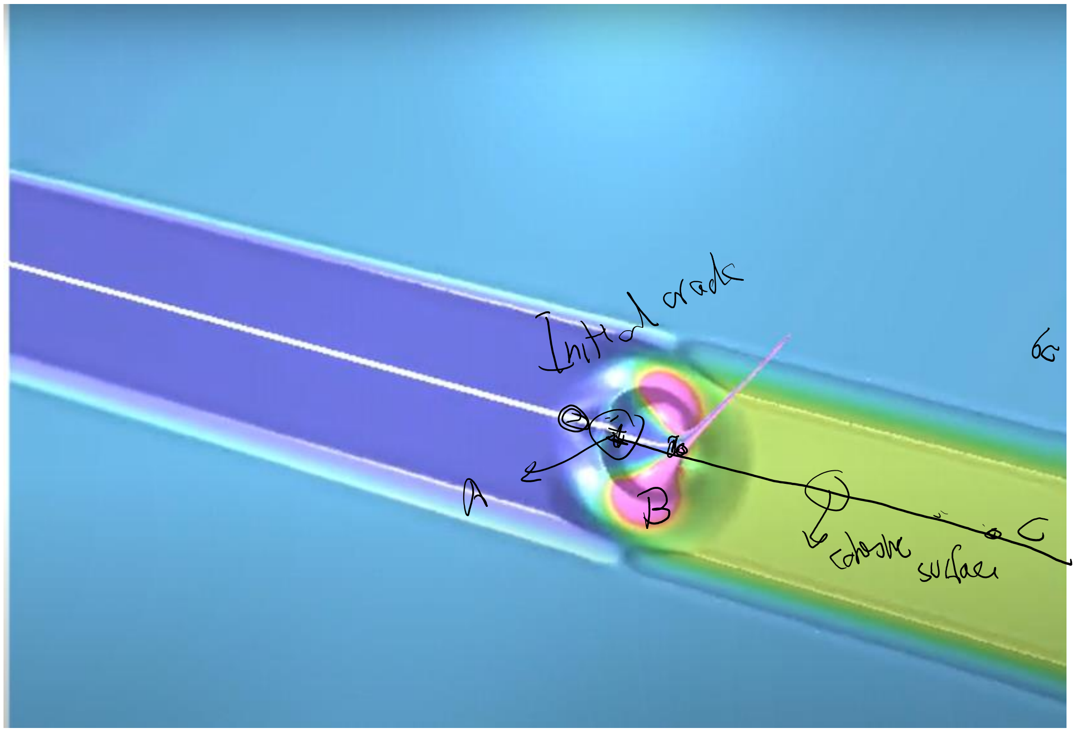
LEFM

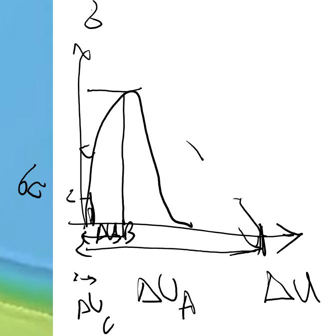
$G = R$



Φ

are related $[\sigma][L]$

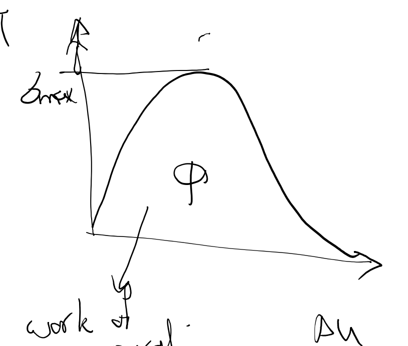




LEFM

$G = R \to [\sigma][L]$

strength ←



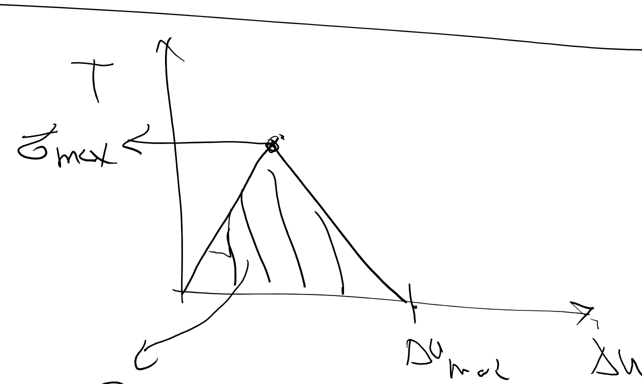
work of separation

$[\sigma][L]$ energy quantity

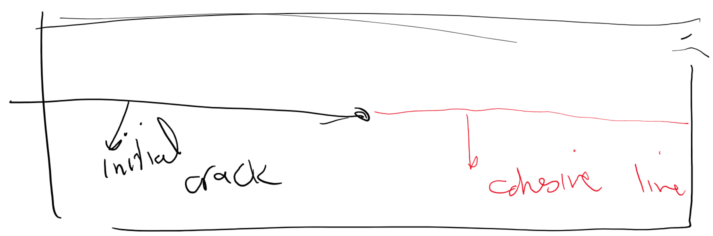
we generally set $\underline{\underline{\phi = R}}$

R

σ_{max}

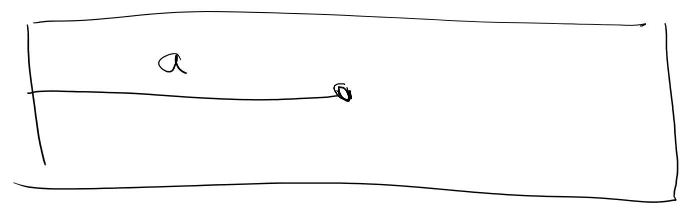


$$\frac{\Delta u_{max}\,\sigma_{max}}{2} = \phi = R$$



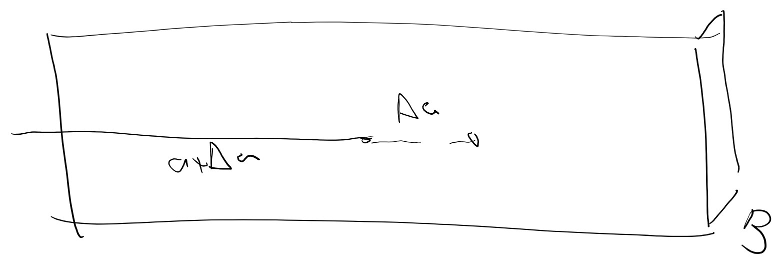
R meaning

LEFM t_0



energy release from t_0 to t_1

t_1



added area $\Delta a B$

$(\Delta a B) R \longrightarrow$ energy released with LEFM

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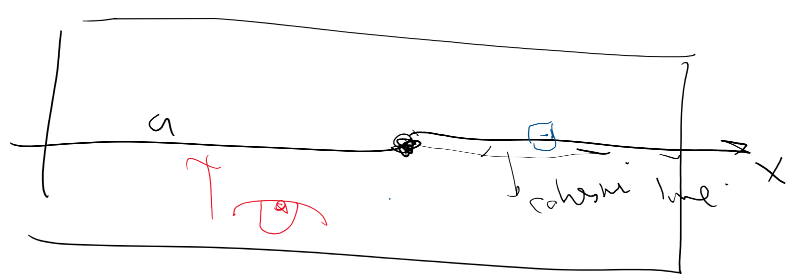
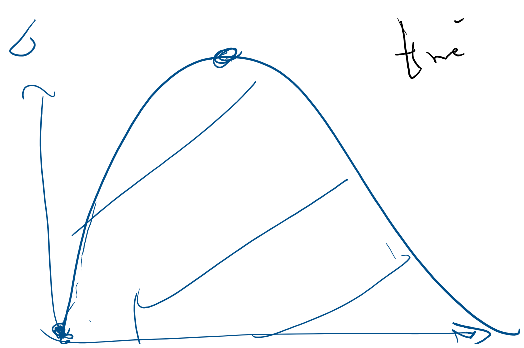
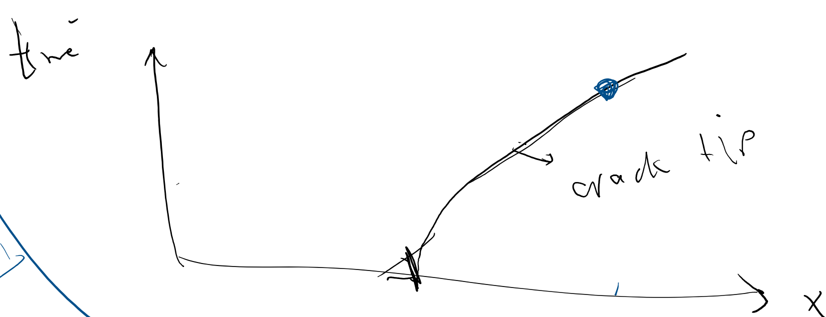
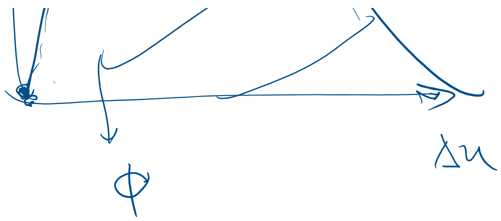
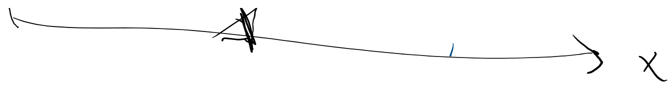
with cohesive models we won't get $(\Delta a B)\cancel{R}\ \phi$ energy released per Δa crack advances

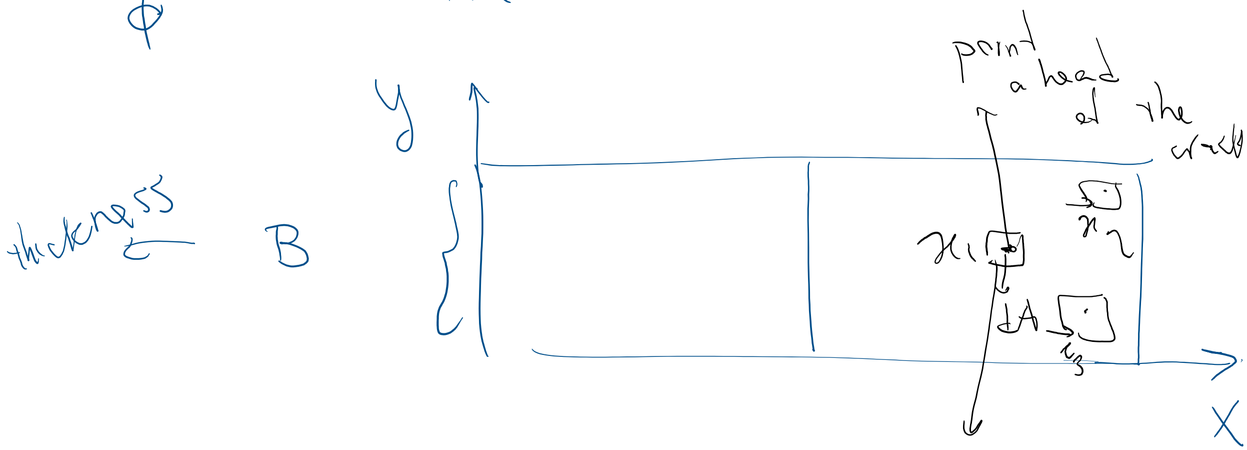










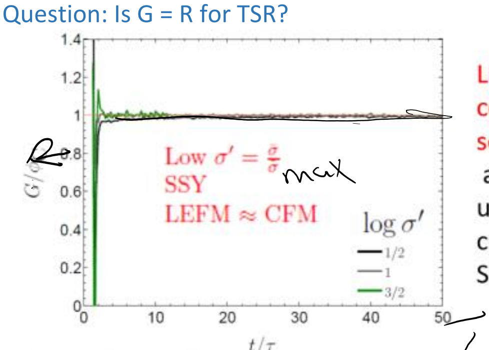


Φ is energy released per surface area at any point as the point transitions from bonded to fully debonded

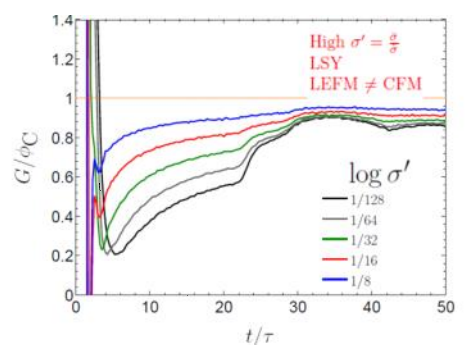
Φ

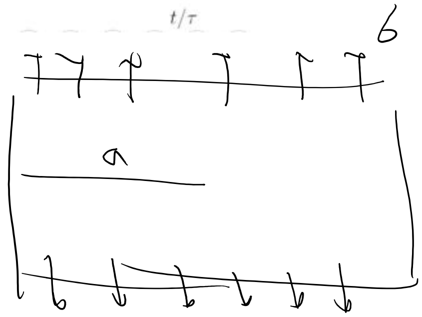
LEFM says G = R
Cohesive model does not have R, but we set phi = R

Question: Is G = R for TSR?



LEFM CFM comparison: set $\Gamma = \phi$ accurate except unsteady / low crack speed OR if SSY is not satisfied

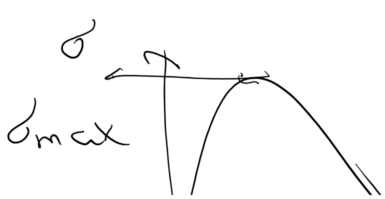


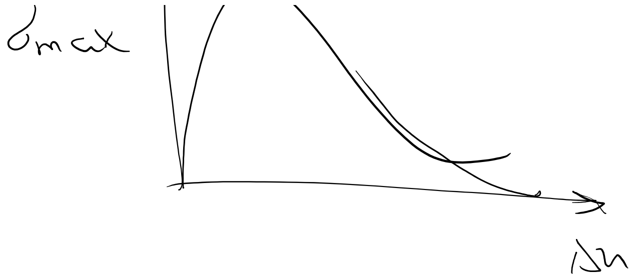


$\frac{\bar{\sigma}}{\sigma_{max}}$ is low

$\frac{\bar{\sigma}}{\sigma_{max}}$ high

SSY does not hold

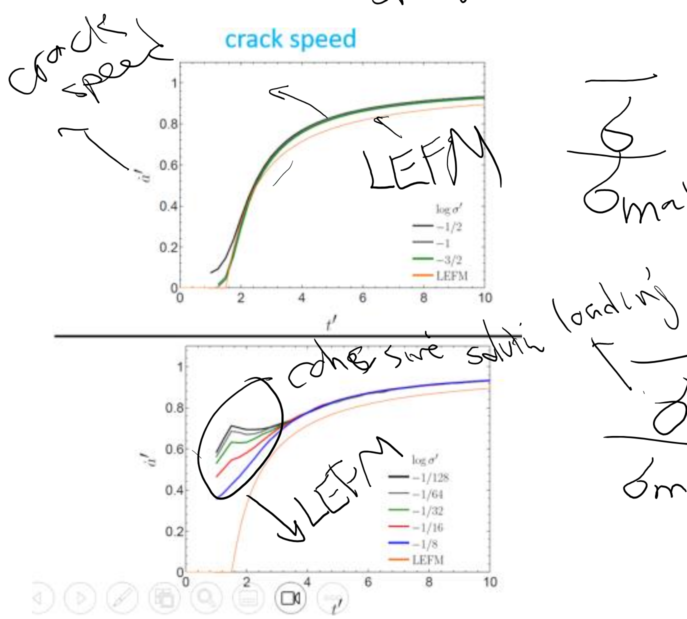




SSY condition

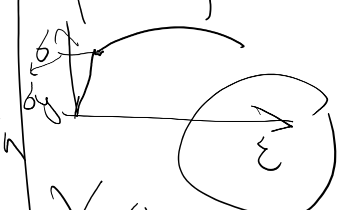
$$\left(\frac{r_p}{r_s}\right) \propto \left(\frac{\bar{\sigma}}{\sigma_{max}}\right)^2$$

σ_{max} ↓ strength



$\frac{\bar{\sigma}}{\sigma_{max}}$ is low, SSY is satisfied

$\frac{\bar{\sigma}}{\sigma_{max}}$ high, SSY is not satisfied

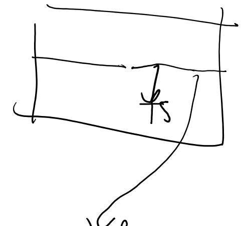
	LEFM	TSR	plasticity
			
energy / area	✓ (R)	✓ (φ)	X (but they have energy diss per Volume)
strength	—	σ_{max}	✓ (σ_y, σ_{max})

necessary … bulk

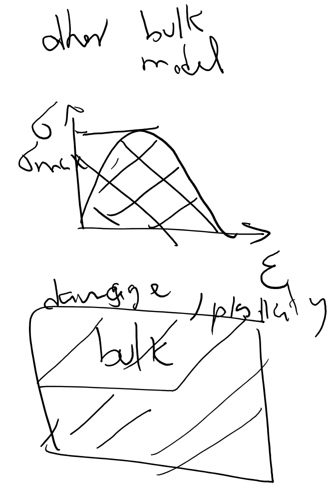
necessary condition

$$\frac{\sigma_{applied}}{\sigma_{max}} \lesssim 0.1 \text{ or even } .3$$

fail

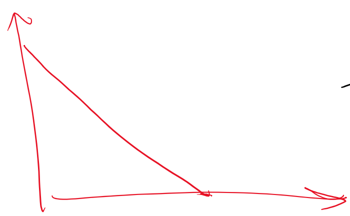


other bulk model

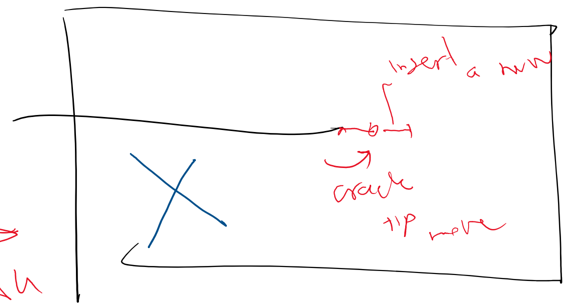


Extrinsic vs. intrinsic

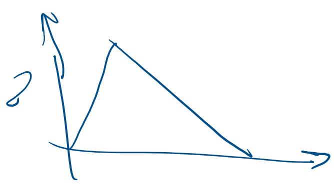
extrinsic: σ

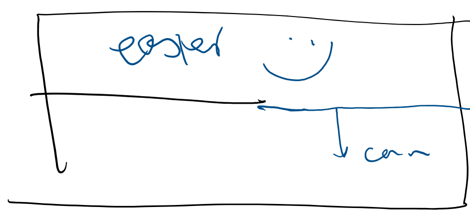


difficult to implement



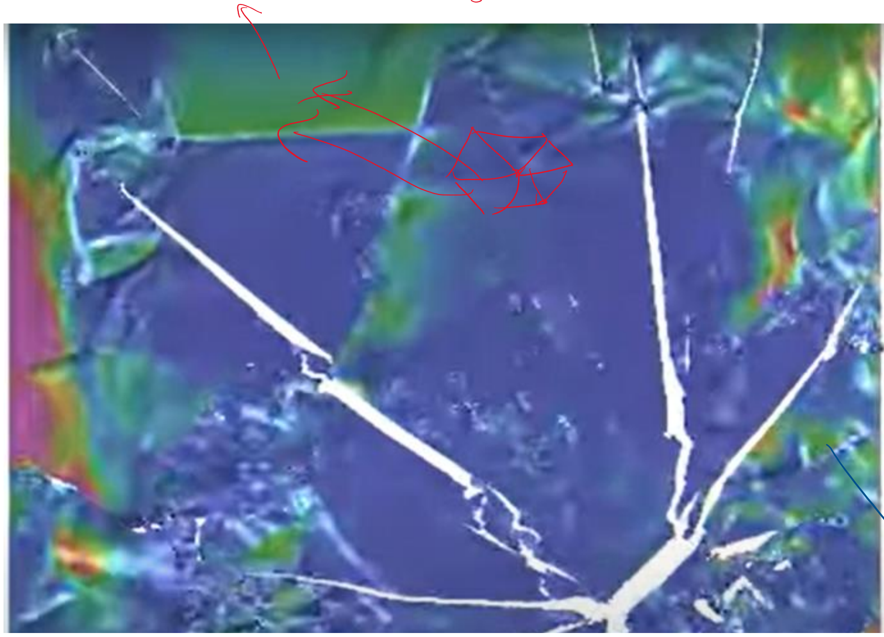
intrinsic





leave it there from the beginning

cohesive surface between all elements

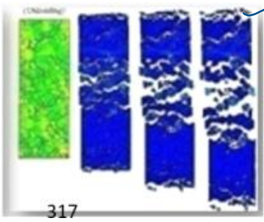


intrinsic

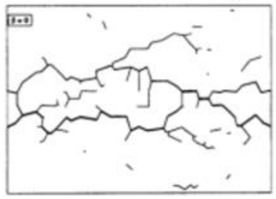
~~intrinsic~~

extrinsic

fragmentation



Microcracking and branching



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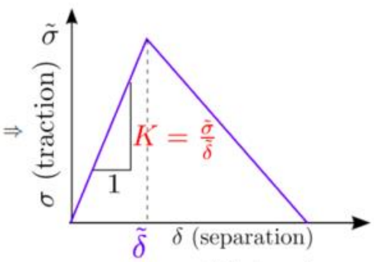
Artificial compliance for intrinsic cohesive models

- Artificial compliance becomes important if cohesive surfaces are added between all elements for intrinsic models to find crack propagation path.
- The artifical compliance is computed as,

$$\Delta = \Delta_e + \Delta_c, \quad \Delta_e = \text{elastic displacement}, \ \Delta_c = \text{cohesive separation} \Rightarrow$$

$$\frac{\sigma}{E_{\text{eff}}}h = \frac{\sigma}{E}h + \frac{\sigma}{K} \Rightarrow$$

$$\frac{1}{E_{\text{eff}}} = \frac{1}{E} + \frac{1}{Kh} \Rightarrow$$



Artificial compliance is,

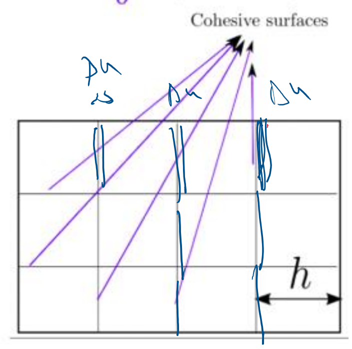
stiffness from cohesive model

material gets softer

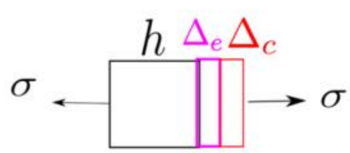
$$C_e = \frac{1}{Kh} = \frac{\tilde{\delta}}{\tilde{\sigma}h} = \frac{1}{E_c}, \text{ where}$$

$$E_c = Kh = \frac{\tilde{\sigma}h}{\tilde{\delta}}, \text{ and effective elastic modulus is}$$

$$\frac{1}{E_{\text{eff}}} = \frac{1}{E} + \frac{1}{E_c}, \quad \Rightarrow E_{\text{eff}} = \frac{EE_c}{E + E_c} < E$$



- That is the smaller element spacing h or softer the initial slope K of TSR the higher artificial compliance (higher errors)
- While extrinsic cohesive models do not have the same problem, adaptive insertion of cohesive surfaces is more challenging for them.



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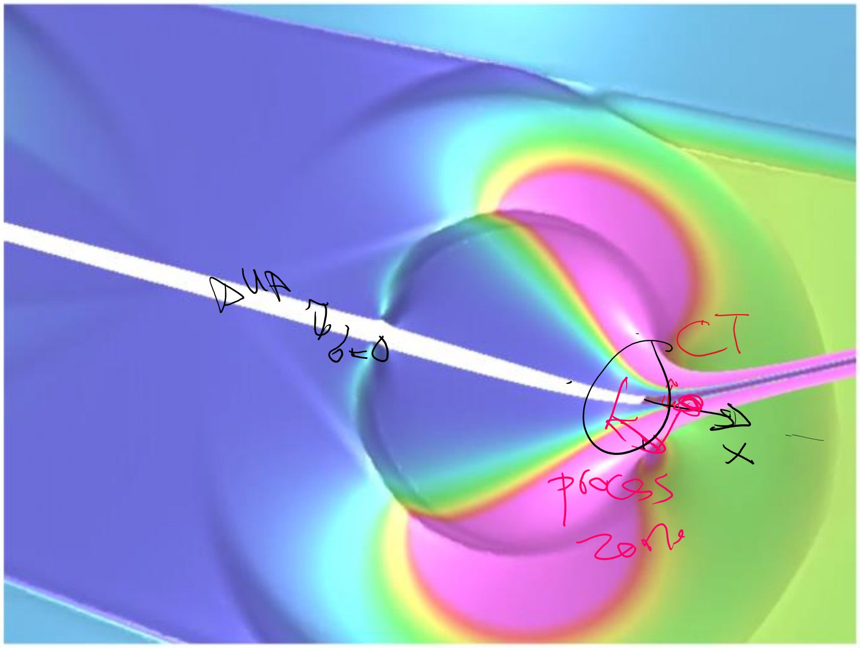
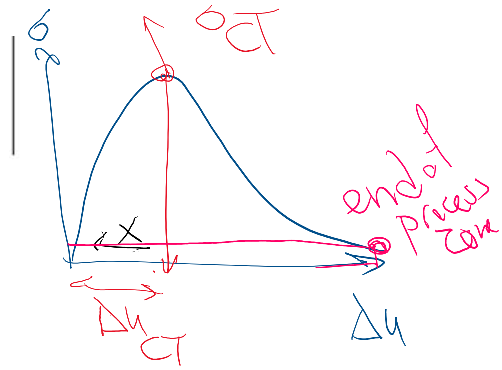
$h \rightarrow 0$ $E_{\text{eff}} \rightarrow 0$

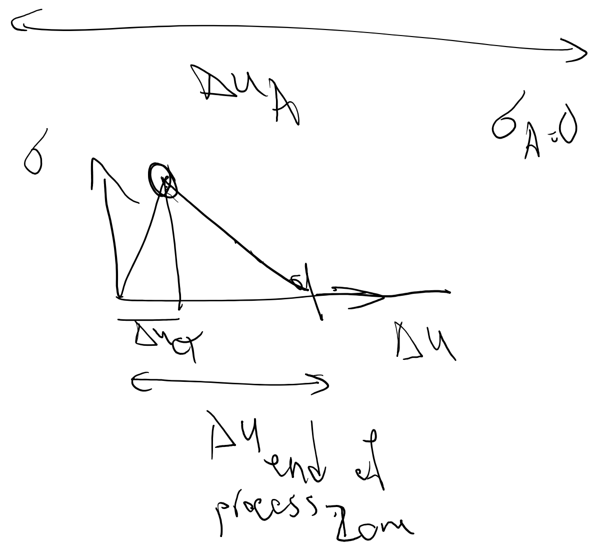
Crack tip and process zone

σ σ_{CT}

Crack tip and process zone



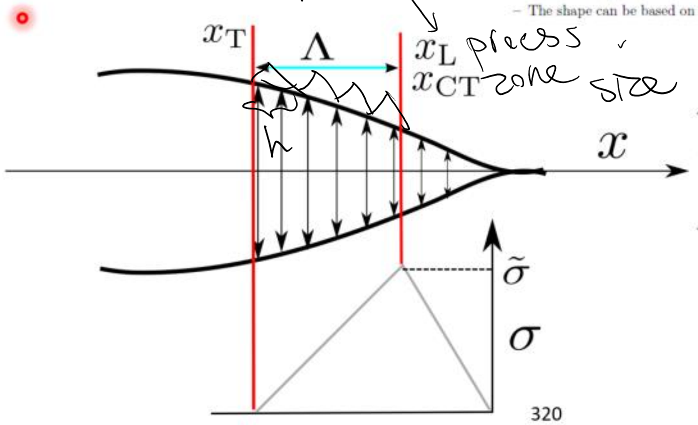


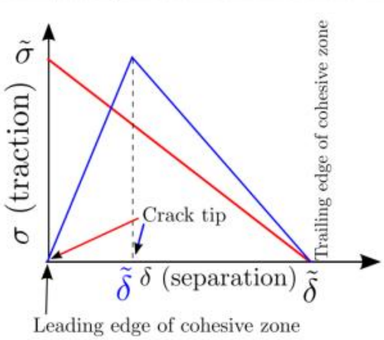


- Process zone Important Points and lengths
  - Nominal crack tip x_{CT} Generally corresponds to the point with maximum traction ($\tilde{\sigma}$)
  - Nominal trailing edge of the process zone x_{T}: The point where traction goes to zero (or if asymptotically goes to zero taken when stress is arbitrary small *e.g.*, 0.01 or 0.001 *sigma*.
  - Nominal leading edge of the process zone x_{L}: When general crack-like (*e.g.*, highly nonlinear response) starts. Often, x_{L} is set to x_{CT}.
  - Process zone size $\Lambda = |x_{\mathrm{L}} - x_{\mathrm{T}}|$: Characteristic length scale corresponding to cohesive models ($\tilde{L}$)

h < Λ

- Cohesive model shape
  - Can have important influence on the response of cohesive model
  - The shape can be based on ductile/brittle response of TSR and can make it intrinsic or extrinsic





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Why process zone size is important?

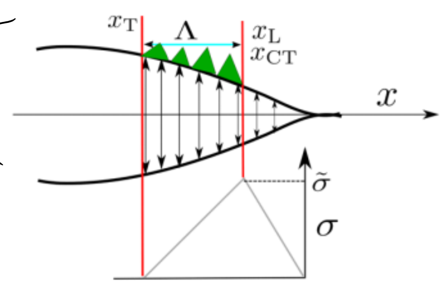
- Importance of process zone size Λ
  - Static estimate:

$$\Lambda = \varsigma\pi\frac{\mu}{1-\nu}\frac{\tilde{\phi}}{\tilde{\sigma}^2} \propto \tilde{L}$$

$$\varsigma = \begin{cases}\frac{1}{4} & \text{Dugdale model}\\ \frac{9}{16} & \text{Potential-based TSRs}\end{cases}$$

work of separation

strength

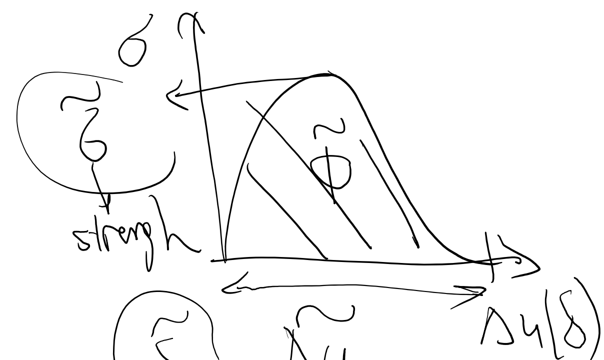


$$h = \frac{1}{6}\Lambda$$

$$\Lambda \propto \frac{E}{\tilde{\sigma}^2}\tilde{\phi}$$

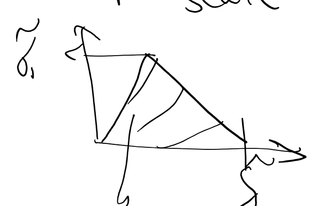
$$\Lambda \propto \frac{E}{\tilde{\sigma}^2}\left(f\tilde{\sigma}\,\tilde{\delta}\right)$$

some factor



$\tilde{\delta} = \Delta\tilde{u}$

separation scale



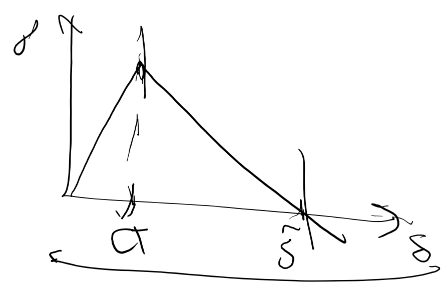
$$\tilde{\phi} = \frac{1}{2}\tilde{\sigma}\,\tilde{\delta}$$

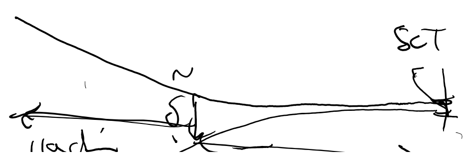
$$\boxed{\begin{aligned}\Lambda &\propto \left(\frac{E}{\tilde{\sigma}}\right)\tilde{\delta}\\ \tilde{\delta} &\propto \left(\frac{\tilde{\sigma}}{E}\right)\Lambda\end{aligned}}$$

$\tilde{\varepsilon}$

typically .001 – .01

small #

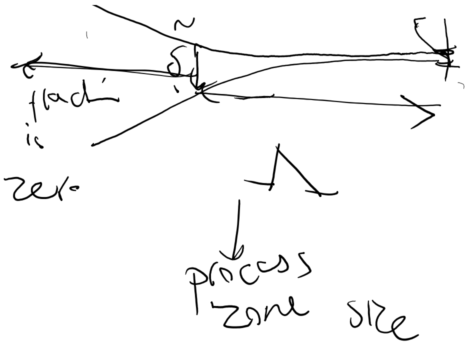




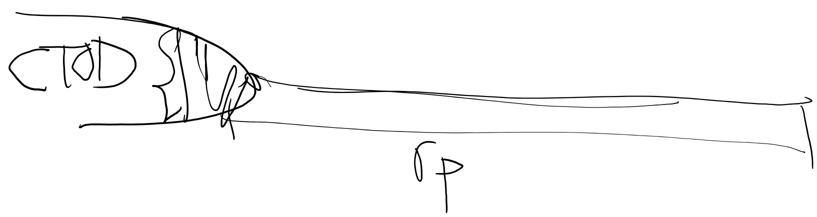
typically .001 – .01

small #

$$\Delta \gg \tilde{\delta}$$



LEFM, PFM



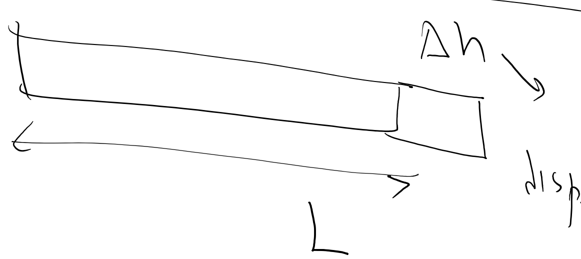
$$r_p \gg CTOD \qquad \frac{r_p}{CTOD} \propto \frac{E}{\sigma}$$

σ_y

$$\Delta u = \varepsilon L$$

displacement — mm [L]

length — mm [L]



$$\varepsilon = \frac{\sigma}{E} \qquad [-]$$

$$\Delta u \propto \tilde{\varepsilon} L$$

$$\boxed{\tilde{\varepsilon} = \frac{\tilde{\sigma}}{E}}$$

displacement << length in general

In dynamic fracture the process zone size goes to ZERO!

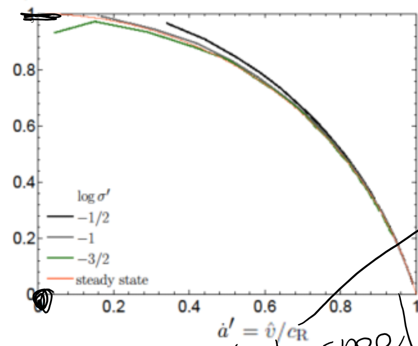
– Dynamic estimate: PZS decreases as crack speed $\hat{v}$ approaches Rayleigh wave speed c_R

$$\Lambda(\hat{v}) = \frac{\Lambda}{A(\hat{v})}, \quad A(\hat{v}) \to 0 \text{ as } \hat{v} \to c_R \quad \Rightarrow$$

Smaller elements are needed in PZT as crack accelerates!

static process zone

dynamic (real) process zone size



Rayleigh crack speed

crack speed

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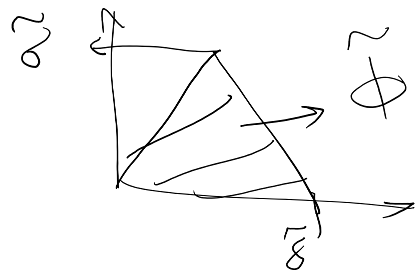
The challenge is that in dynamics process zone size goes to zero and need to keep using finer elements in the FPZ as the crack speed increases

Scales of cohesive model

$$\tilde{\phi} = \tilde{\sigma}\tilde{\delta}$$ Energy

2 out of the three are independent



$$\tilde{p} = \rho\tilde{v} = \frac{\tilde{\sigma}}{c_d}$$ Linear momentum

$$\tilde{\epsilon} = \frac{\tilde{v}}{c_d} = \frac{\tilde{\sigma}}{\rho c_d^2} \propto \frac{\tilde{\sigma}}{\|\mathbf{C}\|}$$ Strain

$$\tilde{v} = \frac{\tilde{\delta}}{\tilde{\tau}} = \frac{\tilde{\sigma}}{\rho c_d}$$ Velocity

$$\boxed{\tilde{L} = c_d\tilde{\tau} = \frac{\rho c_d^2 \tilde{\delta}}{\tilde{\sigma}} \propto \Lambda^0} \propto \tilde{\delta}\left(\frac{E}{\tilde{\sigma}}\right)$$ Length

Process zone size Λ → this determines h

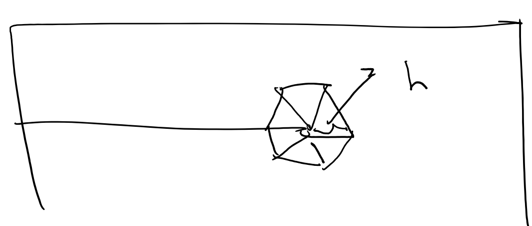
$$\boxed{\tilde{\tau} = \frac{\rho c_d \tilde{\delta}}{\tilde{\sigma}}}$$ Time

longitudinal wave speed

Influences time step for time marching methods

$$h \ll \tilde{L}$$

say 0.1



$$\Delta t < \tilde{\tau}$$

for similar dynamic

$\Delta t < c$ simulat