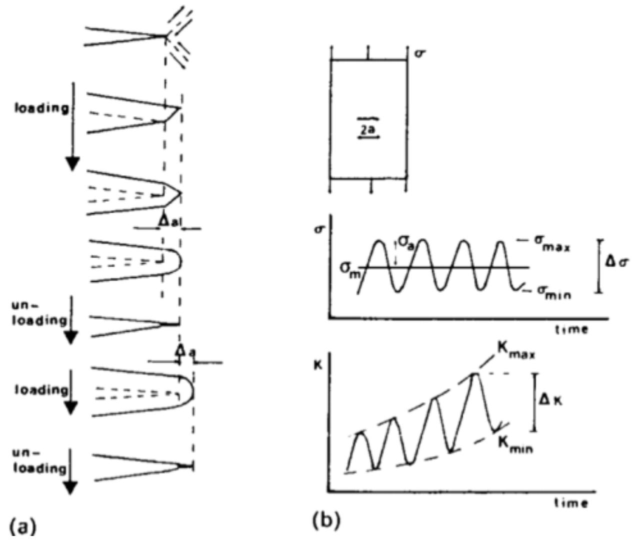
FM20241202

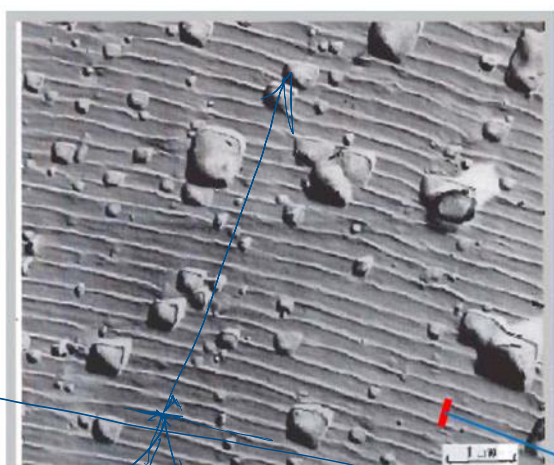
Monday, December 2, 2024 9:41 AM

blunting

resharpening



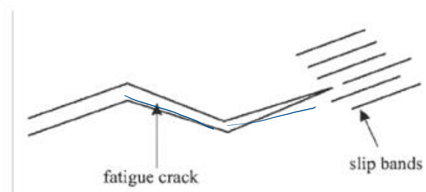
Fatigue striations

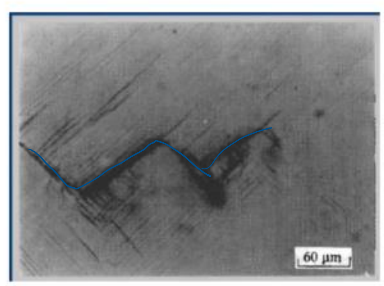


Fracture surface of a 2024-T3 aluminum alloy (source S. Suresh MIT)

Striation caused by individual microscale crack advance incidents

Fatigue crack growth:
Microcrack formation in accumulated slip bands due to repeated loading

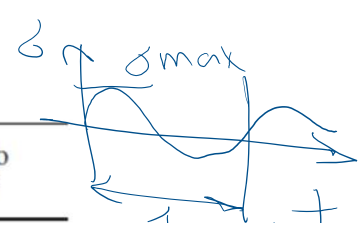




Fatigue Regimes

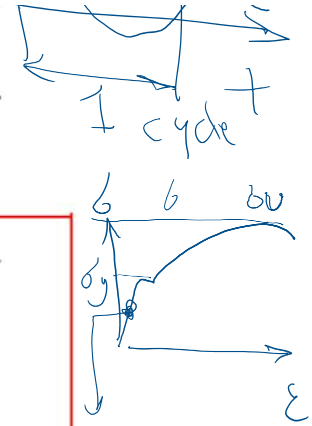
Table 7.1 Classification of fatigue damage

Fatigue	Failure cycles N_R	Pertinent stress	Strain ratio $\Delta\varepsilon^p/\Delta\varepsilon^e$	Energy ratio $\Delta W^p/\Delta W^e$



Fatigue	Failure cycles N_R	Pertinent stress	Strain ratio $\Delta\varepsilon^p/\Delta\varepsilon^e$	Energy ratio $\Delta W^p/\Delta W^e$
Very high cycle fatigue	$> 10^7$	$< \sigma_F$	≈ 0	≈ 0
High cycle fatigue	10^5 to 10^6	$< \sigma_Y$	≈ 0	≈ 0
Low cycle fatigue	10^2 to 10^4	σ_Y to σ_U	1 to 10	1 to 10
Very low cycle fatigue	1 to 20	$\approx \sigma_U$	10 to 100	10 to 100

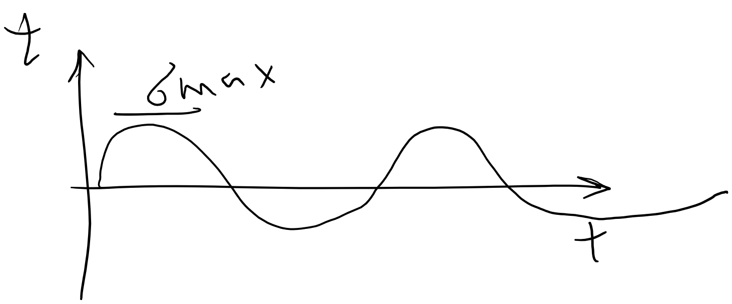
Source: Dufailly and Lemaitre (1995)

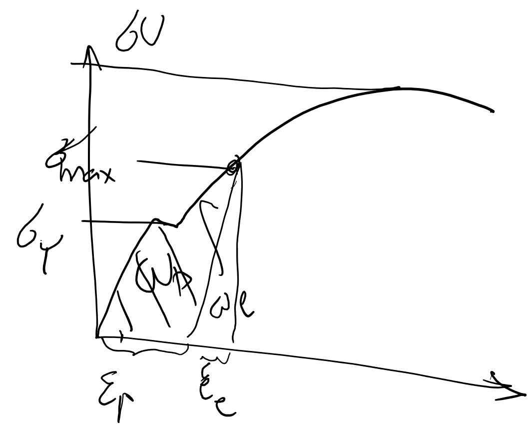


HCF

- Very high cycle and high cycle fatigue:
  - Stresses are well below yield/ultimate strength.
  - There is almost no plastic deformation (in terms of strain and energy ratios)
  - Fatigue models based on LEFM theory (e.g. SIF K) are applicable.
  - Stress-life approaches are used (stress-centered criteria)
- Low cycle and very low cycle fatigue:
  - Stresses are in the order of yield/ultimate strength.
  - There is considerable plastic deformation.
  - Fatigue models based on PFM theory (e.g. J integral) are applicable.
  - Strain-life approaches are used (strain-centered criteria)

LEFM
- stress based

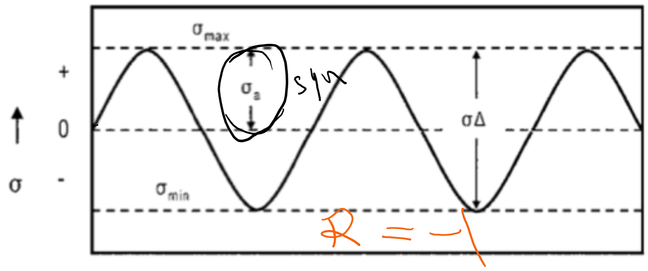




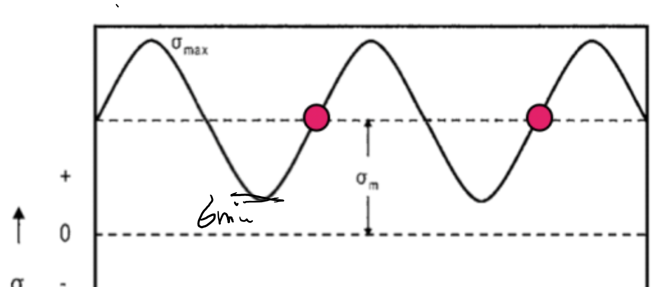
PFM, ...
strain-driven

(not covered here)

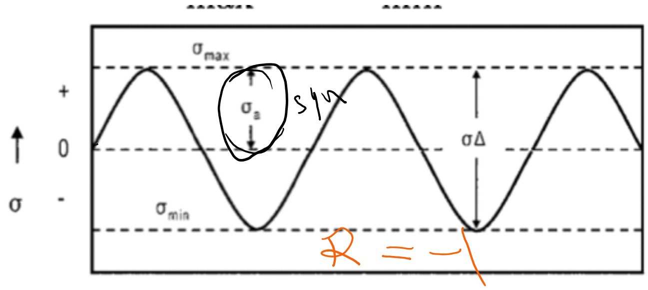
Definitions from a cyclic loading:



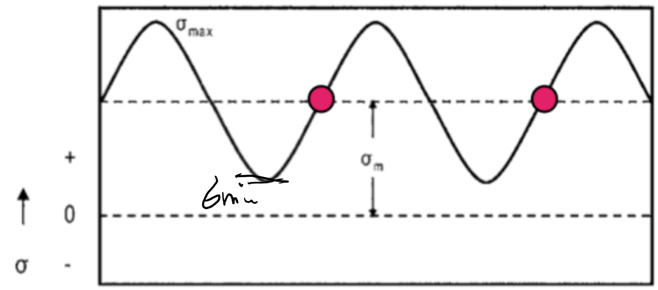
Fully Reversed Loading



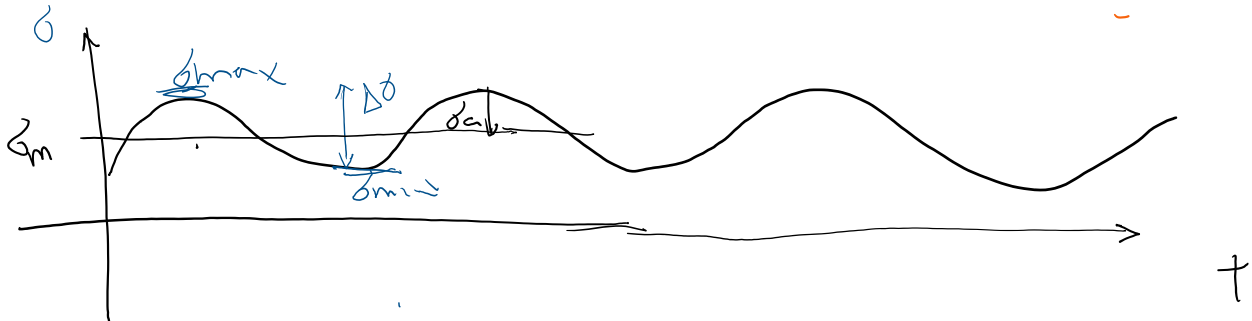
Tension-Tension with Applied Stress



Fully Reversed Loading



Tension-Tension with Applied Stress



$\Delta\sigma = \sigma_{max} - \sigma_{min}$

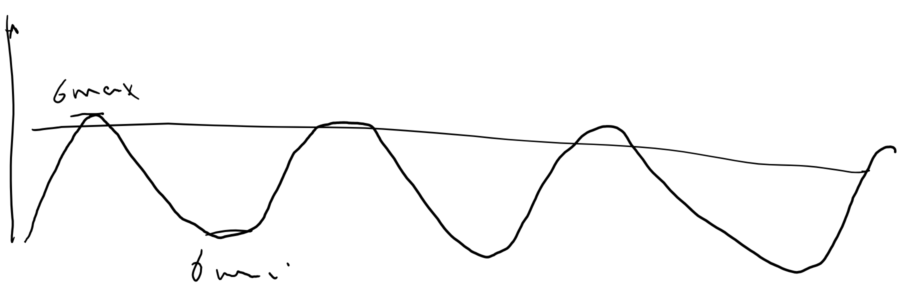
$\sigma_a = \frac{\Delta\sigma}{2} = \frac{\sigma_{max} - \sigma_{min}}{2}$ (measure of span)

$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$ mean stress

$$\boxed{R = \frac{\sigma_{min}}{\sigma_{max}} \in (-\infty, 1) \text{ for fatigue simulations}}$$

lowest R

$R = \frac{\sigma_{min} < 0}{\sigma_{max} \searrow 0^+} \rightarrow -\infty$



best case

$R \nearrow \quad -\infty \rightarrow -1$

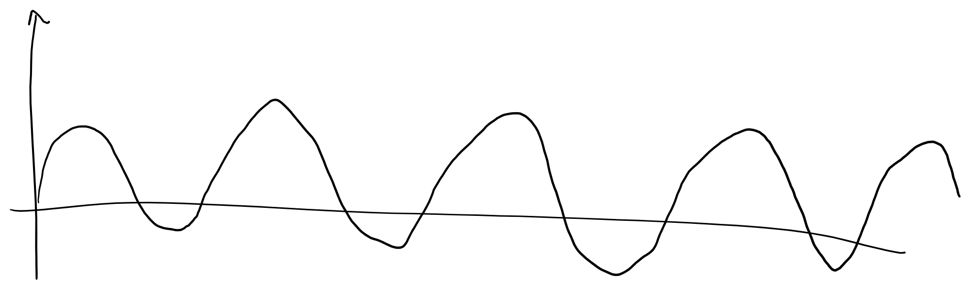
$\boxed{R = -1}$

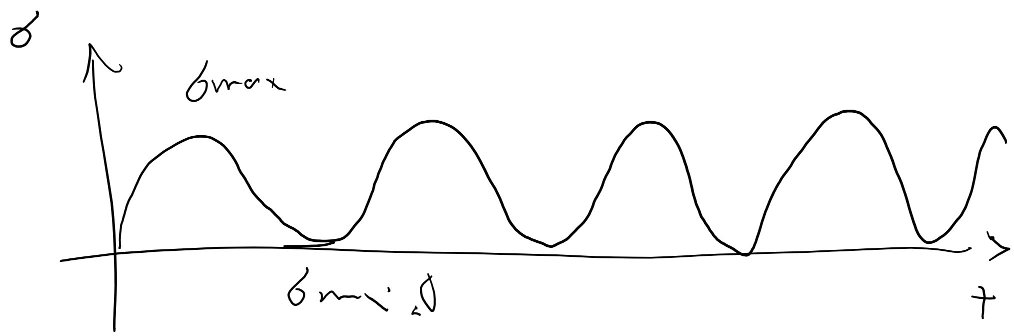
many experiments done

$\boxed{R = -1}$ $\qquad \sigma_{min} = -\sigma_{max}$

many experiments done like this

$-1 < R < 0$

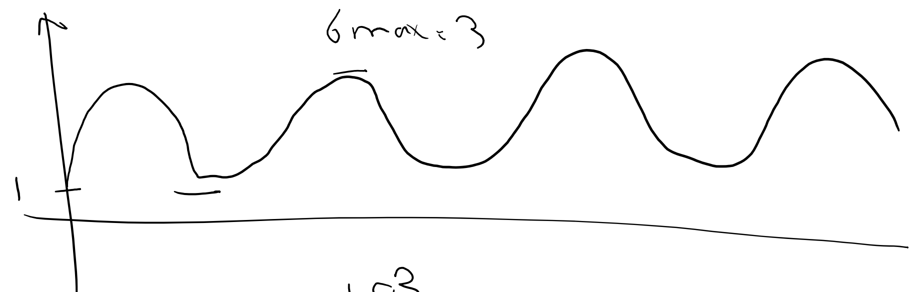




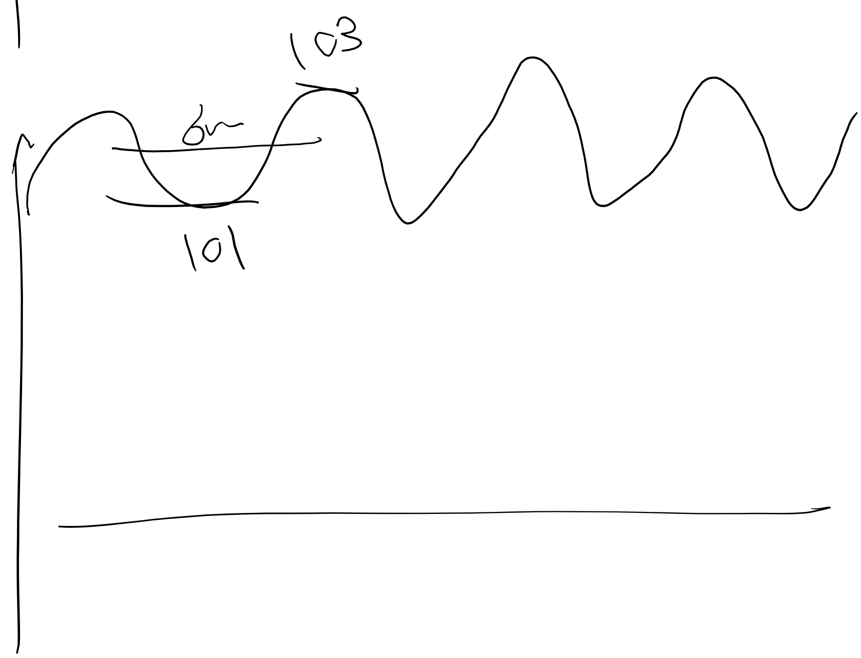
$$\boxed{R = \frac{\sigma_{min}}{\sigma_{max}} = 0}$$

$R > 0$

$$\boxed{R = \frac{1}{3} > 0}$$



$R = \frac{101}{103} \approx 1^-$



as $\sigma_m \to \infty$

$R \to 1^-$

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min}$$

$$\sigma_a = 0.5(\sigma_{\max} - \sigma_{\min})$$

$$\sigma_m = 0.5(\sigma_{\max} + \sigma_{\min})$$

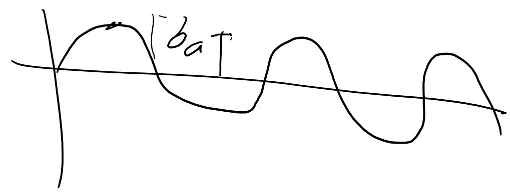
$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \quad \text{load ratio}$$

Two approaches for fatigue:
1) Older one: S-N plots
2) Paris-Erdogan relation

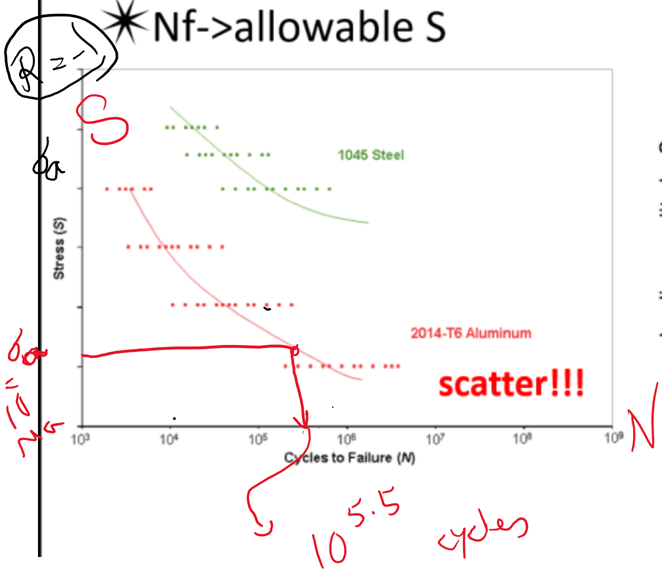
S-N curve

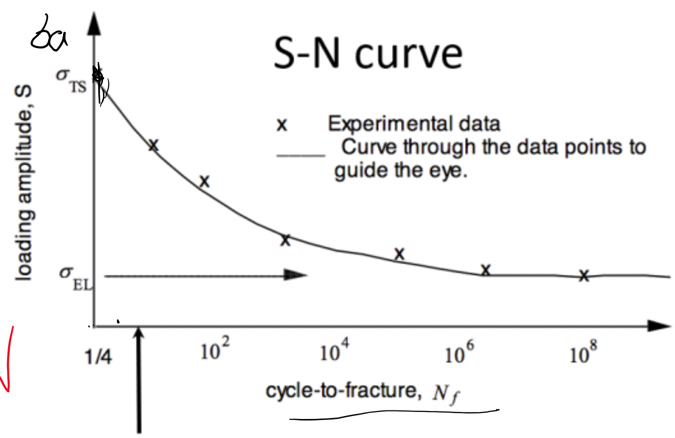
Reminder: ASTM defines *fatigue life*, N_f, as the number of stress cycles of a specified character that a specimen sustains before failure of a specified nature occurs.

$R = -1$



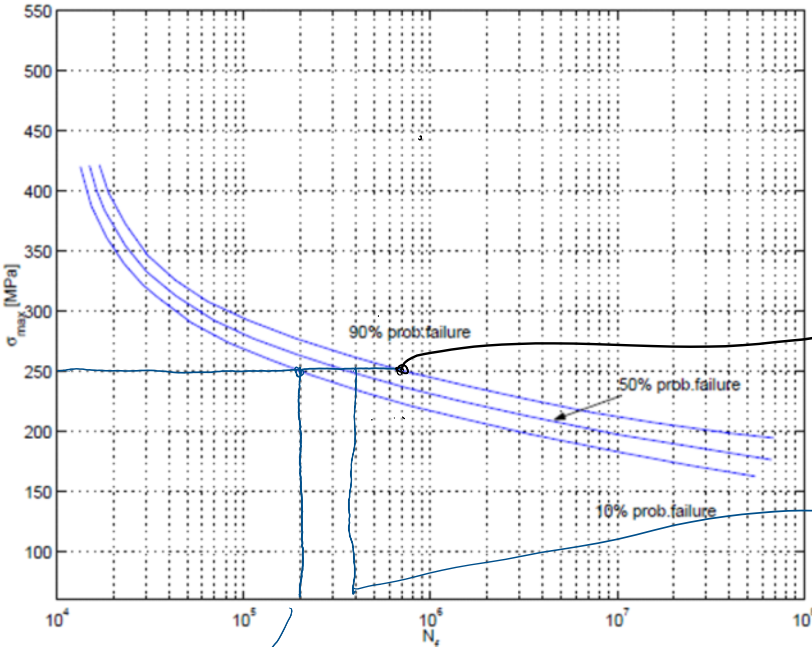
- Stress->Nf
- Nf->allowable S





endurance limit (g.han keo dai)

S-N-P curve: scatter effects

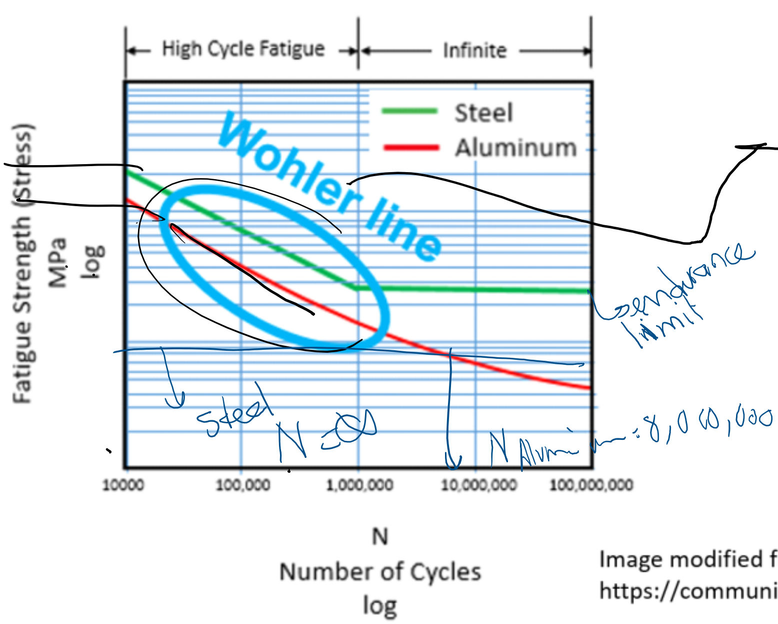


P-S-N-curves

7×10^5 90% failure

4×10^5 50% prob failure

2×10^5 10% prob of failure



$$\log S = A + B \log N$$

$$S = e^A N^B$$

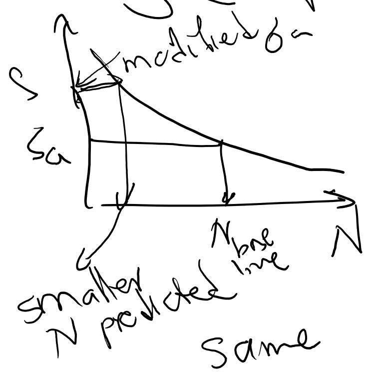
$$(S \propto N^B) \qquad N \propto S^{\frac{1}{B}}$$

Image modified from:
https://community.sw.siemens.com/s/article/what-is-a-sn-curve

S - N plots are calibrated for R = -1

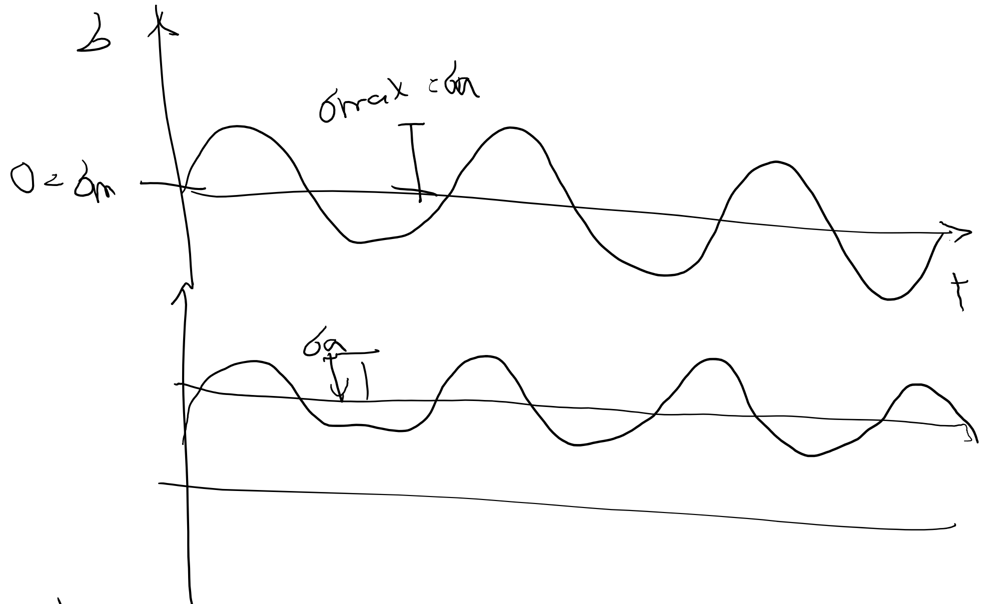
↑ modified σ_a $\sigma \uparrow$

S - N plots [illegible] R = -1



same σ_a

$\sigma_m > 0$

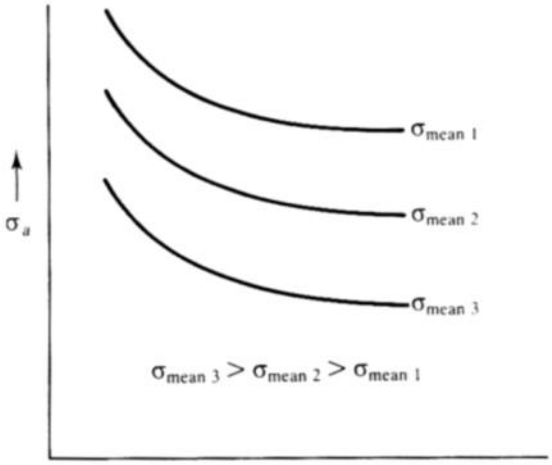


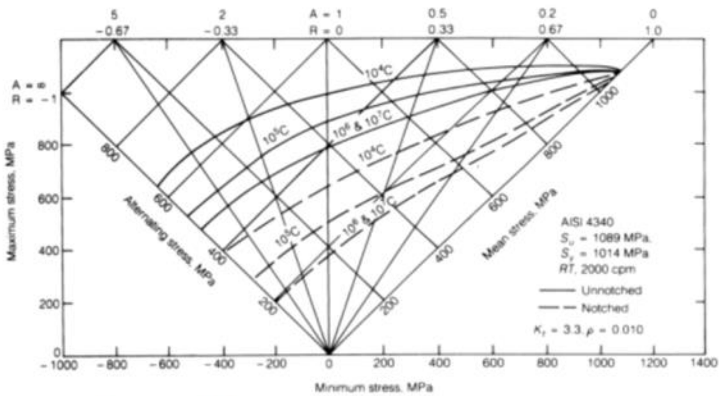
We get fewer cycles now

σ_a we'll take to original S-N

$$\sigma_a = \sigma_{f0}\left[1 - \left(\frac{\sigma_m}{\sigma_u}\right)^{\gamma}\right]$$

Effect of mean stress





Approach 1:
Master diagram

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

$$A = \frac{\sigma_a}{\sigma_m} = \frac{1-R}{1+R}$$

Approach 2:
Correction-factor formulas

$$\sigma_a = \sigma_{f0}\left[1-\left(\frac{\sigma_m}{\sigma_u}\right)^r\right]$$

where σ_a is the amplitude of allowable stress (alternating stress).

σ_{f0} is the stress at fatigue fracture when the material under zero mean stress cycled loading

σ_m is the mean stress of the actual loading.

σ_u is the tensile strength of the material.

r = 1 is called Goodman line which is close to the results of notched specimens.
r = 2 is the Gerber parabola which better represents ductile metals.

Other correction factor

Gerber (1874) $\quad \dfrac{\sigma_a^*}{\sigma_a} = 1 - \left(\dfrac{\sigma_m}{\sigma_u}\right)^2$

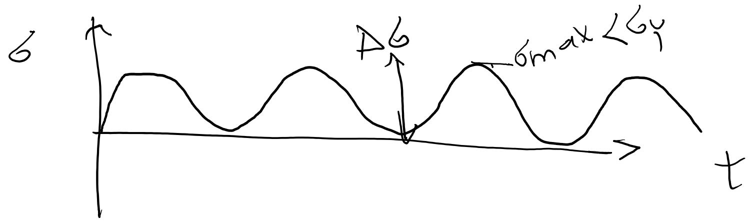
Goodman (1899) $\quad \dfrac{\sigma_a^*}{\sigma_a} = 1 - \dfrac{\sigma_m}{\sigma_u}$

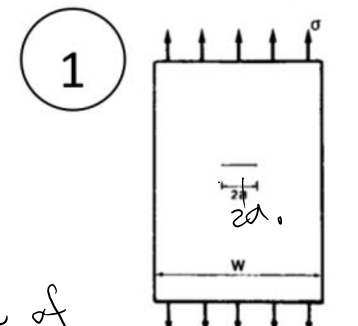
Soderberg (1939) $\quad \dfrac{\sigma_a^*}{\sigma_a} = 1 - \dfrac{\sigma_m}{\sigma_{y0}}$

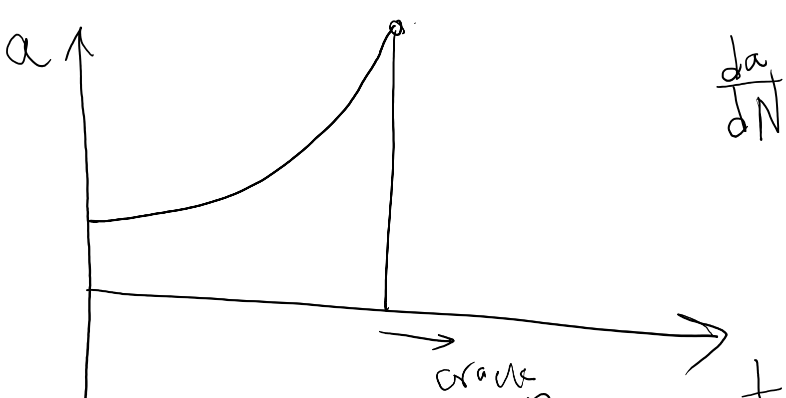
Approach 2) Paris-Erdogan

$R = 0$

$$K = \sigma\sqrt{\pi a}$$







$\frac{da}{dN}$ rate of crack growth

$$\frac{da}{dN} \propto \Delta\sigma$$

crack becomes unstable

t

N

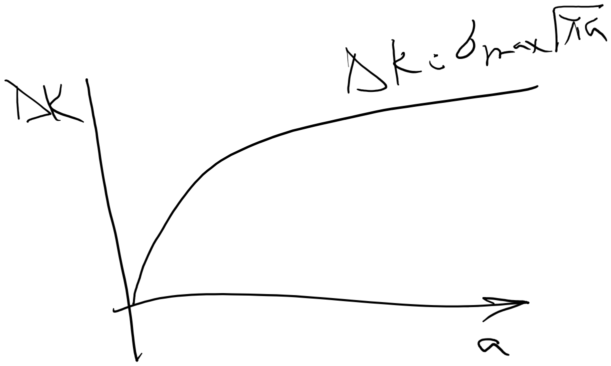
$$K_{max} = \sigma_{max}\sqrt{\pi a}$$

$$K_{min} = \sigma_{min}\sqrt{\pi a} = 0$$

assume an infinite domain

$$\boxed{\Delta K = \sigma_{max}\sqrt{\pi a}}$$

how does it behave versus a

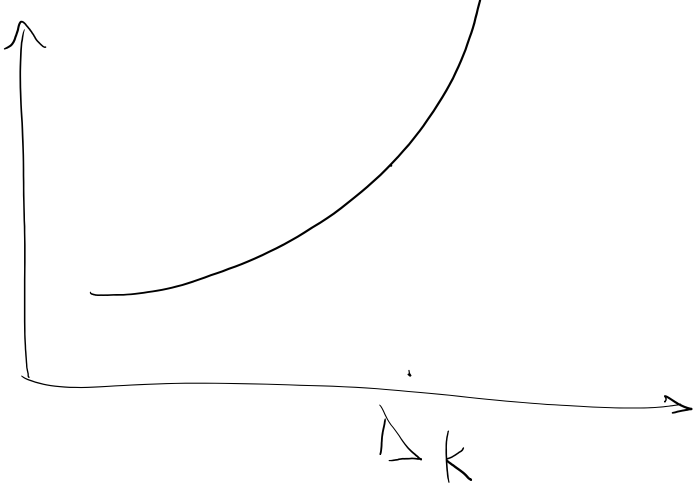


a

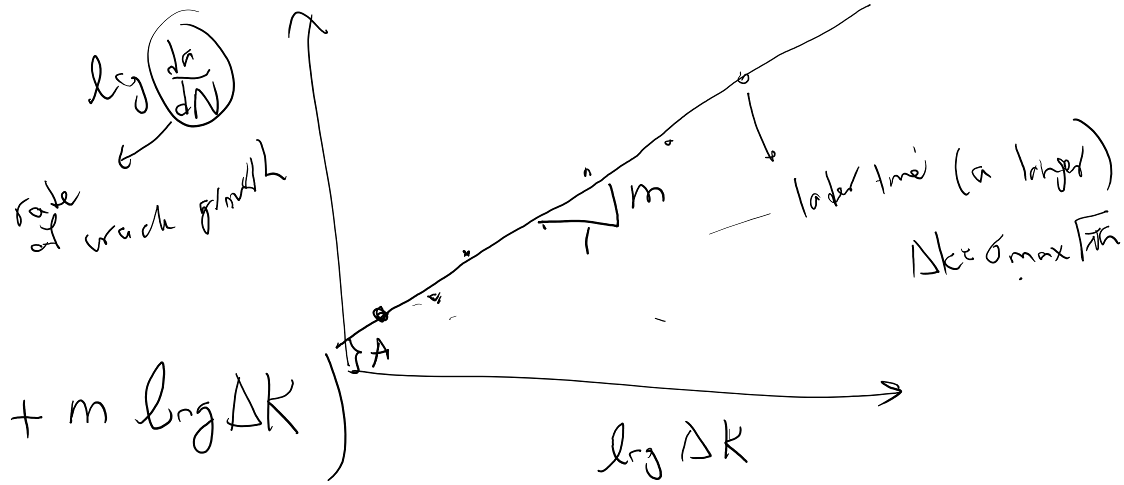
$\frac{1}{2a}$

as the crack grows

$\frac{da}{dN}$



apply log on both axes



$$\left(\log \frac{da}{dN} = A + m \log \Delta K\right)$$

e

$$\frac{da}{dN} = \underbrace{\left(e^{A}\right)}_{C} \Delta K^{m}$$

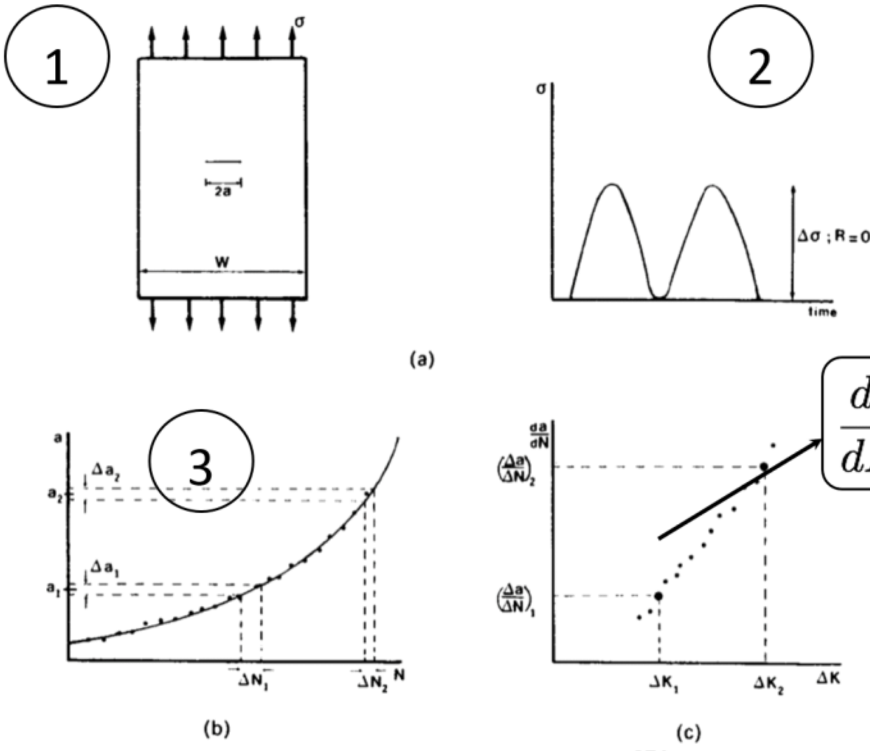
$$\boxed{\frac{da}{dN} = C \Delta K^{m}}$$

Paris – Erdogan Relation

needs an initial crack

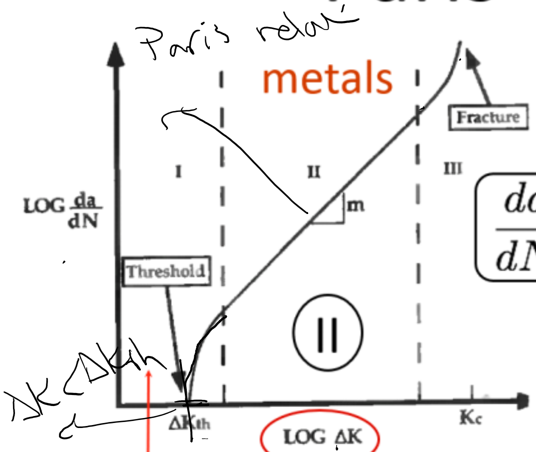
Crack growth data

$$K = \sigma\sqrt{\pi a}$$



$$\frac{da}{dN} = f_1(\Delta K, R)$$

Paris' law (fatigue)



Fatigue crack growth behavior in metals

Paris' law $\boxed{2 \leq m \leq 7}$

$$\boxed{\frac{da}{dN} = C(\Delta K)^m, \ \Delta K = K_{\max} - K_{\min}}$$

(Power law relationship for fatigue crack growth in region II)

N: number of load cycles

base 10 logarithm

I

Paris' law is the most popular fatigue crack growth model

Paris' law can be used to quantify the residual life (in terms of load cycles) of a specimen given a particular crack size.

$\Delta K \leq \Delta K_{\text{th}}$: no crack growth

(dormant period)

10^{-8} mm/cycle

375

not depends on load ratio R

$$\boxed{\frac{da}{dN} = C(\Delta K)^m, \ \ \Delta K = K_{\max} - K_{\min}}$$

Table 1: Numerical parameters in the Paris equation.

alloy	m	A
Steel	3	10^{-11}
Aluminum	3	10^{-12}
Nickel	3.3	4×10^{-12}
Titanium	5	10^{-11}

C, m

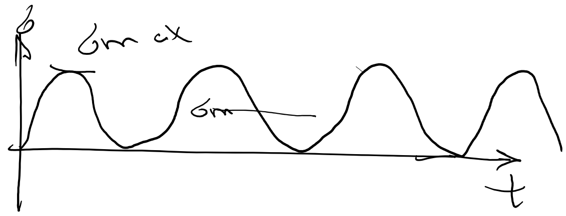
are material properties that must be determined experimentally from a log(delta K)-log(da/dN) plot.

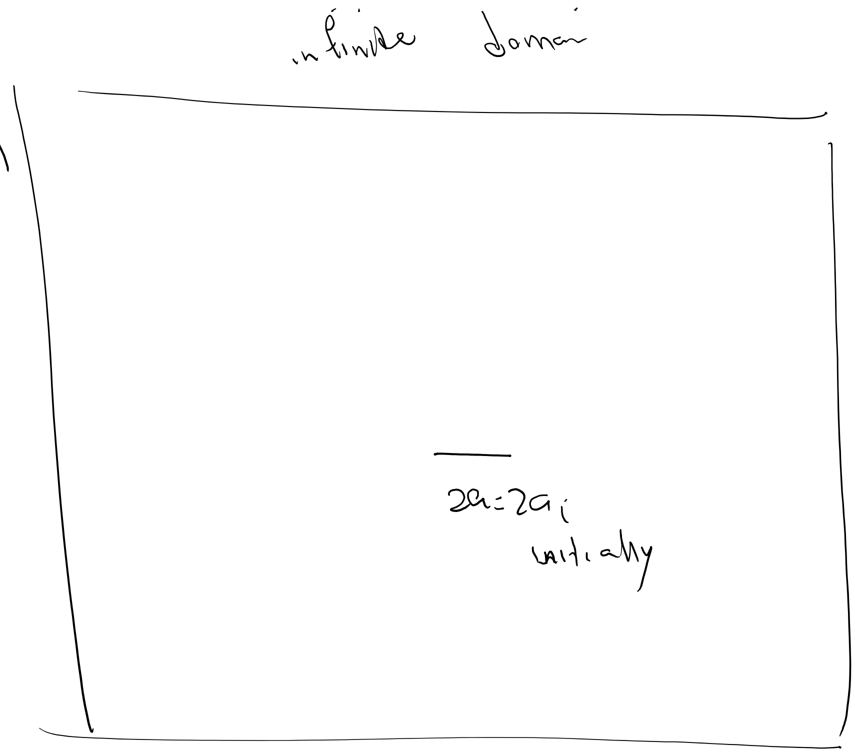
m

2-4 metals

4-100 ceramics/ polymers

infinite domain





$$\Delta K = \sigma_{max}\sqrt{\pi a}$$

$$\frac{da}{dN} = C\,\Delta K^{m}$$

$$= C\left(\sigma_{max}\sqrt{\pi a}\right)^{m}$$

$$\frac{da}{dN} = \underbrace{\left(C\,\sigma_{max}^{m}\,\pi^{m/2}\right)}\,a^{m/2}$$

$$\boxed{A = C\,\sigma_{max}^{m}\,\pi^{m/2}}$$

$$\frac{da}{dN} = A\,a^{m/2} \longrightarrow \frac{da}{a^{m/2}} = A\,dN$$

$$a^{-\frac{m}{2}}\,da = A\,dN \qquad \text{integrate}$$

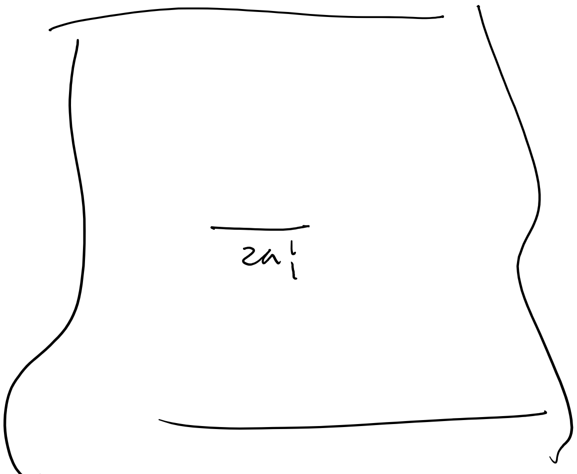
$$\int_{a_i}^{a} a^{-m/2}\,da = \int_0^N A\,dN \Rightarrow \frac{1}{1-\frac{m}{2}}\,a^{1-\frac{m}{2}}\Big|_{a_i}^{a} = A\,N(a)$$

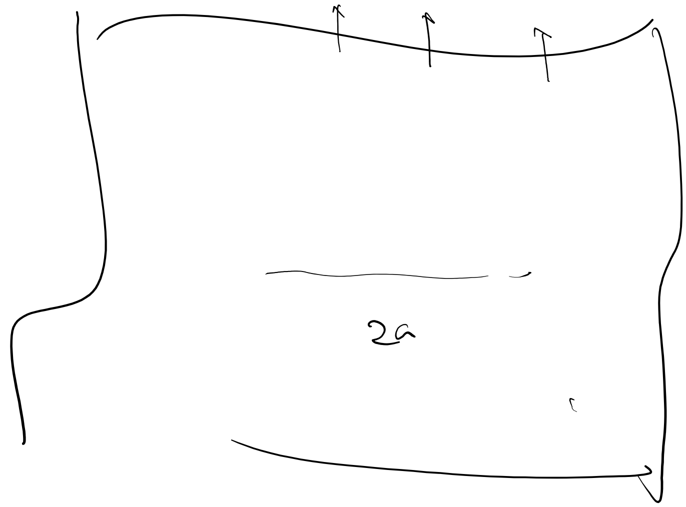
$$\Longrightarrow \quad \frac{1}{\frac{m}{2}-1}\,\frac{1}{a_i^{\frac{m}{2}-1}} - \frac{1}{\frac{m}{2}-1}\,\frac{1}{a^{\frac{m}{2}-1}} = A\,N(a)$$

(2)
$$\boxed{N(a) = \frac{1}{\left(\frac{m}{2}\right)C\,\pi^{m/2}(\Delta\sigma)^{m}}\left(\frac{1}{a_i^{\frac{m}{2}-1}} - \frac{1}{a^{\frac{m}{2}-1}}\right) \qquad m>2}$$

$m > 2$

cycles to reach crack length a

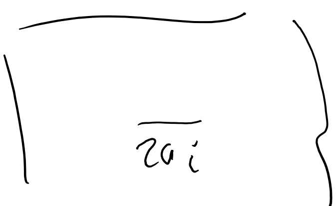




$K_{max} = \sigma_{max}\sqrt{\pi a}$ we'll end up with sudden fracture when $K_{max} = K_{Ic}$

$$\sigma_{max}\sqrt{\pi \underbrace{a_f}_{\text{failure}}} = K_{Ic} \Rightarrow \boxed{a_f = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_{max}}\right)^2} \quad (3)$$

$$N(a) = \frac{1}{\left(\frac{m}{2}-1\right)\pi^{m/2}\Delta\sigma^{m/2}}\left(\frac{1}{a_i^{m/2}} - \frac{1}{a^{m/2}}\right) \quad \text{eq } (2)$$



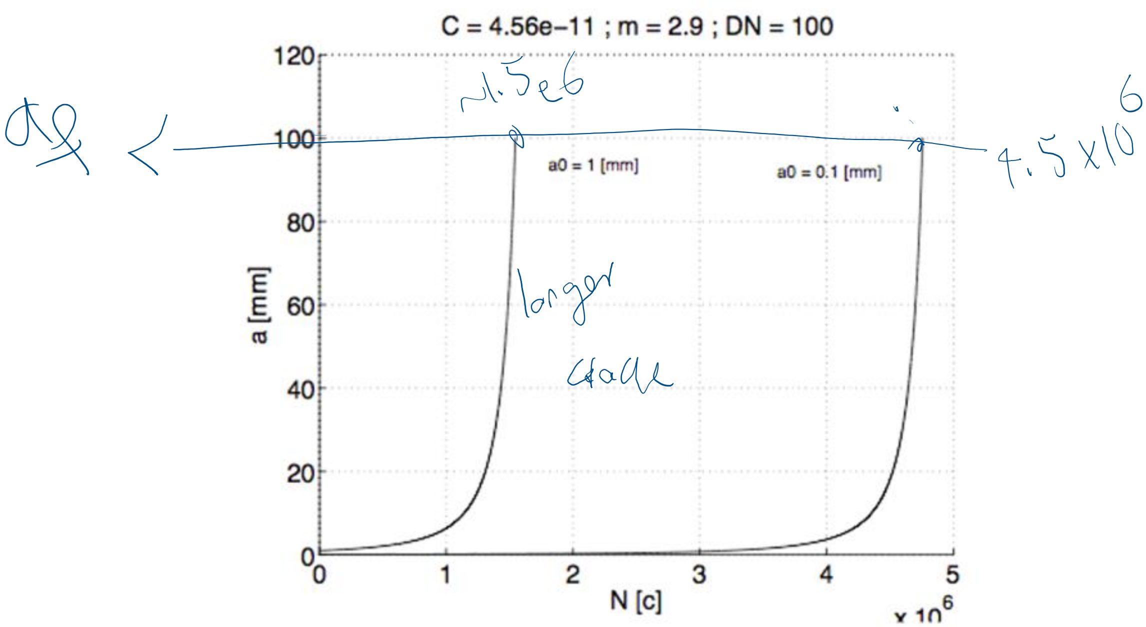
How long does it take to fail?

$$(4) \quad \boxed{N_f(a_i) = \frac{1}{\left(\frac{m}{2}-1\right)\pi^{m/2}\Delta\sigma^{m/2}}\left(\frac{1}{a_i^{m/2}} - \frac{1}{a_f^{m/2}}\right)}$$

4

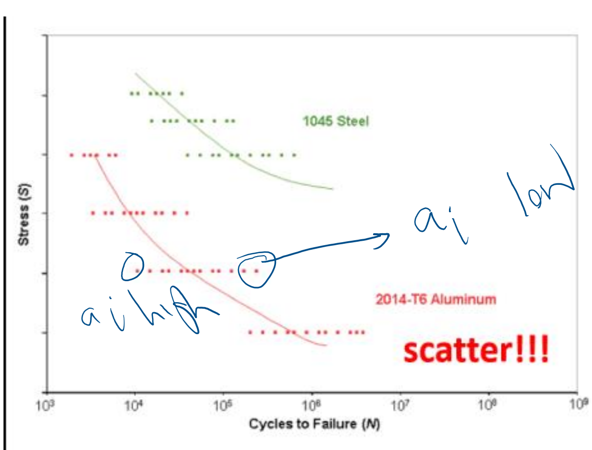
$$a_f = \frac{1}{\pi}\left(\frac{K_{IC}}{\sigma_{max}}\right)^2$$

Importance of initial crack length



aluminum ; $\Delta\sigma = 50$ [MPa]

391



coarse

Cycles to Failure (N)

$$a_i = \begin{cases} \text{worse crack length we observe} \\ a_{tol} \rightarrow \text{the measurement tolerance} \end{cases}$$