FM20241203

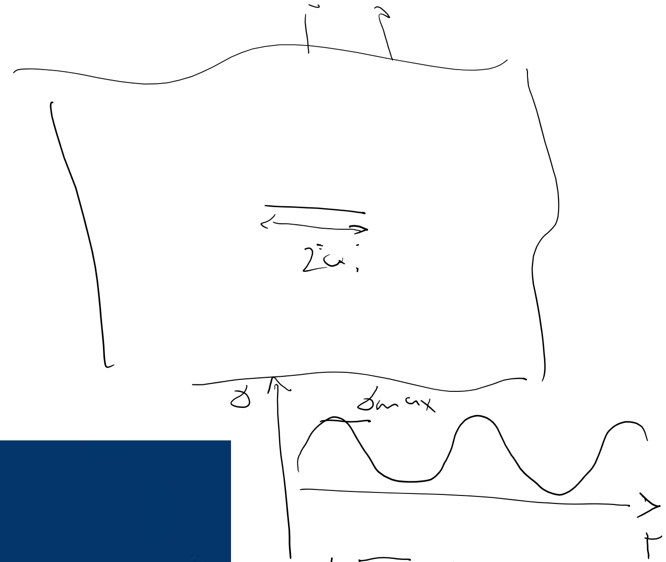
Tuesday, December 3, 2024 9:42 AM

$$\frac{da}{dN} = C\,\Delta K^{m}$$

$$\Delta K = Y \Delta\sigma \sqrt{\pi a}$$

Correction factor (if assumed independent of crack)

$\Rightarrow$



$$Y\sqrt{\pi a_f}\,\sigma_{max} = K_{IC}$$

$$a_f = \frac{1}{\pi}\left(\frac{K_{IC}}{Y\sigma_{max}}\right)^2$$

For $m > 2$:

$$N_f = \underbrace{\frac{2}{(m-2)\,CY^m\,(\Delta\sigma)^m\,\pi^{m/2}}}_{B}\left[\frac{1}{(a_0)^{(m-2)/2}} - \frac{1}{(a_f)^{(m-2)/2}}\right]$$

For $m = 2$:

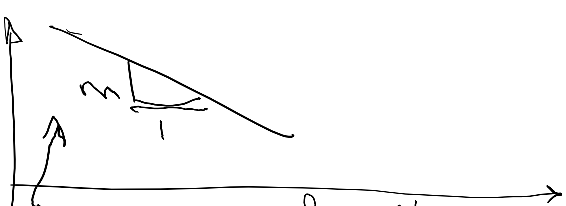
$$N_f = \frac{1}{CY^2\,(\Delta\sigma)^2\,\pi}\ln\frac{a_f}{a_0}$$

Some observations assume $a_f \gg a_i$

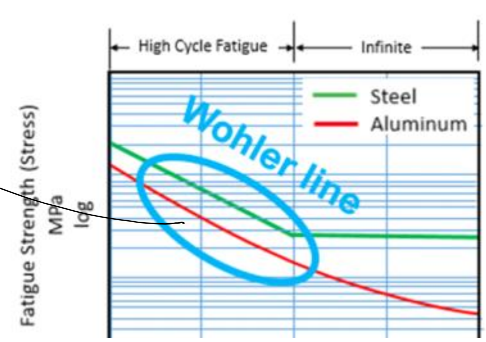
$$N_f \approx \frac{B}{(\Delta\sigma)^m}\;\frac{1}{a_0^{\frac{m}{2}-1}}$$

$$\log N_f = \log B - m \log \underbrace{\Delta\sigma}_{\sigma_{max}} - \left(\frac{m}{2}-1\right)\log a_0$$

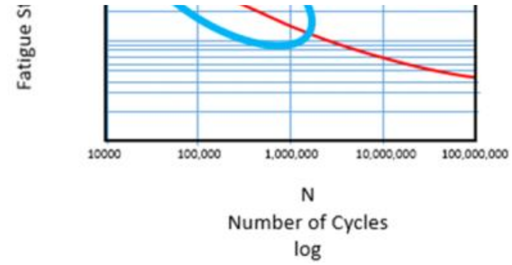
$\Delta\sigma = \sigma_{max}$



power relation in S-N plot



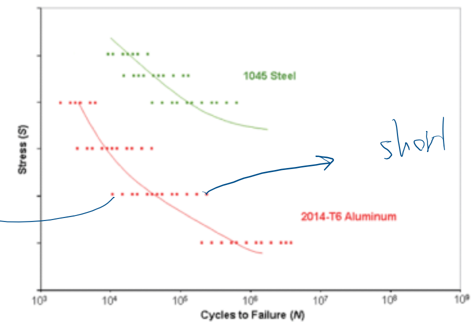
in S-N plot



$$\boxed{N_f \propto \frac{1}{a_0^{\frac{m}{2}-1}}}$$

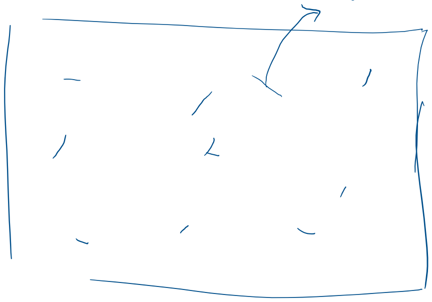
$a_0 \to 0$

$N_f \to \infty$

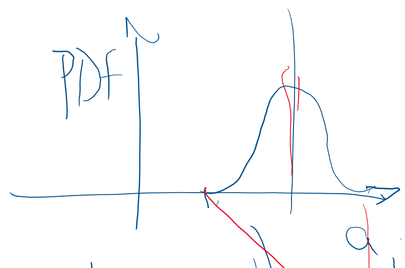


long a_i

short a_i specimen

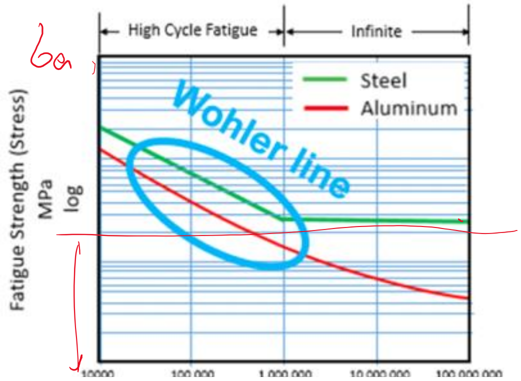


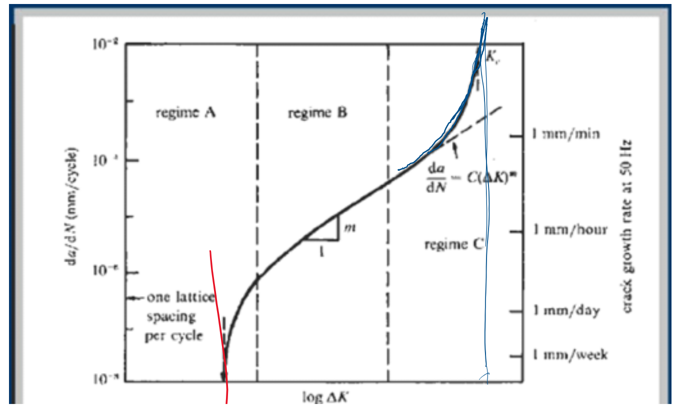
assume crack length has a distribution

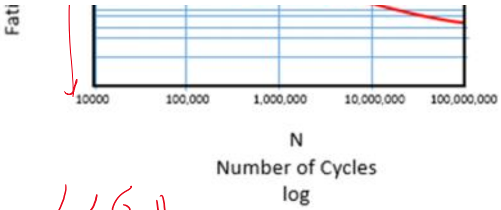


$$\Rightarrow \quad N_f(a_i) = \frac{B}{\Delta\sigma^m}\left(\frac{1}{a_i^{\frac{m}{2}-1}} - \frac{1}{a_f^{\frac{m}{2}-1}}\right)$$

random variable

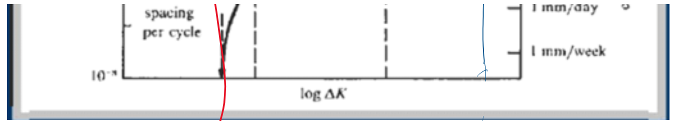






$\sigma_a < \sigma_{a,th}$

$N_f = \infty$



$\Delta K < \Delta K_{th} \rightarrow \frac{da}{dN} = 0$

as $a \rightarrow a_f$

$\frac{da}{dN} \rightarrow \infty$

the crack quickly becomes unstable

correction factor

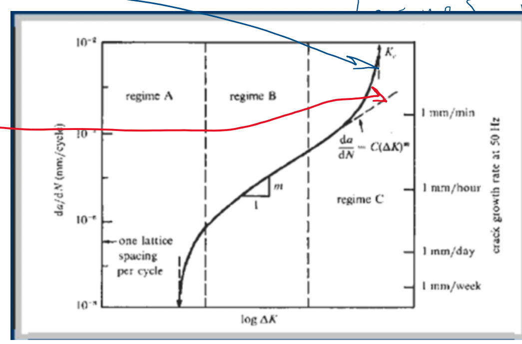
Other fatigue models

Forman's model (stage II-III)

$$\frac{da}{dN} = \frac{C(\Delta K)^n}{(1-R)K_c - \Delta K}$$

Paris' model

$$\frac{da}{dN} = C(\Delta K)^m$$



$$R = K_{\min}/K_{\max}$$

$$\frac{K_{\max} - K_{\min}}{K_{\max}} K_c - (K_{\max} - K_{\min}) \longrightarrow K_{\max} = K_c : \frac{da}{dN} = \infty$$

$(1 - R) K_{Ic} - \Delta K =$

$\left(1 - \frac{K_{min}}{K_{max}}\right) K_{Ic} - (K_{max} - K_{min})$

$= (K_{max} - K_{min}) \left(\frac{K_{Ic}}{K_{max}} - 1\right) = \frac{\Delta K}{K_{max}} (K_{Ic} - K_{max})$

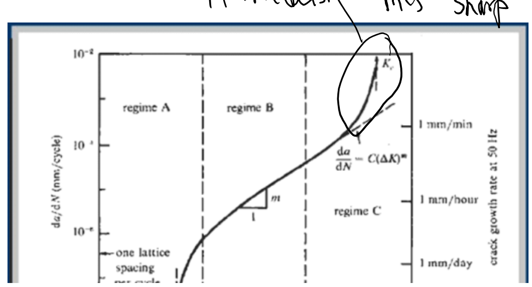
$R = \frac{\sigma_{min}}{\sigma_{max}} = \frac{K_{min}}{K_{max}}$

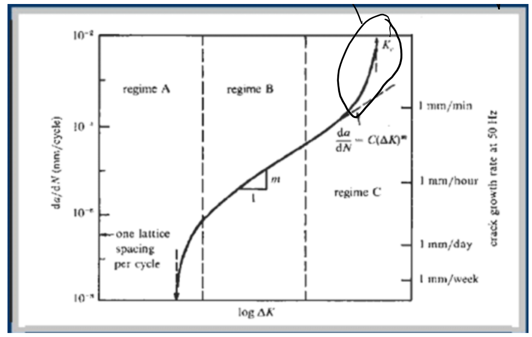
$K = \sigma \sqrt{\pi a}$

$\frac{da}{dN} = C \Delta K^m \left(\frac{1}{\frac{\Delta K}{K_{max}} (K_{Ic} - K_{max})} \right)$

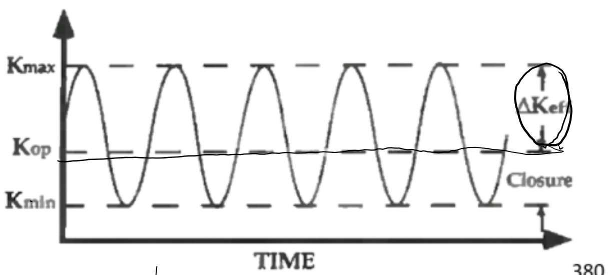
as $a \rightarrow a_f$ $K_{max} \rightarrow K_{Ic}$ ⇒ this term → 0 → $\frac{da}{dN} \rightarrow \infty$

It models this sharp growth





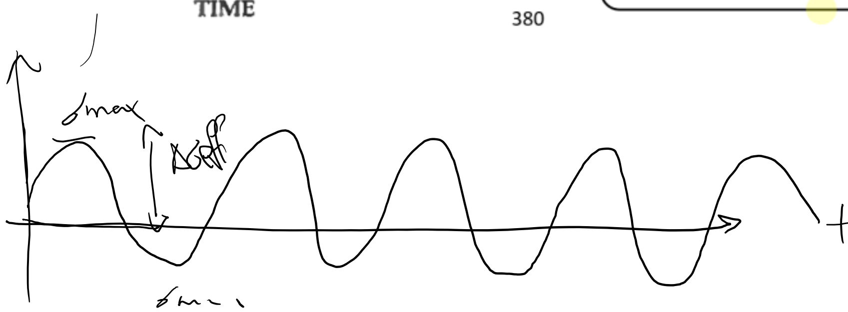
Final point: in ΔK we only need to consider K's that are positive & crack is open



$$\Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{op}}$$

K_{op}: opening SIF

$$\boxed{\frac{da}{dN} = C\Delta K_{\text{eff}}^m}$$

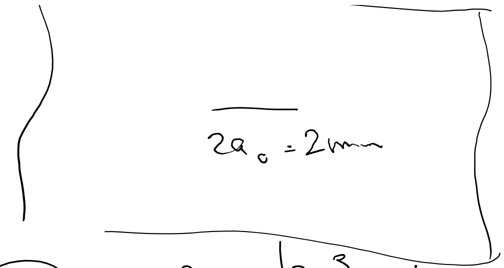


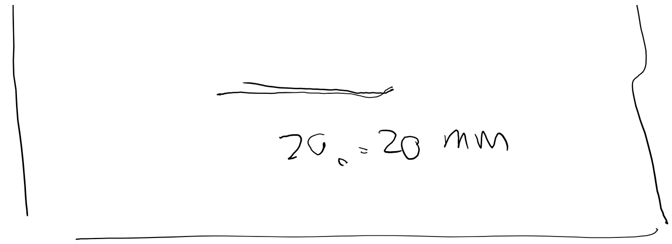
simplified uses

$\Delta K = Y \Delta\sigma_{eff} \sqrt{\pi a}$ $\Delta\sigma_{eff} = \sigma_{max} - \max(0, \sigma_{min})$

Examples for Fatigue

A large plate contains a crack of length $2a_0$ and is subjected to a constant-amplitude tensile cyclic stress normal to the crack which varies between 100 MPa and 200 MPa. The following data were obtained: for $2a_0 = 2$ mm it was found that $N = 20,000$ cycles were required to grow the crack to $2a_f = 2.2$ mm, while for $2a = 20$ mm it was found that $N = 1000$ cycles were required to grow the crack to $2a_f = 22$ mm. The critical stress intensity factor is $K_c = 60$ MPa $\sqrt{m}$. Determine the constants in the Paris (Equation (9.3)) and Formam (Equation (9.4)) equations.

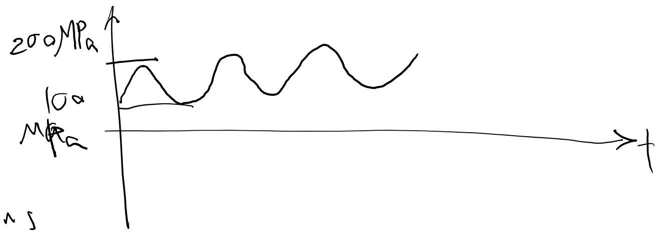




eqn 1 $\quad a_0 = 1e{-3}$, $a_1 = 1.1e{-3}$ $\Big\}$ $N = 20{,}000$

eqn 2 $\quad a_0 = 10e{-3}$ m, $a_1 = 11e{-3}$ m $\quad N = 1000$ cycles

$K_{Ic} = 60\ MPa\sqrt{m}$



$$\frac{da}{dN} = C(\Delta K)^m$$

2 unknowns : $C = ?$ $m = ?$

(1) $\log \frac{da}{dN} = \log C + m \log \Delta K$

case A (pt A) $\quad a_i = 1e{-3}$ m $\quad a_1 = 1.1e{-3}$ $\quad N = 20{,}000$

$$\Delta K_A \approx (\sigma_{max} - \sigma_{min})\sqrt{\pi a_i} = (200 - 100)\,MPa\,\sqrt{\pi\ 1e{-3}\,m} = 5.6\ MPa\sqrt{m}$$

$$\left(\frac{da}{dN}\right)_A \approx \frac{\Delta a}{\Delta N} = \frac{1.1e{-3} - 1e{-3}}{20{,}000} = 5e{-9}\ m$$

case B $\quad a_i = 10e{-3}$ $\quad a_1 = 11e{-3}$ m $\quad N = 1000$

$$(\Delta K)_B = (\sigma_{max} - \sigma_{min})\sqrt{\pi a_i} = (200 - 100)\,MPa\,\sqrt{\pi \times 10e{-3}\,m} = 17.72\ MPa\sqrt{m}$$

$$\left(\frac{da}{dN}\right)_B = \frac{\Delta a}{\Delta N} = \frac{11e{-3} - 10e{-3}}{1000} = 10^{-6}\ m$$

case (A) $\quad \log 5e{-9} = \log C + m \log 5.6$

case (B) $\quad \log 10^{-6} = \log C + m \log 17.72$

$$\Rightarrow \quad \boxed{\begin{aligned} -8.3 &= \log C + m \times .748 \\ -6 &= \log C + m \times 1.248 \end{aligned}} \quad \Longrightarrow$$

$$\boxed{\begin{aligned} & m = 4.6 \qquad C = 1.82e{-12} \frac{m}{(MPa\sqrt{m})^{4.6}} \\ & \underbrace{\frac{da}{dN}}_{m} = C\underbrace{(\Delta K)^m}_{(SIF)^m} \Rightarrow [C] = \frac{L}{(SIF)^m} \end{aligned}}$$

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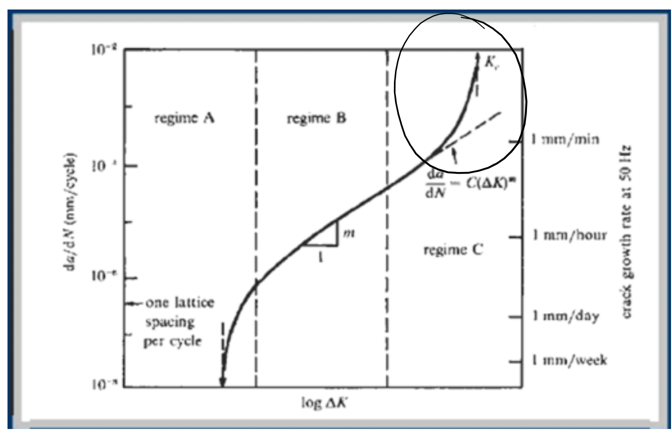
what if we wanted to make the Forman's correction



$$\frac{da}{dN} = \frac{C\Delta K^m}{[(1-R)K_{Ic} - \underbrace{\Delta K}_{\text{we had this for case A \& B}}]} \rightarrow \text{correction}$$

$\sigma_{min} = 100\,MPa$

$\sigma_{max} = 200\,MPa$

$R = \frac{100}{200} = .5$

$K_{Ic} = 60\,MPa\sqrt{m}$

$$\overbrace{[(1-R)K_{Ic} - \Delta K]}^{\text{different from prev case}} \frac{da}{dN} = C\Delta K^m$$

case A $\quad [(1-.5)\,60 - 5.6]\,5e{-9} = C(5.6)^m$

case B $\quad [(1-.5)\,60 - 17.72] \times 1e{-6} = C(17.72)^m$

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take the $\log_{10}$ now

$$\begin{cases} -6.914 = \log C + .748\,m \\ -4.911 = \log C + 1.248\,m \end{cases}$$

$$\boxed{m = 4.006 < \underbrace{4.6}_{\downarrow}}$$

w/o Forman's correct.

$$C = 1.22e{-12}\ \frac{m}{(MPa\sqrt{m})^{4.006}}$$

HW

4. A crack growth at a rate $(\frac{da}{dN})_1 = 8.84 \times 10^{-7} \frac{m}{cycle}$ when the stress intensity factor is $(\Delta K)_1 = 50$ MPa$\sqrt{m}$ and at a rate $(\frac{da}{dN})_2 = 4.13 \times 10^{-5} \frac{m}{cycle}$ when $(\Delta K)_2 = 150$ MPa$\sqrt{m}$. Determine the parameters C and m in Paris equation. (**60 Points**)

$$\frac{da}{dN} = C \Delta K^m$$

$$\log \frac{da}{dN} = \log C + m \log \Delta K$$

(1) $\log 8.84e{-7} = \log C + m \log 50$

(2) $4.13e{-5} = \text{"} + m \log 150$

$\log C$ & m → C, m

How to choose ai?

If we observe a crack use ai = factor * observed crack length

2,3

a_o

else a_i = factor * measurement tol

build safety

observence

$2a_{obs}$

Another way to build safety $\sqrt{\pi a_f}\, \sigma_{max} = K_{Ic}$

→ a_f

we go to some shorter crack rather than a_f

$a_i \longrightarrow a_f$

$a_i \longrightarrow a_f$

$\underbrace{(SF)_1,\ a_i}_{\text{large}} \longrightarrow \dfrac{a_f}{(SF)_2}$

how many cycles needed $\longrightarrow$ part life

Nondestructive testing (NDT)

Nondestructive Evaluation (NDE), nondestructive Inspection (NDI)

NDT is a wide group of analysis techniques used in science and industry to evaluate the properties of a material, component or system without causing damage

NDT: provides input (e.g. crack size) to fracture analysis

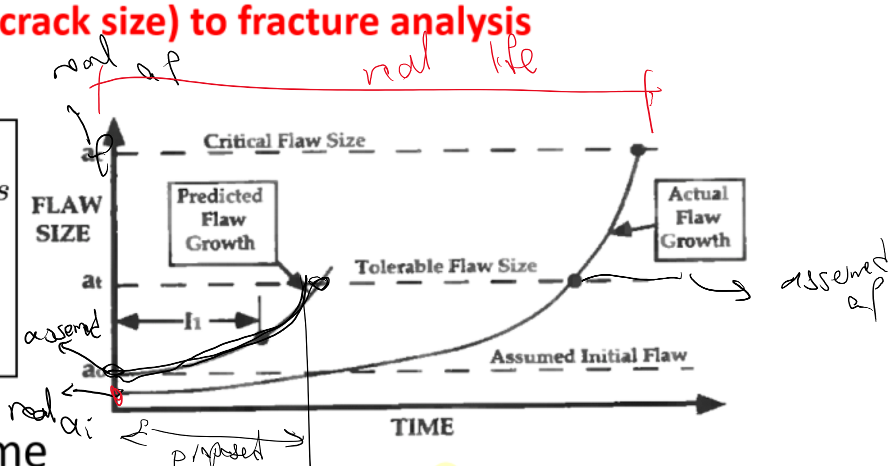
safety factor s

$K(a,\sigma) = K_c \rightarrow a_c -> a_t * s$

$\mathrm{NDT} \rightarrow a_o$

$t:\quad a_o \rightarrow a_t$ (Paris)

inspection time

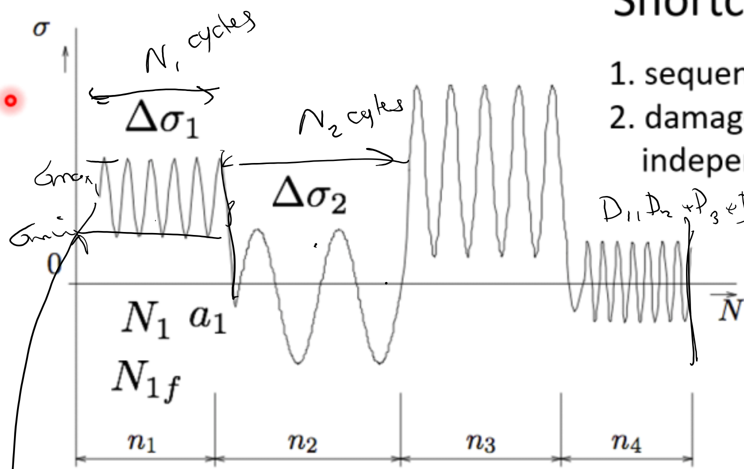


386 (a) Determination of first inspection interval, I_1.

life

Miner's rule for variable load amplitudes

1945



Shortcomings:

1. sequence effect not considered
2. damage accumulation is independent of stress level

$D_1 + D_2 + D_3 + D_4 = .75$

N_i/N_{if} : damage

$$\boxed{\sum_{i=1}^{n} \frac{N_i}{N_{if}} = 1}$$

$\Delta\sigma_i$

N_i number of cycles a_0 to a_i

N_{if} number of cycles a_0 to a_c

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$$N_{1f} = \frac{1}{(\frac{m}{2}-1) C \pi^{m/2} (\sigma_{max,1} - \sigma_{min,1})^m} \left(\frac{1}{a_i^{m/2-1}} - \frac{1}{(a_{f_1})^{\frac{m}{2}}} \right)$$

$$Y (\sigma_{max})_1 \sqrt{\pi a_{f_1}} = K_{IC}$$

$$D_1 = \frac{N_1}{N_{1f}}$$

$D_1 = 1$ done failed

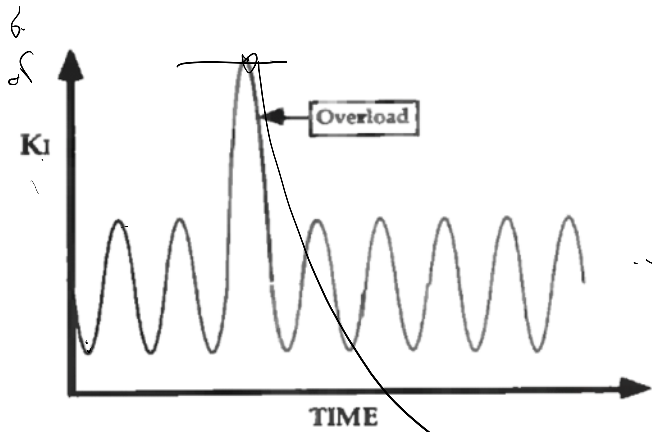
$D_1 = 0$ no fatigue damage

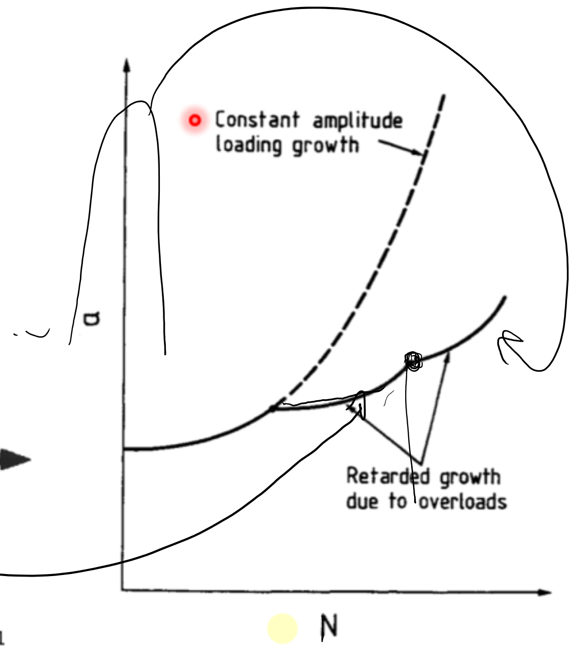
$$D = \underbrace{\frac{N_1}{N_{f_1}}}_{D_1} + \underbrace{\frac{N_2}{N_{f_2}}}_{D_2} + \dots \le 1$$

- The Miner's model ignores the order of loading (sequencing effect)
- In reality having higher simga_max earlier is worse in terms of fatigue life.

Overload and crack retardation

It was recognized empirically that the application of a tensile overload in a constant amplitude cyclic lo
leads to crack retardation following the overload; that is, the crack growth rate is smaller than it would
have been under constant amplitude loading.





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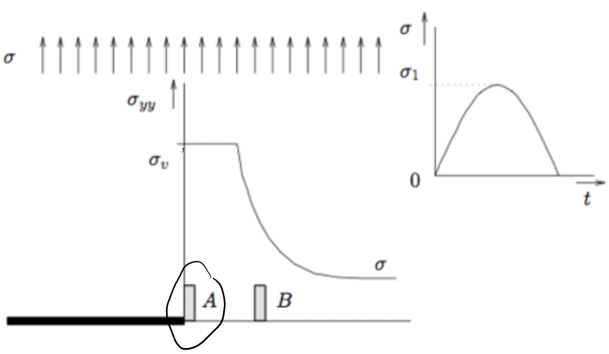
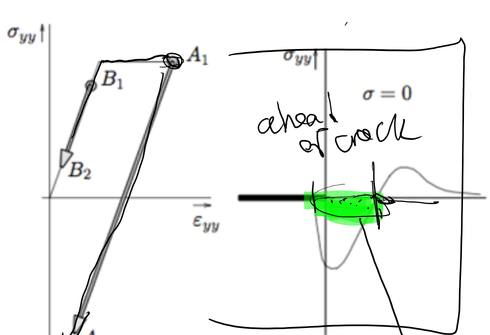
Crack retardation

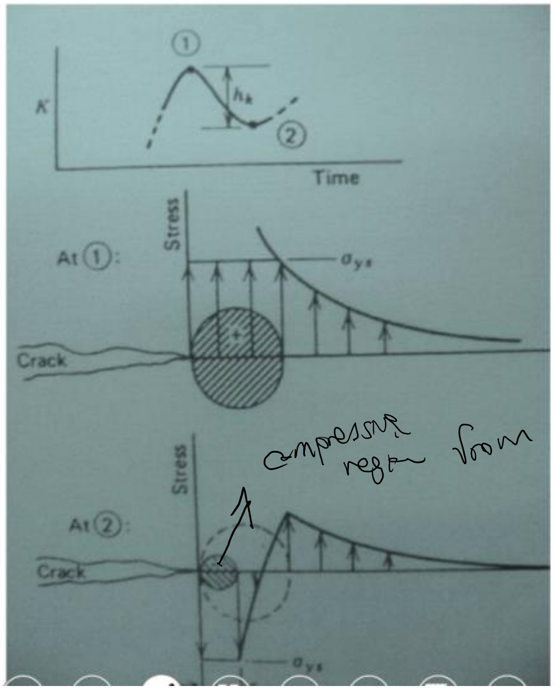


Point A: plastic
point B: elastic



After unloading: point A and B has more or less the same strain ->
point A : compressive stress.

under compressive state :)



a large plastic zone at overload has left behind

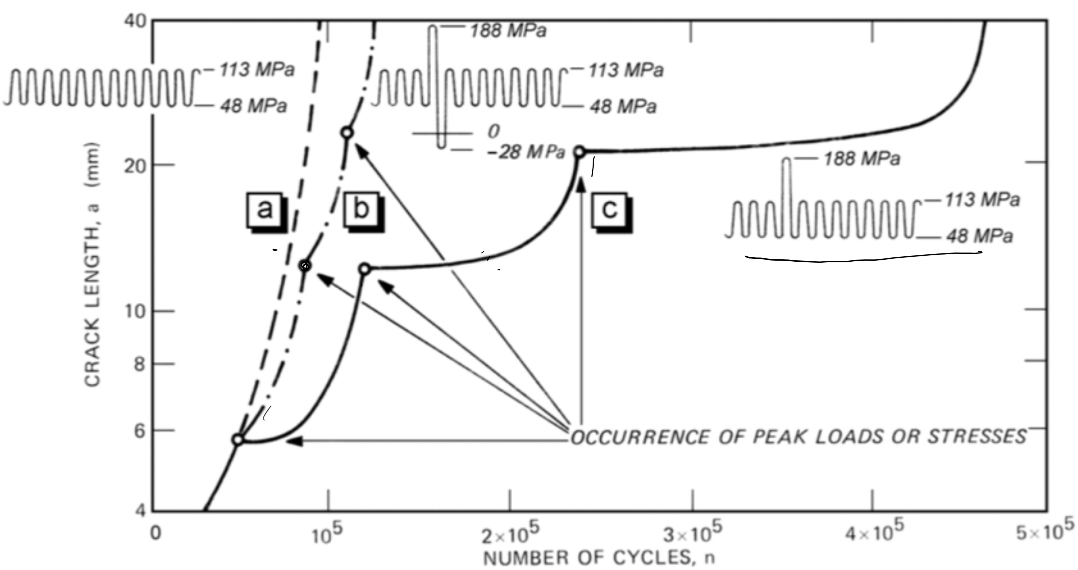
compressive region from overload

residual compressive plastic zone

close the crack->crack retards

Overload and crack retardation

It was recognized empirically that the application of a tensile overload in a constant amplitude cyclic load leads to crack retardation following the overload; that is, the crack growth rate is smaller than it would have been under constant amplitude loading.



Fatigue crack inhibition: Shot-peening

Shot peening is a cold working process in which the surface of a part is bombarded with small spherical media called *shot*. Each piece of shot striking the surface acts as a tiny peening hammer, imparting to the surface a small indentation or dimple. The net result is a layer of material in a state of residual compression. It is well established that cracks will not initiate or propagate in a compressively stressed zone.

(source Course presentation Hanlon, S. Suresh MIT)

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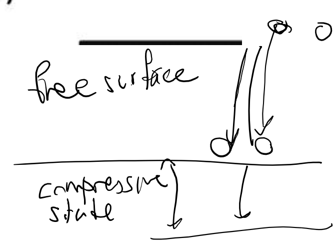
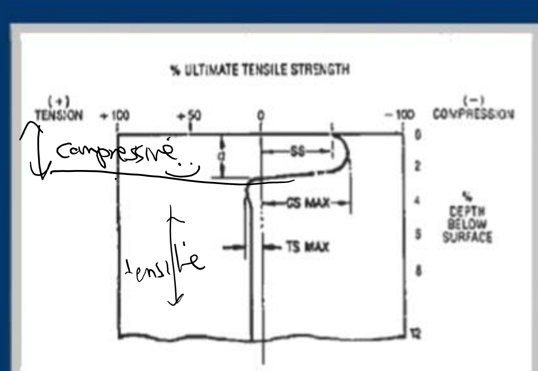
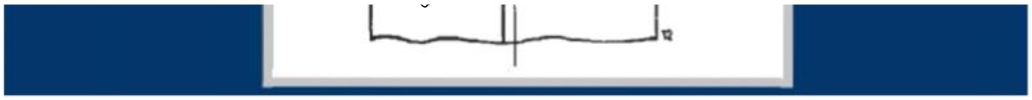


Fatigue crack inhibition: Shot-peening

A typical residual stress profile created by shot peening is shown below:





(source Course presentation Hanlon, S. Suresh MIT)